# Weather Derivatives Valuation and Market Price of Weather Risk

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This paper has two objectives: (1) to propose and implement a valuation framework for temperature derivatives (a specific class of weather derivatives); and (2) to study the significance of the market price of weather risk. The objectives are accomplished by generalizing the Lucas model of 1978 to include the weather as another fundamental source of uncertainty in the economy. Daily temperature is modeled by incorporating such key properties as seasonal cycles and uneven variations throughout the year. The temperature variable is related to the aggregate dividend or output through both contemporaneous and lagged correlations, as corroborated

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## INTRODUCTION

It is estimated that about one-seventh of the U.S. economy is weather sensitive (Challis, 1999; Hanley, 1999). Weather conditions directly affect agricultural outputs and the demand for energy products, and indirectly affect retail businesses. For instance, earnings of the power industry depend on the retail prices and the sales quantities of electricity, which in turn are affected by weather conditions. Until 1997, earnings stabilization for utility firms was primarily achieved through price hedging mechanisms while volumetric risks were largely left unhedged. However, increasing competition due to ongoing deregulations made it necessary for companies to hedge the volumetric risk caused by unexpected weather conditions. Such needs have created a new class of derivatives, weather derivatives. Since its inception in the late 1990s, the market for weather derivatives has grown steadily. Among all the weather derivative transactions, temperature-related deals are the most prevalent, accounting for more than 80% of all transactions.[1]

Although the impetus of the weather derivatives market comes from the power and energy sectors, weather derivatives can be and have been used by other industries such as the retail business and the tourist industry. For instance, the inventory level of winter coats at department stores depends on the weather forecast for the coming winter and the eventual sales depend on the actual weather condition (Agins & Kranhold, 1999). To avoid loss of sales, contracts can be struck to hedge against unfavorable weather conditions.

Despite the rapid growth of the weather derivatives market, the bid/ask spread is still large. There is not yet an effective pricing method and some key valuation issues have not been addressed. For example, insofar as weather variables are not tradeable, weather risk will command a risk premium; it is therefore important to ask if the market price of weather risk is a significant factor in valuations.

[1]According to the Weather Risk Management Association (WRMA), a total of 3,937 weather derivative transactions were completed in the weather risk industry for the period from April 1, 2001, to March 31, 2002. This represents a 43% increase from the previous year, which recorded a total of 2,759 transactions.

In the meantime, the trading volume of temperature derivatives listed on the Chicago Merchantile Exchange has also grown rapidly. The total number of contracts traded was 4,165 in 2002 and 14,234 in 2003.

This paper is aimed at filling the aforementioned gaps in the literature. Using an equilibrium valuation model for temperature derivatives, the paper establishes whether the market price of temperature risk is significant in the valuation of weather derivatives. Specifically, Lucas' (1978) equilibrium asset-pricing model is extended so that the fundamental uncertainties in the economy are generated by the aggregate dividend and a state variable representing the weather condition, i.e., the temperature. The model is calibrated with temperature and consumption data, and the market price of weather risk is then analyzed and quantified.

It is found that the risk premium can represent a significant part of the temperature derivative's price evaluated with risk aversion and aggregate dividend process parameters conforming to empirical reality. The market price of weather risk is more pronounced in option prices than in forward prices due to non-linearity in the option's payoff. Discounting the derivative's payoff at the risk-free rate can therefore lead to significant pricing errors. The only time for discounting at the risk-free rate to be a valid approximation is when the correlation between the aggregate dividend and the temperature is very low and/or the investor's risk aversion is low, none of which is supported by empirical evidence.

The paper proceeds as follows. The next section briefly describes temperature derivatives. The section *Valuation Framework and Temperature Modeling* lays out the modelling framework. Valuation formulas for temperature derivatives and general discussions on the market price of temperature risk are given in the section *Valuing HDD/CDD Derivatives.* The *Empirical Estimation* section contains the estimation results for the temperature process and the correlation between monthly average temperature and consumption. The penultimate section quantifies and analyzes the market price of risk via simulations. The last section contains concluding remarks.

## DESCRIPTION OF TEMPERATURE DERIVATIVES

The underlying variables for weather contracts include temperature, rainfall, snowfall, and humidity, to name a few.[2] However, the most commonly contracted weather variable is temperature. Most contracts are written on heating degree day (HDD) and cooling degree day (CDD). The daily HDD

[2]For a complete survery, see Hanley (1999).

and CDD are calculated as max[65F − $y$, 0] and max[$y$ − 65F, 0], respectively, where $y$ is the daily average temperature defined as the arithmetic average of the daily maximum and minimum temperatures. For brevity, the daily average temperature is referred to as daily temperature hereafter. For a typical northern or midwest city in the United States, an HDD season includes winter months from November to March and the CDD season (or summer season) includes months from May to September. April and October are commonly referred to as the shoulder months.

Most contracts are written on the accumulation of HDD or CDD over a calendar month or a season. There are four basic elements in a contract: (i) the underlying variable: HDD or CDD; (ii) the accumulation period; (iii) a specific weather station reporting daily temperatures for a particular city; and (iv) the tick size: the dollar amount attached to each HDD or CDD.

The fictitious example in Table I presents the typical elements of a swap and an option. In the New York HDD swap, the tick size is set at \$5,000 per HDD. XYZ Co. agrees to pay ABC Co. a fixed rate of 1,000 HDD and in return for a floating rate, which is the actual accumulated HDD during January, 2002. The realized HDD for January, 2002 is 956. Then the payoff for XYZ Co. at maturity is \$5000 × (956 − 1000) = −\$220,000.

The Chicago CDD call option works in a similar fashion. A cap or maximum payoff is typically specified for an option contract. For

**TABLE I**
Examples of HDD- and CDD-based Swap and Option

| | *HDD swap* | *CDD call option* |
|---|---|---|
| Location | La Guardia Airport, New York | O'Hare Airport, Chicago |
| Buyer | XYZ Co. (paying fixed rate) | XYZ Co. (paying call premium) |
| Seller | ABC Co. (paying floating rate) | ABC Co. |
| Accumulation period | January 1–31, 2002 | June 1–30, 2002 |
| Tick size | \$5,000 per HDD | \$5,000 per CDD |
| Fixed rate | 1,000 HDD | |
| Strike level | | 190 CDD |
| Floating rate | the actual HDD for January, 2002 = 956 HDD | |
| Settlement price | | the actual CDD for June, 2002 = 196 CDD |
| Payoffs at maturity for the buyer | (956 − 1000) × 5000 = −\$220,000 | (196 − 190) × 5000 = \$30,000 |

instance, the payoff function for a call on CDD with a cap is, min[cap, tick $\times$ max(0, CDD $-$ strike)]. For brevity, the cap is ignored in this study.

## VALUATION FRAMEWORK AND TEMPERATURE MODELING

### The Basic Valuation Framework

Because temperature is not an asset, let alone tradable, the traditional no-arbitrage, risk-neutral valuation cannot be applied to temperature derivatives. To study the market price of risk attached to the temperature variable, an extension of the Lucas (1978) pure-exchange economy is used. Specifically, a discrete setting is adopted where the fundamental uncertainties in the economy are driven by two state variables: the aggregate dividend $\delta_t$ and the temperature $Y_t$. Aggregate dividends can be viewed as aggregate outputs. For a representative investor, the equilibrium conditions imply that total consumption is equal to the aggregate dividend, and the time $t$ price of a contingent claim with a payoff $q_T$ at a future time $T$, denoted by $X(t, T)$, is

$$X(t, T) = \frac{1}{U_c(\delta_t, t)} E_t(U_c(\delta_T, T) q_T, \quad \forall\, t \in (0, T) \tag{1}$$

where $U_c(\delta_T, T)$ is the first derivative of the period-$T$ utility function on consumption, $c_T$: $U(c_T, T)$. Contingent claims on the temperature variable can be valued via Equation (1) once the temperature process, the agent's preference, and the dividend process are specified.

### Modeling the Daily Temperature Behavior

In order to model the temperature variable, historical daily temperature data, covering the period from January 1, 1979, to December 31, 1998, for Atlanta, Chicago, Dallas, New York, and Philadelphia are obtained from the National Climate Data Center (NCDC), a subsidiary of the National Oceanic Atmospheric Administration (NOAA). Table II presents summary statistics.[3] The following observations are in order: (1) the sample means of the two southern cities (Atlanta and Dallas) are higher than those of the three northern counterparts, as expected; (2) northern cities generally have larger standard deviations, with Chicago having the

[3]The sample size is 7300 (= 365 $\times$ 20) since, for simplicity, we have omitted the observations for February 29.

**TABLE II**
Summary Statistics for Daily Temperature

| | *Atlanta* | *Chicago* | *Dallas* | *New York* | *Philadelphia* |
|---|---|---|---|---|---|
| Mean | 63 | 50 | 66 | 56 | 56 |
| Median | 64 | 50 | 67 | 56 | 56 |
| Mode | 79 | 70 | 86 | 72 | 75 |
| Standard deviation | 15 | 20 | 16 | 17 | 18 |
| Minimum | 5 | −17 | 9 | 3 | 1 |
| Maximum | 92 | 93 | 97 | 93 | 92 |
| Sample size | 7,300 | 7,300 | 7,300 | 7,300 | 7,300 |
| | | *Correlation* | | | |
| Atlanta | 1.0000 | | | | |
| Chicago | 0.8847 | 1.0000 | | | |
| Dallas | 0.8777 | 0.9038 | 1.0000 | | |
| New York | 0.8966 | 0.8964 | 0.8443 | 1.0000 | |
| Philadelphia | 0.9125 | 0.8970 | 0.8455 | 0.9853 | 1.0000 |
| | | *Auto correlation* | | | |
| *k*-lags | | | | | |
| 1 | 0.9402 | 0.9421 | 0.9354 | 0.9448 | 0.9462 |
| 2 | 0.8690 | 0.8809 | 0.8680 | 0.8896 | 0.8926 |
| 3 | 0.8281 | 0.8494 | 0.8318 | 0.8654 | 0.8678 |
| 4 | 0.8069 | 0.8304 | 0.8132 | 0.8533 | 0.8550 |
| 5 | 0.7952 | 0.8181 | 0.8005 | 0.8470 | 0.8486 |
| 6 | 0.7867 | 0.8091 | 0.7918 | 0.8431 | 0.8437 |
| 7 | 0.7804 | 0.8022 | 0.7855 | 0.8394 | 0.8380 |
| 8 | 0.7764 | 0.7973 | 0.7813 | 0.8346 | 0.8330 |
| 9 | 0.7728 | 0.7925 | 0.7773 | 0.8297 | 0.8283 |
| 10 | 0.7687 | 0.7894 | 0.7731 | 0.8246 | 0.8228 |
| 11 | 0.7665 | 0.7870 | 0.7718 | 0.8197 | 0.8175 |
| 12 | 0.7652 | 0.7857 | 0.7720 | 0.8164 | 0.8142 |
| 13 | 0.7614 | 0.7835 | 0.7683 | 0.8124 | 0.8098 |
| 14 | 0.7562 | 0.7793 | 0.7608 | 0.8099 | 0.8054 |
| 15 | 0.7534 | 0.7759 | 0.7558 | 0.8070 | 0.8017 |

*Note.* Sample period is from January 1, 1979, to December 31, 1998.

largest sample standard deviation of 20 degrees, indicating large temperature swings (in contrast, Atlanta has the smallest sample standard deviation of 15 degrees); (3) correlations among the five cities are very high, with New York and Philadelphia having the highest correlation, 0.9853; and (4) daily temperatures exhibit strong autocorrelations.[4]

To fix notation, let yr index the years in the sample period, thus yr = 1 for 1979, yr = 2 for 1980, ..., and yr = 20 for 1998. In addition, January 1 is indexed as $d = 1$, January 2 as $d = 2$, and so on for 365 days

[4]Although not shown here, plots of daily temperature also reveal a clear global warming trend for each city.

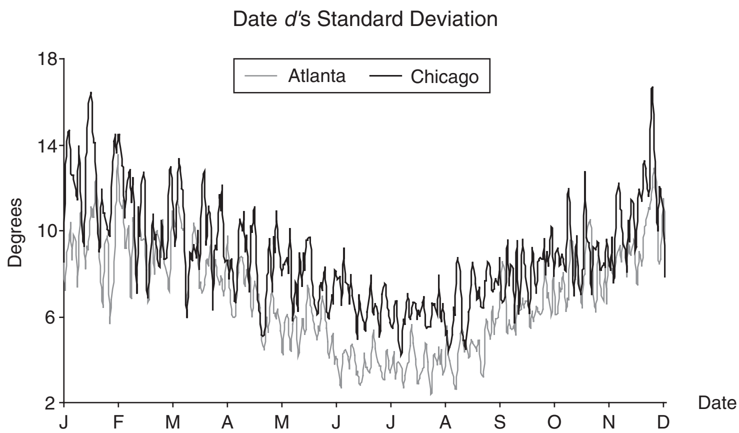


**FIGURE 1**
Standard Deviation of Date $d$'s Temperature ($\psi_d$).

*Note.* The graph shows the standard deviation for each of the 365 calendar days. For each calendar day, the standard deviation is calculated from the corresponding 20 observations in the sample (January 1, 1979–December 31, 1998).

in a year. Denote $y_{\mathrm{yr},d}$ as the temperature on date $d$ in year yr. The mean ($\bar{y}_d$) and the standard deviation ($\psi_d$) for date $d$ can be calculated as

$$\bar{y}_d = \frac{1}{20}\sum_{\mathrm{yr}=1}^{20} y_{\mathrm{yr},d} \quad \text{and} \quad \psi_d = \sqrt{\frac{1}{19}\sum_{\mathrm{yr}=1}^{20}(y_{\mathrm{yr},d} - \bar{y}_d)^2};$$

$$\forall \quad d = 1, 2, \ldots, 365 \quad \text{and} \quad [\bar{y} = [\bar{y}_1 \quad \bar{y}_2 \quad \bar{y}_3 \quad \ldots \quad \bar{y}_{365}]'$$

The plot of daily standard deviations ($\psi_d$) for Atlanta and Chicago in Figure 1 shows a clear seasonal pattern: the temperature variation in the HDD season is larger than that in the CDD season. This is common for all cities under consideration.

In light of the above observations, a model for the daily temperature should possess the following features. First, it must capture the seasonal cyclical patterns; second, the daily variations in temperature must be around some average "normal" temperature; third, it should incorporate the autoregressive property in temperature changes (i.e., a warmer day is most likely to be followed by another warmer day, and vice versa); fourth, the extent of variation must be bigger in the winter and smaller in the summer; and fifth, the model must reflect the global warming trend.[5]

[5]The finance literature on weather modelling is very scanty. One exception is Campbell and Diebold (2003). They use a time-series approach to modeling and forecasting daily average temperature for 10 U.S. cities. They find that the time-series approach is surprisingly successful in describing and capturing the distributional properties of the temperature variable. The model proposed below is also based on the time-series approach, However, the specification is different from theirs.

For ease of exposition, the daily temperature observations are stacked in vector $Y_t$ for $t = 1, 2, 3, \ldots, T = 20 \times 365 = 7300$ and the corresponding historical average temperature for each day are stacked in vector $\overline{Y}_t = \bar{y}_{365\times 1} \otimes l_{20\times 1}$ where $l_{20\times 1}$ is a $20 \times 1$ vector of ones. After removing the mean and the trend, the residual daily temperature can be expressed as

$$U_t = Y_t - \left(\frac{\beta}{365}\left(t - \frac{T}{2}\right) + \overline{Y}_t\right) \tag{2}$$

where $\beta$ is the warming trend parameter and $t = 1, 2, \ldots, T$.

**Assumption 1.** *The daily temperature residual, $U_t$, follows a k-lag autocorrelation process:*

$$\begin{aligned} U_t &= \sum_{i=1}^{k} \rho_t U_{t-i} + \sigma_t * \xi_t \\ \sigma_t &= \sigma_0 - \sigma_1 \left|\sin(\pi t/365 + \phi)\right| \\ \xi_t &\sim i.i.d\ N(0, 1) \\ \forall \quad t &= 1, 2, \ldots, T \end{aligned} \tag{3}$$

*where $\xi_t$ presents the randomness in the temperature changes.*

In the above, the volatility specification using the sine wave reflects the fourth requirement and the feature in Figure 1. The parameter $\phi$ captures the proper starting point of the sine wave. The autocorrelation setup reflects the third feature. The other features such as the global warming trend and seasonal variations and are captured by $\beta$ and $\overline{Y}_t$. Since $\overline{Y}_t$ represents the daily historical average temperature within the sample, the daily temperatures over the first half of the sample period must be, on average, lower than the historical average temperature; and the opposite is true for the second half of the sample. This is captured by the term in Equation (2) involving $\beta$.

## Modeling the Investor's Preference and Aggregate Dividend Behavior

Following the convention in the literature, the representative investor is assumed to have a risk preference characterized by constant relative risk aversion:

**Assumption 2**. *The representative agent's period utility is described by*

$$U(c_t, t) = e^{-\rho t}\frac{c_t^{\gamma+1}}{\gamma + 1} \tag{4}$$

*with the rate of time preference, $\rho > 0$ and the risk parameter $\gamma \in (-\infty, 0]$.*

For the aggregate dividend process, the estimation results of Marsh and Merton (1987) suggest mean-reversion in the rate of aggregate dividend changes. Specifically,

**Assumption 3.** *The aggregate dividend, $\delta_t$, evolves according to the following Markov process:*

$$\ln \delta_t = \alpha + \mu \ln \delta_{t-1} + \nu_t, \quad \forall \quad \mu \leq 1 \tag{5}$$

*where $1 - \mu$ measures the speed of mean reversion, and the error term takes the following form*

$$\nu_t = \sigma\epsilon_t + \sigma\left[\frac{\varphi}{\sqrt{1-\varphi^2}}\xi_t + \eta_1\xi_{t-1} + \eta_2\xi_{t-2} + \eta_3\xi_{t-3} + \cdots + \eta_m\xi_{t-m}\right],$$

$$0 \leq m \leq +\infty \tag{6}$$

In the above, $\epsilon_t$ is an i.i.d. standard normal variable which captures the randomness due to all factors other than the temperature uncertainty; $\xi_t$ and its lagged terms are innovations of the temperature variable defined in Equation (3). By construction, the contemporaneous correlation between the dividend process and the temperature process is $\varphi$. The lagged terms capture the lagged effects of the temperature on the aggregate dividend or output of the economy. By necessity and assumption, $\sum_{j=1}^{m} \eta_j^2$ ($\forall\ m$) is bounded.[6] When $t$ represents a future time and when all the lags are beyond the present time, the conditional variance of $\nu_t$ is $\sigma^2[1 + \frac{\varphi^2}{1-\varphi^2} + \sum_{j=1}^{m}\eta_j^2]$, which can be broken down into three parts: the part due to all factors other than the temperature, $\sigma^2$; the part due to the contemporaneous impact of the temperature, $\sigma^2\frac{\varphi^2}{1-\varphi^2}$; and the part due to the lagged impact of the temperature, $\sigma^2\Sigma_{j=1}^{m}\eta_j^2$. The correlation between the dividend innovation at time $t$ and the temperature innovation at time $t - j\ \forall\ j$ is $[\eta_j/(1 + \frac{\varphi^2}{1-\varphi^2} + \sum_{j=1}^{m}\eta_j^2)]$. When $\varphi = 0$ and $\eta_j = 0, \forall\ j$, the dividend process is totally independent of the temperature innovation.

The specifications for the representative agent's preference in Equation (4), the dividend process in Equations (5) and (6), and temperature dynamics in Equations (2) and (3), together with the general pricing equation in Equation (1), enable the valuation of any claim contingent upon the temperature variable.

[6] In general, the correlation parameters, $\varphi$, $\eta_1$, $\eta_2$, $\eta_3$,..., can be time dependent throughout the year, and this will not add any difficulty in modeling. To streamline notation, the time index to those parameters is omitted, although all results still hold when time dependency is introduced.

## VALUING HDD/CDD DERIVATIVES

### Valuation of HDD/CDD Forward Contracts and Options

Consider an HDD forward contract with a tick size of \$1 and a delivery price, $K$. The accumulation period starts at $T_1$ and ends at $T_2 > T_1$. Denote $\mathrm{HDD}(T_1, T_2)$ as the accumulation of heating degree days between dates $T_1$ and $T_2$. Then, by applying the general pricing equation in Equation (1) and the utility function in Equation (4), the value of the HDD forward contract at time $t$, $f_{\mathrm{HDD}}(t, T_1, T_2, K)$, can be expressed as

$$\begin{aligned} f_{\mathrm{HDD}}(t, T_1, T_2, K) &= E_t\left(\frac{U_c(\delta_{T_2}, T_2)}{U_c(\delta_t, t)}[\mathrm{HDD}(T_1, T_2) - K]\right) \\ &= e^{-\rho(T_2 - t)}E_t\left(\frac{\delta_{T_2}^{\gamma}}{\delta_t^{\gamma}}[\mathrm{HDD}(T_1, T_2) - K]\right) \end{aligned} \tag{7}$$

By definition, the forward price at time $t$, $F_{\mathrm{HDD}}(t, T_1, T_2)$, is the value of $K$ which makes $f = 0$. That is,

$$F_{\mathrm{HDD}}(t, T_1, T_2) = \frac{E_t(\delta_{T_2}^{\gamma}\mathrm{HDD}(T_1, T_2))}{E_t(\delta_{T_2}^{\gamma})} \tag{8}$$

Similar expressions for $f_{\mathrm{CDD}}(t, T_1, T_2, K)$ and $F_{\mathrm{CDD}}(t, T_1, T_2)$ can be obtained by replacing the notation "HDD" in Equations (7) and (8) with "CDD."

Now consider a European option written on $\mathrm{HDD}(T_1, T_2)$ with maturity $T_2$ and strike price $X$. Denote the call and put prices at time $t$ as $C_{\mathrm{HDD}}(t, T_1, T_2, X)$ and $P_{\mathrm{HDD}}(t, T_1, T_2, X)$, respectively. Again, by Equations (1) and (4), the call and put values can be expressed as

$$C_{\mathrm{HDD}}(t, T_1, T_2, X) = e^{-\rho(T_2 - t)}\delta_t^{-\gamma}E_t(\delta_{T_2}^{\gamma}\max(\mathrm{HDD}(T_1, T_2) - X, 0)) \tag{9}$$

$$P_{\mathrm{HDD}}(t, T_1, T_2, X) = e^{-\rho(T_2 - t)}\delta_t^{-\gamma}E_t(\delta_{T_2}^{\gamma}\max(X - \mathrm{HDD}(T_1, T_2), 0)) \tag{10}$$

Similar formulas for $C_{\mathrm{CDD}}(t, T_1, T_2, X)$ and $P_{\mathrm{CDD}}(t, T_1, T_2, X)$ can be obtained by replacing "HDD" with "CDD" in Equations (9) and (10).

The specifications for the dividend and temperature processes render closed-form solutions impossible. Monte Carlo simulations must be used.

### Market Price of Weather Risk

The inherent market price of risk for the temperature variable results from the inability to hedge the temperature risk. In other words, the correlation between the temperature variable and the aggregate dividend plays an important role in determining the risk premium embedded in the value of a temperature derivative contract. To begin with, the following statement about the market price of risk can be inferred from the pricing equations: the risk premium in the value of a derivative security would be zero if the dividend process and the temperature process are completely independent, i.e., if $\varphi = 0$ and $\eta_j = 0, \forall j$. In this case, any contingent claim can be valued by discounting its payoff at the risk-free rate. In certain special cases, it is also possible to make some specific statements about the market price of risk for forward prices. Take the forward price for HDD in Equation (8) as an example. The forward price can be re-expressed as

$$F_{\text{HDD}}(t, T_1, T_2) = \frac{E_t(\delta_{T_2}^{\gamma}\text{HDD}(T_1, T_2))}{E_t(\delta_{T_2}^{\gamma})} = E_t(\text{HDD}(T_1, T_2)) + \frac{\text{cov}(\delta_{T_2}^{\gamma}, \text{HDD}(T_1, T_2))}{E_t(\delta_{T_2}^{\gamma})} \quad (11)$$

where $\text{cov}(\cdot\,, \cdot)$ stands for covariance. The first term represents the expected future spot value of HDD, and the second term represents forward premium. Similar results can be obtained for the CDD forward price. Since HDD is negatively related to the temperature, Equation (11) implies that the forward price is smaller (larger) than the expected HDD when $\varphi < 0$ and $\eta_j \leq 0, \forall j$ ($\varphi > 0$ and $\eta_j \geq 0, \forall j$). The reverse is true for CDDs.

## EMPIRICAL ESTIMATION

### Estimation of the Temperature Process

Our setup calls for a joint estimation of the aggregate dividend and the daily temperature processes. Macroeconomic variables such as GNP or aggregate consumption can be used as proxies for the aggregate dividend. Unfortunately, the frequency of such data is usually low (at most monthly), making it impossible to carry out the joint estimation with daily data. To get around this difficulty, the temperature process is estimated independently using daily data, and then the correlation between the dividend and temperature processes is estimated using the monthly consumption data.

The parameters in Equations (2) and (3) are estimated using the maximum likelihood method. In order to determine $k$ in Equation (3), sequential estimations are performed by including, respectively, $\rho_1$, $\rho_2$, $\rho_3$, and so on, and for each estimation, the Schwarz criterion, SC = [$-2$ ln(maximum likelihood) + (ln $T$)(number of parameters)] is calculated. The optimal number of lags is the one which minimizes SC. It turns out that three lags describe the data the best for all cities. The estimation results are reported in Table III.

Almost all parameters are estimated with very low standard errors, implying the proper specification of the temperature process. Furthermore, the first order autoregressive behavior tends to be stronger for southern cities, and $\rho_1$ has the highest value for Atlanta. Roughly

**TABLE III**
Maximum Likelihood Estimation Results

| $\sigma_0$ | $\sigma_1$ | $\phi$ | $\beta$ | $\rho_1$ | $\rho_2$ | $\rho_3$ | *Log-likelihood* | *SC value* |
|---|---|---|---|---|---|---|---|---|
| | | | | *Atlanta* | | | | |
| 7.6501 | 5.0697 | −0.1939 | 0.0235 | 0.9487 | −0.3186 | 0.0839 | −20753.6 | 41569.5 |
| (0.1144) | (0.1130) | (0.0139) | (0.0264) | (0.0117) | (0.0157) | (0.0117) | | |
| | | | | *Chicago* | | | | |
| 7.9283 | 3.1183 | −0.1999 | 0.0682 | 0.8605 | −0.2666 | 0.0929 | −23266.2 | 46594.7 |
| (0.1455) | (0.1718) | (0.0247) | (0.0371) | (0.0117) | (0.0151) | (0.0117) | | |
| | | | | *Dallas* | | | | |
| 9.0128 | 6.3318 | −0.1495 | 0.0311 | 0.8720 | −0.2523 | 0.0706 | −21513.7 | 43089.8 |
| (0.1274) | (0.1207) | (0.0104) | (0.0263) | (0.0117) | (0.0152) | (0.0117) | | |
| | | | | *New York* | | | | |
| 6.7025 | 2.8063 | −0.2432 | 0.0979 | 0.8093 | −0.2596 | 0.0929 | −21864.5 | 43791.3 |
| (0.1186) | (0.1372) | (0.0264) | (0.0267) | (0.0117) | (0.0148) | (0.0117) | | |
| | | | | *Philadelphia* | | | | |
| 7.0430 | 3.2261 | −0.2026 | 0.1047 | 0.8274 | −0.2565 | 0.0966 | −21941.7 | 43945.7 |
| (0.1247) | (0.1423) | (0.0176) | (0.0288) | (0.0117) | (0.0149) | (0.0117) | | |

*Note.* 1. The estimated system is:

$$U_t = \rho_1 U_{t-1} + \rho_2 U_{t-2} + \rho_3 U_{t-3} + \cdots + \rho_k U_{t-k} + \sigma_t {}^* \xi_t$$
$$\text{with } U_t = Y_t - \left(\tfrac{\beta}{365}\left(t - \tfrac{7300}{2}\right) + \bar{Y}_t\right)$$
$$\sigma_t = \sigma_0 - \sigma_1 \left|\sin(\pi t/365 + \phi)\right|$$
$$\xi_t \sim i.i.d.\ N(0,1), \quad = \quad t = 1, 2, \ldots, 7300$$

2. The numbers in parentheses are standard errors.
3. "SC value" stands for Schwarz criterion value which is defined as $-2$ ln(maximum likelihood) + (ln $T$)(number of parameters). This criterion is used to determine the optimal number of lags, $k$, in the error term, $U_t$. This value is to be minimized. It turns out that $k$ is 3 for all five cities.

speaking, a stronger autocorrelation means less dramatic changes in temperature, and vice versa. As shown in Table II, Atlanta does have the lowest overall standard deviation in the sample period. Incidentally, Atlanta has the lowest trending parameter $\beta$, and Philadelphia has the highest. Over the twenty-year period, a $\beta$ value of, say 0.1047 (for Philadelphia), indicates that the temperature has risen by about two degrees Fahrenheit.

## Correlation Estimation Using Monthly Data

Historical monthly consumption data are obtained from the Federal Reserve Bank of St. Louis (http://www.stls.frb.org/fred). Total personal consumption is used to proxy the aggregate consumption. The consumption data are matched with monthly average temperature. Again, the sample period is from 1979 to 1998 and the total number of observations is $20 \times 12 = 240$.

Joint dynamics for the temperature variable and total consumption similar to those in Equations (2), (3), (5), and (6) are postulated, all on a monthly scale. To avoid the cumbersome evaluation of all the Schwarz criterion permutations for the joint system, the temperature system and the consumption process are estimated separately, and the resulting residuals are used to estimate the parameters in Equation (6). Panel A of Table IV reports the temperature estimation results.

Next, the constant correlation between the consumption residuals and the temperature residuals is calculated. This is done for each city, and for the average residual over the five cities. Panel B of Table IV summarizes the results. It is seen that, when the correlation is forced to be constant throughout the year, it tends to be negative and not very high. However, a simple dichotomy of CDD and HDD months (by putting April and October into HDD months) shows a clear seasonal pattern in the correlation. The results indicate that, in the summer/winter months, higher/lower temperatures are associated with more consumptions. This is certainly intuitive for energy and power products.[7]

With the above insights, the lagged correlations can be investigated. Based the HDD/CDD dichotomy, it is assumed that the contemporaneous correlations keep the same absolute values but can change sign across the two seasons. As for the lagged coefficients in Equation (6), two simplifying

[7]Although not reported, the correlation for each month of the year is also calculated, and the overall structure is consistent with the HDD/CDD season dichotomy, but the change in sign and magnitude is not very smooth. A smooth fitting procedure could be applied to the monthly correlation estimates.

**TABLE IV**

Estimation Results for Monthly Average Temperature and Consumption

*Panel A: Parameter estimates for monthly average temperature*

| *City* | $\sigma_0$ | $\sigma_1$ | $\phi$ | $\beta$ | $\rho_1$ | $\rho_2$ | $\rho_3$ |
|---|---|---|---|---|---|---|---|
| Atlanta | 4.1563 | 2.2222 | −0.2796 | 0.0293 | 0.2384 | 0.1140 | −0.0639 |
| | (0.4055) | (0.4519) | (0.0925) | (0.0406) | (0.0659) | (0.0659) | (0.0617) |
| Chicago | 5.6138 | 3.2856 | −0.2959 | 0.0433 | 0.1880 | 0.1324 | −0.0323 |
| | (0.4836) | (0.4992) | (0.0817) | (0.0502) | (0.0651) | (0.0659) | (0.0602) |
| Dallas | 3.6349 | 1.6489 | −0.2281 | 0.0362 | 0.2178 | −0.0313 | 0.0624 |
| | (0.3650) | (0.4263) | (0.1205) | (0.0373) | (0.0719) | (0.0666) | (0.0619) |
| New York | 4.1563 | 2.6084 | −0.2590 | 0.0837 | 0.1569 | 0.1431 | −0.0686 |
| | (0.3354) | (0.3409) | (0.0734) | (0.0320) | (0.0653) | (0.0631) | (0.0572) |
| Philadelphia | 4.3236 | 2.4967 | −0.2620 | 0.0959 | 0.1877 | 0.1403 | −0.0466 |
| | (0.3791) | (0.4100) | (0.0962) | (0.0389) | (0.0657) | (0.0631) | (0.0585) |

*Panel B: Correlation between temperature and total consumption*

| *City* | *Over all months* | *Over HDD months 1,2,3,4,10,11,12* | *Over CDD months 5,6,7,8,9* |
|---|---|---|---|
| Atlanta | −0.1086 | −0.2166 | 0.2268 |
| Chicago | −0.0281 | −0.1439 | 0.3458 |
| Dallas | −0.0149 | −0.1254 | 0.2602 |
| New York | −0.0657 | −0.1464 | 0.2322 |
| Philadelphia | −0.0906 | −0.1758 | 0.1903 |
| Average | −0.0739 | −0.1882 | 0.3382 |

*Panel C: Lagged correlation between temperature and total consumption*

| | *Zero lag* | *One lag* | *Two lags* | *Three lags* |
|---|---|---|---|---|
| Multiple correlation | 0.2188 | 0.2199 | 0.2209 | 0.2260 |
| $R^2$ | 0.0479 | 0.0484 | 0.0488 | 0.0511 |
| Adjusted $R^2$ | 0.0436 | 0.0400 | 0.0362 | 0.0344 |
| Contemporaneous correlation ($\varphi$) | 0.2298 | 0.2278 | 0.2254 | 0.2238 |
| ($t$ value) | (3.4237) | (3.3840) | (3.3372) | (3.3122) |
| Lag-one correlation ($\eta_1$) | N/A | −0.0229 | −0.0247 | −0.0203 |
| ($t$ value) | | (−0.3391) | (−0.3641) | (−0.2975) |
| Lag-two correlation ($\eta_2$) | N/A | N/A | −0.0219 | −0.0177 |
| ($t$ value) | | | (−0.3230) | (−0.2601) |
| Lag-three correlation ($\eta_3$) | N/A | N/A | N/A | 0.0507 |
| ($t$ value) | | | | (0.7447) |

*Note.* 1. In panel A, the estimated system is:

$$U_t = \rho_1 U_{t-1} + \rho_2 U_{t-2} + \cdots + \rho_k U_{t-k} + \sigma_t{}^* \xi_t$$
$$\text{with } U_t = Y_t - \left(\tfrac{\beta}{12}\left(t - \tfrac{240}{2}\right) + \bar{Y}_t\right)$$
$$\sigma_t = \sigma_0 - \sigma_1 \left|\sin(\pi \cdot t/12 + \phi)\right|$$
$$\xi_t \sim i.i.d.\ N(0, 1), \quad \forall \quad t = 1, 2, \ldots, 240$$

The data frequency is monthly. The monthly temperature is calculated as the average daily temperature within the month. $\bar{Y}_t$ is the average of the 20 monthly temperatures for each calendar month. Similar to the estimation in Table III, the Schwarz criterion is also used to determine the optimal number of lags, which happens to be three for all cities.

2. In panel B, correlations are calculated between monthly temperature and monthly total consumption. When calculating correlations for HDD and CDD seasons, the shoulder months, April and October, are added to the HDD season.
3. In panel C, an OLS regression is run according to Equation (5) using monthly consumption data. The residuals from Equation (5) are then regressed on the residual terms from the system in panel A of this table according to Equation (6). See the text for details.
4. The CDD season is the base season for signing the parameters. For example, in the zero lag case, the contemporaneous correlation is 0.2298 for the summer season and −0.2298 for the winter season.
5. In panel C, the temperature is the average across the five cities.

assumptions are made: (1) their magnitude remains constant, and (2) they will adopt the sign of the contemporaneous correlation belonging to the season where the lagged error term is from. For instance, if the current month is June, then $\varphi$ and $\eta_1$ will be positive, but $\eta_2$ will be negative. With the above structure, simple OLS regressions in Equation (6) are run (with the intercept suppressed) to estimate the parameters. The regression coefficients are scaled by the residual's standard deviation to obtain $\varphi$, $\eta_1$, $\eta_2$, and so on. For brevity, panel C of Table IV reports only the results for the average temperature across five cities.

It is seen that only the contemporaneous correlation is significant (with a $t$ value larger than 3). Adding lagged terms will reduce the adjusted $R^2$ and none of the estimated coefficients are significant. Results for individual cities are the same qualitatively. Taken altogether, the above analysis seems to indicate that total consumption is indeed affected by the temperature as evidenced by the significant correlation estimate which is around 0.22 in absolute terms. However, the lagged impacts seem to be weak, and the magnitude of lagged coefficients is much smaller.

## MARKET PRICE OF RISK FOR TEMPERATURE DERIVATIVES: NUMERICAL ANALYSIS

### Simulation Design

To jointly generate the temperature and aggregate dividend processes, parameters are initialized as follows. First, throughout the simulations, the rate of time preference, $\rho$ is set at 0.03, which mirrors the historical level of the real interest rate.

Second, the base value of the mean reversion parameter for the dividend process, $\mu$ is set at 0.9. This is based on the empirical findings in the literature. Shiller (1983) estimated $\mu$ to be 0.807, while Marsh and Merton (1987) estimated $\mu$ to be as high as 0.945.

Third, as for the number of lagged error terms in Equation (6), the analysis in the previous section seems to indicate a maximum of 30 (i.e., one month). Two cases are simulated, one with only the contemporaneous correlation and the other with 30 lagged error terms. The coefficients, $\eta_j \forall j$ are set in conjunction with the contemporaneous correlation $\varphi$ by using a simple geometric decay function, to be discussed later. The upper bound for the contemporaneous correlation $\varphi$ is set at 0.25, in light of the estimation results. For simplicity, $\varphi$ and $\eta_j \forall j$ are assumed to share the same sign which remains unchanged across seasons. A robustness check is performed for mixed correlation signs.

Fourth, for a given structure of $\eta_j$ and a contemporaneous correlation $\varphi$, $\sigma$ is set such that the overall volatility of the dividend process, $\sigma\sqrt{1 + \frac{\varphi^2}{1-\varphi^2} + \Sigma_{j=1}^{90}\eta_j^2}$, is 20%, a magnitude similar to that for a stock market index.

Fifth, the risk aversion parameter, $\gamma$ is used as a comparative static variable. Three cases are examined: $\gamma = -2.0$, $-10.0$, and $-40.0$. Most empirical studies indicate that $\gamma$ ranges between zero and $-2.0$, although some recent studies suggest a higher risk aversion to accommodate the so-called equity premium puzzle. Mehra and Prescott (1985) found that a risk aversion between $-30.0$ and $-40.0$ is required to explain the historical equity premium. The case of $\gamma = -40.0$ is therefore included to accommodate the observed equity premium. For each risk aversion scenario, two correlation levels are examined, each of which can be positive or negative.

Sixth, given the choice of volatility parameters and a value for $\gamma$, the average dividend growth rate $\alpha$ and the initial dividend $\delta_t$ are set according to $e^{-r(t,T)(T-t)} = E_t(\frac{U_c(\delta_T)}{U_c(\delta_t)})$ such that the risk-free interest rate or yield, $r(t, T)$ is maintained at 6%.

Finally, the parameter estimates in Table III are used for the temperature process. January 1, 1999 is taken as the valuation date, which is the first day following the last observation date in the sample. Since HDD contracts are mirror image of CDD contracts in nature, for brevity, only simulation results for CDD contracts are reported, which cover the period of May 1, 1999 to September 30, 1999. Each temperature derivative's value is averaged over 10,000 realizations. The antithetic variable technique is used to reduce simulation errors.

Since $\varphi = 0.0$ and $\eta_j = 0, \forall j$ amount to a zero market price of risk for the temperature variable (irrespective of the risk-aversion level), forward and option values in this case are called "risk-neutral" values. Derivatives values under other correlation/risk aversion scenarios are compared against these "risk-neutral" values. The difference in values directly gauges the impact of market price of risk.

## Correlations and Market Price of Risk

### *Contemporaneous Correlations Only*

Two contemporaneous correlation levels (0.15 and 0.25) are used for the analysis. With zero-lagged correlations, a contemporaneous correlation

**TABLE V**

Risk Premium in Forward Prices: Contemporaneous Correlations Only

| | *Risk-neutral forward* | $\gamma = -2.0$ | | $\gamma = -10.0$ | | $\gamma = -40.0$ | |
|---|---|---|---|---|---|---|---|
| | | $\phi = -a$ | $\phi = a$ | $\phi = -a$ | $\phi = a$ | $\phi = -a$ | $\phi = a$ |
| *Panel A: a = 0.15* | | | | | | | |
| Atlanta | 1802.6 | 0.02 | −0.01 | 0.08 | −0.04 | 0.31 | −0.15 |
| Chicago | 826.7 | 0.03 | −0.01 | 0.13 | −0.05 | 0.40 | −0.27 |
| Dallas | 2416.7 | 0.01 | −0.01 | 0.07 | −0.04 | 0.24 | −0.13 |
| New York | 1181.0 | 0.02 | −0.01 | 0.10 | −0.04 | 0.33 | −0.20 |
| Philadelphia | 1233.4 | 0.02 | −0.01 | 0.10 | −0.04 | 0.33 | −0.18 |
| *Panel B: a = 0.25* | | | | | | | |
| Atlanta | 1802.6 | 0.03 | −0.02 | 0.12 | −0.08 | 0.48 | −0.27 |
| Chicago | 826.7 | 0.04 | −0.03 | 0.18 | −0.10 | 0.66 | −0.44 |
| Dallas | 2416.7 | 0.02 | −0.02 | 0.10 | −0.07 | 0.39 | −0.23 |
| New York | 1181.0 | 0.03 | −0.02 | 0.14 | −0.09 | 0.53 | −0.33 |
| Philadelphia | 1233.4 | 0.03 | −0.02 | 0.14 | −0.08 | 0.53 | −0.31 |

*Note.* 1. "Risk-neutral forward" prices are calculated by setting the correlation between the temperature and the dividend processes to zero. Other entries are percentage differences between the risk-neutral price and the price under risk aversion. For example, for Atlanta, when the correlation is −0.25 and the risk aversion is −10, the price under risk aversion is 0.12% higher than the risk-neutral one.
2. All prices are for a CDD season which covers the period of May 1, 1999, to September 30, 1999.
3. In panel A, the contemporaneous correlation is either −0.15 or 0.15 under each risk aversion; in panel B, it is either −0.25 or 0.25.

of ±0.15 or ±0.25 means that about 2.3% or 6.3% of the total variance in outputs is due to temperature variations (the percentages are calculated from $(\frac{\varphi^2}{1-\varphi^2})/(1+\frac{\varphi^2}{1-\varphi^2}) = \varphi^2$). Table V reports the risk premium for forward prices, which is defined as the percentage difference between the forward price under various correlation/risk aversion scenarios and the risk-neutral price. Several observations are in order. First, the risk premium is very small for all cases. The largest percentage price difference, 0.66%, is for Chicago when the correlation is −0.25 and the risk aversion parameter is −40.0. Second, the correlation's sign determines the sign of the risk premium or market price of risk, which is consistent with the theoretical results discussed earlier. Third, for a fixed correlation, a higher risk aversion leads to a bigger risk premium, which makes intuitive sense. In addition, other things being equal, a stronger correlation leads to a bigger risk premium, which again makes intuitive sense.

Table VI reports risk premiums for CDD call and put options. To facilitate discussions, the strike price is set equal to the risk-neutral forward price so that the risk-neutral call and put options are exactly at-the-money and have the same value. The rest of the setup is the same as in

**TABLE VI**

Risk Premium in Option Prices: Contemporaneous Correlations Only

| | | Strike price | RN option value | $\gamma = -2.0$ | | $\gamma = -10.0$ | | $\gamma = -40.0$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $\phi = -a$ | $\phi = a$ | $\phi = -a$ | $\phi = a$ | $\phi = -a$ | $\phi = a$ |
| *Panel A: $a = 0.15$* | | | | | | | | | |
| Atlanta | Call | 1802.6 | 45.9 | 0.34 | −0.20 | 1.74 | −0.97 | 5.90 | −5.92 |
| | Put | | 45.9 | −0.34 | 0.19 | −1.86 | 0.80 | −8.52 | 3.77 |
| Chicago | Call | 826.7 | 52.1 | 0.22 | −0.09 | 1.15 | −0.39 | 3.32 | −2.99 |
| | Put | | 52.1 | −0.17 | 0.10 | −0.93 | 0.47 | −3.87 | 2.77 |
| Dallas | Call | 2416.7 | 46.2 | 0.37 | −0.25 | 1.90 | −1.22 | 6.69 | −7.05 |
| | Put | | 46.2 | −0.37 | 0.24 | −2.03 | 1.09 | −9.30 | 5.07 |
| New York | Call | 1181.0 | 44.7 | 0.27 | −0.15 | 1.40 | −0.68 | 4.37 | −4.45 |
| | Put | | 44.7 | −0.25 | 0.14 | −1.34 | 0.61 | −5.83 | 3.29 |
| Philadelphia | Call | 1233.4 | 47.8 | 0.27 | −0.14 | 1.38 | −0.63 | 4.14 | −4.32 |
| | Put | | 47.8 | −0.24 | 0.12 | −1.35 | 0.53 | −5.92 | 2.83 |
| *Panel B: $a = 0.25$* | | | | | | | | | |
| Atlanta | Call | 1802.6 | 45.9 | 0.52 | −0.38 | 2.65 | −1.86 | 10.18 | −9.53 |
| | Put | | 45.9 | −0.51 | 0.36 | −2.73 | 1.70 | −12.44 | 8.01 |
| Chicago | Call | 826.7 | 52.1 | 0.32 | −0.20 | 1.66 | −0.89 | 5.73 | −4.83 |
| | Put | | 52.1 | −0.26 | 0.20 | −1.39 | 0.94 | −6.17 | 4.90 |
| Dallas | Call | 2416.7 | 46.2 | 0.58 | −0.46 | 2.95 | −2.25 | 11.61 | −11.29 |
| | Put | | 46.2 | −0.58 | 0.45 | −3.04 | 2.15 | −13.79 | 10.13 |
| New York | Call | 1181.0 | 44.7 | 0.41 | −0.29 | 2.10 | −1.36 | 7.63 | −7.11 |
| | Put | | 44.7 | −0.37 | 0.27 | −1.99 | 1.27 | −8.87 | 6.31 |
| Philadelphia | Call | 1233.4 | 47.8 | 0.40 | −0.27 | 2.05 | −1.29 | 7.29 | −6.85 |
| | Put | | 47.8 | −0.37 | 0.25 | −1.97 | 1.16 | −8.84 | 5.73 |

*Note.* 1. The strike price is set at the risk-neutral forward price. “RN option value” stands for “risk-neutral option value” which is calculated by setting the correlation between the temperature and the dividend processes to zero. Other entries are percentage differences between the risk-neutral price and the price under risk aversion. For example, for Atlanta, when the contemporaneous correlation is −0.25 and the risk aversion is −10, the call option’s price under risk aversion is 2.65% higher than the risk-neutral one.
2. All prices are for a CDD season which covers the period of May 1, 1999, to September 30, 1999.
3. In panel A, the contemporaneous correlation is either −0.15 or 0.15 under each risk aversion; in panel B, it is either −0.25 or 0.25.

Table V. Again, the risk-neutral option values do not contain risk premiums, and all other option values are compared against these values to assess the market price of risk.

All the patterns associated with the forward prices also apply to call options. The opposite patterns apply to put options, which is intuitive since the value of a put is inversely related to the level of CDD. In fact, all the explanations for the patterns in forward prices also apply to options. However, the risk premiums in option prices are many-fold larger than those in the forward prices. The marked difference in risk premium between the two types of instruments is mainly due to the different payoff features: linear for forward contracts and non-linear for options.

For options, when the risk aversion is −10.0 or −40.0, the risk premium is no longer insignificant. The largest risk premium, 11.61% for call and −13.79% for put, is observed for the combination of $\gamma = -40.0$ and $\varphi = -0.25$ for the city of Dallas. It means that the CDD call (put) option contains a risk premium of 11.61% (−13.79%) purely due to the market price of risk associated with the temperature variable.

The overall results in Tables V and VI indicate that, by and large, the market price of risk is not a significant factor if the dividend and temperature processes are only contemporaneously correlated and if the risk aversion is low (i.e., $\gamma = -2.0$). For forward prices, it is not significant even when the risk aversion is high. However, for options, the market price of risk is no longer negligible, especially when the risk aversion is high.

*Contemporaneous and Diminishing Lagged Correlations*

Lagged correlations are introduced to fully assess the significance of risk premium. Our empirical analysis indicates that correlation diminishes significantly after one month, suggesting that daily correlations most likely follow the same diminishing pattern. To reflect this feature, a simple geometric decay structure for the coefficients, $\eta_j$ is assumed. Specifically, once the contemporaneous correlation, $\varphi$ is set, $\eta_1$ is calculated as $q\varphi$, $\eta_2$ as $q^2\varphi$, $\eta_3$ as $q^3\varphi$, and so on with $0 < q < 1$. The decay factor $q$ is chosen such that $|\eta_{30}| = |q^{30}\varphi| = 0.0001$, a level arbitrarily chosen to signify the eventual diminution of the lagged effect. With such a structure, the portion of the variance attributable to the temperature variations is calculated as:

$$\frac{\frac{\varphi^2}{1-\varphi^2} + \sum_{j=1}^{30} \eta_j^2}{1 + \frac{\varphi^2}{1-\varphi^2} + \sum_{j=1}^{30} \eta_j^2}.$$

With $|\varphi| = 0.15$ and $|\varphi| = 0.25$, the corresponding proportions are 5.56% and 13.64%. Given that one-seventh (or 14.3%) of the GNP is believed to be weather sensitive, the case of $|\varphi| = 0.25$ roughly corresponds to reality. Interestingly, this is also the correlation level revealed in panel C of Table IV. Lastly, without loss of generality, the realized,

**TABLE VII**
Risk Premium in Forward Prices: Contemporaneous and Diminishing Lagged Correlations

| | *Risk-neutral forward* | $\gamma = -2.0$ | | $\gamma = -10.0$ | | $\gamma = -40.0$ | |
|---|---|---|---|---|---|---|---|
| | | $\phi = -a$ | $\phi = a$ | $\phi = -a$ | $\phi = a$ | $\phi = -a$ | $\phi = a$ |
| *Panel A: $a = 0.075$* | | | | | | | |
| Atlanta | 1802.6 | 0.12 | −0.10 | 0.64 | −0.38 | 3.22 | −1.06 |
| Chicago | 826.7 | 0.23 | −0.20 | 1.19 | −0.87 | 5.83 | −2.91 |
| Dallas | 2416.7 | 0.10 | −0.08 | 0.51 | −0.27 | 2.66 | −0.64 |
| New York | 1181.0 | 0.16 | −0.14 | 0.85 | −0.58 | 4.24 | −1.82 |
| Philadelphia | 1233.4 | 0.16 | −0.14 | 0.86 | −0.58 | 4.27 | −1.82 |
| *Panel B: $a = 0.15$* | | | | | | | |
| Atlanta | 1802.6 | 0.21 | −0.18 | 1.23 | −0.53 | 7.99 | −1.03 |
| Chicago | 826.7 | 0.38 | −0.34 | 2.15 | −1.32 | 12.75 | −3.80 |
| Dallas | 2416.7 | 0.17 | −0.14 | 1.02 | −0.35 | 7.02 | −0.53 |
| New York | 1181.0 | 0.28 | −0.24 | 1.59 | −0.86 | 9.83 | −2.22 |
| Philadelphia | 1233.4 | 0.28 | −0.24 | 1.60 | −0.86 | 9.89 | −2.22 |

*Note.* 1. "Risk-neutral forward" prices are calculated by setting the correlation between the temperature and the dividend processes to zero. Other entries are percentage differences between the risk-neutral price and the price under risk aversion. For example, for Atlanta, when the correlation is −0.25 and the risk aversion is −10, the price under risk aversion is 0.12% higher than the risk-neutral one.
2. All prices are for a CDD season which covers the period of May 1, 1999, to September 30, 1999.
3. In panel A, the contemporaneous correlation is either −0.15 or 0.15 under each risk aversion; in panel B, it is either −0.25 or 0.25.
4. The lag period is 30 days. The lagged correlation diminishes to zero according to a geometric series specified in the text.

lagged innovations of the temperature process, $\hat{\xi}_{t-m} = 0 (1 \leq m \leq 30)$ are set to zero.

Table VII and VIII are counterparts of Table V and VI, with diminishing lagged correlations. To begin with, it is seen that the qualitative relations between forward/option prices and the model parameters remain the same when lagged correlations are introduced. But there is a marked increase in the significance of the risk premium. For forward prices, comparisons between Table VII and Table V reveal that the risk premium increases by many times. When the risk aversion is −40, the risk premium can be as high as 5.47%. This surge in magnitude is partly due to the larger proportion of output variance attributable to temperature variations.[8]

[8]But the larger proportion is not the only factor. Although not shown here, the calculations in Table VII are repeated by lowering the correlation level while keeping 30 lag terms such that the proportions are aligned to those in Table VI, i.e., 2.3% or 6.3%. It turns out that the risk premium is still larger when lagged correlations are present. This implies that, other things being equal, the presence of lagged correlations between the temperature and dividend processes would magnify the impact of the market price of weather risk.

**TABLE VIII**
Risk Premium in Option Prices: Contemporaneous and Diminishing Lagged Correlations

| | | Strike price | RN option value | $\gamma = -2.0$ | | $\gamma = -10.0$ | | $\gamma = -40.0$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $\phi = -a$ | $\phi = a$ | $\phi = -a$ | $\phi = a$ | $\phi = -a$ | $\phi = a$ |
| *Panel A: $a = 0.15$* | | | | | | | | | |
| Atlanta | Call | 1802.6 | 45.9 | 1.44 | −1.28 | 7.40 | −6.15 | 33.72 | −25.32 |
| | Put | | 45.9 | −1.39 | 1.26 | −7.08 | 6.39 | −30.61 | 33.60 |
| Chicago | Call | 826.7 | 52.1 | 0.90 | −0.76 | 4.71 | −3.52 | 21.12 | −13.19 |
| | Put | | 52.1 | −0.78 | 0.72 | −4.00 | 3.64 | −18.17 | 19.27 |
| Dallas | Call | 2416.7 | 46.2 | 1.58 | −1.44 | 8.13 | −6.95 | 37.26 | −28.91 |
| | Put | | 46.2 | −1.55 | 1.44 | −7.82 | 7.37 | −33.32 | 39.07 |
| New York | Call | 1181.0 | 44.7 | 1.18 | −1.04 | 6.10 | −4.91 | 27.61 | −19.51 |
| | Put | | 44.7 | −1.08 | 0.99 | −5.54 | 5.02 | −24.57 | 26.42 |
| Philadelphia | Call | 1233.4 | 47.8 | 1.15 | −1.00 | 5.95 | −4.73 | 26.73 | −18.79 |
| | Put | | 47.8 | −1.06 | 0.94 | −5.41 | 4.78 | −24.14 | 25.11 |
| *Panel B: $a = 0.25$* | | | | | | | | | |
| Atlanta | Call | 1802.6 | 45.9 | 2.13 | −1.95 | 11.14 | −9.26 | 54.55 | −35.17 |
| | Put | | 45.9 | −2.04 | 1.94 | −10.21 | 10.02 | −42.13 | 54.44 |
| Chicago | Call | 826.7 | 52.1 | 1.32 | −1.16 | 7.03 | −5.27 | 35.19 | −17.43 |
| | Put | | 52.1 | −1.14 | 1.08 | −5.81 | 5.57 | −26.30 | 29.62 |
| Dallas | Call | 2416.7 | 46.2 | 2.35 | −2.19 | 12.28 | −10.45 | 60.09 | −40.42 |
| | Put | | 46.2 | −2.29 | 2.21 | −11.34 | 11.53 | −45.68 | 63.85 |
| New York | Call | 1181.0 | 44.7 | 1.74 | −1.58 | 9.16 | −7.35 | 45.22 | −26.54 |
| | Put | | 44.7 | −1.59 | 1.51 | −8.01 | 7.76 | −34.70 | 41.49 |
| Philadelphia | Call | 1233.4 | 47.8 | 1.69 | −1.52 | 8.92 | −7.08 | 43.90 | −25.44 |
| | Put | | 47.8 | −1.54 | 1.44 | −7.80 | 7.41 | −34.02 | 39.54 |

*Note.* 1. The strike price is set at the risk-neutral forward price. "RN option value" stands for "risk-neutral option value" which is calculated by setting the correlation between the temperature and the dividend processes to zero. Other entries are percentage differences between the risk-neutral price and the price under risk aversion. For example, for Atlanta, when the contemporaneous correlation is −0.25 and the risk aversion is −10, the call option's price under risk aversion is 11.14% higher than the risk-neutral one.
2. All prices are for a CDD season which covers the period of May 1, 1999, to September 30, 1999.
3. In panel A, the contemporaneous correlation is either −0.15 or 0.15 under each risk aversion; in panel B, it is either −0.25 or 0.25.
4. The lag period is 30 days. The lagged correlation diminishes to zero according to a geometric series specified in the text.

Very similar results are observed for option prices, and the impact of the market price of risk is much more pronounced. For both correlation levels, the risk premium is generally larger than 5% when the risk aversion is −10. When the risk aversion is −40, the risk premium is extremely large. Clearly, the market price of risk for the temperature variable can not be ignored. The practice of discounting temperature derivative payoffs at the risk-free rate is highly questionable. It would be acceptable only when the correlations, both contemporaneous and lagged, are very low and/or the risk aversion of investors is low, none of which is supported by empirical evidence.

*Lagged Correlations with Mixed Signs*

As mentioned before, a more refined way to model the correlations is to make it time dependent. This will allow both the magnitude and the sign of the lagged correlations to change. However, as far as their impact on the risk premium is concerned, it can be seen intuitively that as long as the signs of $\eta_j$ are not uniform, the overall impact of the market price of risk will decline, simply due to a lower overall covariance between the dividend process and the temperature process. The calculations in Tables VII and VIII are repeated by assuming alternating signs in $\eta_j \; \forall \, j$, and the resulting risk premia are much lower. Therefore, the conclusions based on uniformly signed correlations err on the side of caution.

## Mean Reversion of the Aggregate Dividend and Market Price of Risk

So far, $\mu$ is set at 0.9 which corresponds to a mean reversion rate of 0.1. To see how sensitive the results are to this mean reversion parameter, the calculations in Tables VII and VIII are repeated by assuming four other levels of $\mu$: 0.80, 0.85, 0.95, 0.99. Note that $\mu = 0.99$ roughly corresponds to a random walk. Since the results are very similar across all five cities, only those for Chicago are reported in Table IX (Chicago has the smallest risk premium in option values in Table VIII). It is seen that a higher value of $\mu$, or a lower mean reversion speed, leads to a bigger risk premium in forward and option values. This makes intuitive sense since a higher $\mu$ means bigger variations in the aggregate dividends. What is striking is the nonlinearity of the impact. To illustrate, for options, when $\mu$ increases from 0.8 to 0.9, the risk premium slightly more than double for all cases; however, when $\mu$ increases from 0.9 to 0.99, the risk premium increases by more than 10-fold. With a near-random walk, the risk premium is more than 10% for all option values. An obvious conclusion is that, in determining the significance of the market price of risk for the temperature variable, the degree of mean reversion in the aggregate dividend process must be carefully determined.

**TABLE IX**
Impact of Mean Reversion in the Dividend Process: City of Chicago

*Panel A: Forward prices*

| Mean reversion | Risk-neutral forward | $\gamma = -2.0$ | | $\gamma = -10.0$ | | $\gamma = -40.0$ | |
|---|---|---|---|---|---|---|---|
| | | $\phi = -0.25$ | $\phi = 0.25$ | $\phi = -0.25$ | $\phi = 0.25$ | $\phi = -0.25$ | $\phi = 0.25$ |
| 0.80 | 826.7 | 0.07 | −0.05 | 0.39 | −0.14 | 2.87 | −0.27 |
| 0.85 | 826.7 | 0.10 | −0.08 | 0.55 | −0.24 | 3.79 | −0.52 |
| 0.90 | 826.7 | 0.16 | −0.14 | 0.90 | −0.49 | 5.47 | −1.26 |
| 0.95 | 826.7 | 0.43 | −0.39 | 2.26 | −1.58 | 8.73 | −4.88 |
| 0.99 | 826.7 | 2.11 | −2.03 | 10.85 | −9.77 | 40.58 | −37.30 |

*Panel B: Option prices*

| Mean reversion | | Strike price | RN option value | $\gamma = -2.0$ | | $\gamma = -10.0$ | | $\gamma = -40.0$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $\phi = -0.25$ | $\phi = 0.25$ | $\phi = -0.25$ | $\phi = 0.25$ | $\phi = -0.25$ | $\phi = 0.25$ |
| 0.80 | Call | 826.7 | 52.1 | 0.53 | −0.43 | 2.81 | −1.92 | 14.19 | −5.82 |
| | Put | | 52.1 | −0.44 | 0.40 | −2.24 | 2.04 | −10.29 | 10.46 |
| 0.85 | Call | 826.7 | 52.1 | 0.76 | −0.64 | 4.07 | −2.89 | 20.52 | −9.05 |
| | Put | | 52.1 | −0.64 | 0.60 | −3.30 | 3.06 | −15.16 | 15.76 |
| 0.90 | Call | 826.7 | 52.1 | 1.32 | −1.16 | 7.03 | −5.27 | 35.19 | −17.43 |
| | Put | | 52.1 | −1.14 | 1.08 | −5.81 | 5.57 | −26.30 | 29.62 |
| 0.95 | Call | 826.7 | 52.1 | 3.41 | −3.12 | 18.43 | −14.28 | 90.55 | −49.22 |
| | Put | | 52.1 | −3.03 | 2.99 | −14.97 | 15.64 | −60.53 | 91.58 |
| 0.99 | Call | 826.7 | 52.1 | 17.12 | −15.16 | 73.75 | −59.86 | 103.15 | −89.76 |
| | Put | | 52.1 | −14.55 | 15.78 | −59.41 | 92.56 | −90.23 | 220.34 |

*Note.* Forward and option prices are all for CDD contracts for the city of Chicago. Except for the mean reversion parameter, all other aspects of the calculations are the same as in Tables VII and VIII.

## SUMMARY AND CONCLUSION

This paper proposes a valuation framework for temperature derivatives and studies the market price of weather risk therein. The framework is the generalized Lucas's model of 1978. The underlying variables in the economy are the aggregate dividend and the weather uncertainty, and the two are allowed to correlate with one another both contemporaneously and in a lagged fashion. The study leads to the following important observations.

First and foremost, the market price of risk associated with the temperature variable is significant. Based on investors' risk aversion level and the parameters governing the aggregate dividend process that conform to empirical realities, it is found that the risk premium can represent a significant portion of the derivative's price. Risk-neutral valuation, or using the risk-free rate to discount derivatives payoffs, is a good approximation only when the correlations (both contemporaneous and lagged) between the temperature process and the dividend process are very low and/or the risk aversion is low, none of which is supported by empirical evidence.

Second, the market price of risk affects option values much more than forward prices, mainly due to the payoff specification. Because of the linear payoff structure for forward contracts, much of the impact is "integrated" out. For options, the truncation in payoffs leaves room for the market price of risk to manifest its impact.

In addition to addressing the issue of market price of risk, the framework has many other advantages. To start with, the model not only allows easy estimation of the temperature system, but also incorporates key features of the daily temperature such as seasonal cycles and uneven variations throughout the year. Moreover, since the starting point is the daily temperature, the framework is capable of handling temperature contracts of any maturity, for any season, and it requires only a one-time estimation. In contrast, if one directly models the cooling degree days (CDDs) or heating degree days (HDDs), then by nature of the temperature behavior, the CDDs or HDDs will necessarily be season and maturity specific, which implies that each contract requires a separate estimation procedure. This not only creates potential inconsistency in pricing, but also renders the whole idea impractical if many different contracts are dealt with or if the valuation is to be done on an on-going basis.

As for future research, one obvious avenue is to adapt the framework to other weather variables such as snowfall and rainfall. This is challenging in that the weather variable such as rainfall is not a continuous variable. Moreover, the cumulation of such a variable in a season is far more important than the realized level within, say, one day. Nonetheless, derivative contracts on such variables will have direct appeal to users such as farmers and ski resort operators. Another avenue is to test the model using the market data when such data become available in the future.

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