Optimal Insurance Against Climatic Experience

Olivier Mahul

An optimal insurance contract against a climatic risk is derived in the presence of an uninsurable and dependent aggregate production risk. The optimal design depends on the stochastic dependency between both sources of uncertainty and on the producer's attitude towards risk, especially on his prudent behavior. Rational weather insurance purchasing decisions are also derived. The prudent producer responds to actuarially fair weather insurance by increasing his exposure towards risk.

Key words: agricultural insurance, background risk, climatic risk, prudence, stochastic dominance, weather derivatives.

The U.S. multiple peril crop insurance program, in which indemnity payments are based on the producer's individual yields, has been disappointing in that premiums have not been sufficient to cover indemnity payments and administrative costs. This failure has induced the U.S. government to promote alternative insurance policies. Since 1993, the Risk Management Agency has offered area-yield contracts under its group risk plan in selected counties (Skees, Black and Barnett). Another alternative is to base the indemnity on a weather index, such as rainfall or temperature. This policy was proposed in the past by Sanderson but has failed to gain acceptance among policy makers. The difficulties of rainfall insurance were a subject of debate in Australia (e.g., Bardsley, Abey and Davenport; Quiggin 1986; Patrick). The recent development of weather based instruments on financial markets, known as weather derivatives, for which the payoff depends on the cause of risk, rather than on its effect on yield, provide new opportunities for covering exposure to unfavorable weather events.

The objective of this article is threefold. First, a stochastic production function affected by a climatic risk and by an aggregate production risk, where both sources of risk are stochastically dependent, is defined. The design of an optimal insurance contract against a weather variable, the first-best solution, is derived when the aggregate production shock cannot be insured. This uninsurable background risk creates a source of incompleteness in the crop insurance market. The optimal indemnity schedule depends on the stochastic dependence between the insurable climatic risk and the uninsurable aggregate production risk, and on the producer's attitude towards risk. The concept of stochastic dominance and the behavioral property of prudence, introduced by Kimball, are used to derive the optimal insurance design. Second, a functional weather insurance contract in which the indemnity is contingent on the intensity of the weather variable is proposed. In this context of incomplete markets the first-best optimum is used to examine the producer's weather insurance purchasing decision. The consequences of stochastic dependence on rational insurance purchasing decisions are highlighted using recent works on optimal insurance in the presence of an uninsurable, additive, and independent background risk (Mahul 1999). Third, the impact of actuarially fair weather insurance on the optimal level of input use is analyzed. Previous comparative statistical results obtained by Leathers and Quiggin and by Ramaswami are re-examined under the behavioral assumption of prudence.

After describing the stochastic production function in the next section, the above objectives are examined successively in the following three sections and concluding comments are provided in the last section.

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The Stochastic Production Function

Contrary to the standard crop insurance contracts, indemnity payments of weather insurance are based on the intensity of the weather index rather than on its effect on yield. Nevertheless, the producer seeks to insure revenue that is affected by these climatic changes. Consequently, the first step is to specify a model of crop yield to characterize the impact of weather uncertainty and other production risks on yield. Biophysical models of yield have been developed to examine changes in yield that are generated by changes in the physical climate (Kaufmann and Snell). Crop weather models simulate the crop-climate interface and multiple regression is used to estimate the effect of physical variables on yield from empirical observations. Here our purpose is more modest. We aim to define a stochastic production function which models the yield impact of an insurable random weather variable, of an uninsurable aggregate production shock, and of the input level selected by the producer.

The first feature of this production function is that it allows yield to depend linearly on two sources of risk

$$(1) \qquad \tilde{y} = g(x)\tilde{\omega} + k(x)\tilde{\varepsilon} + h(x)$$

where $\tilde{y}$ is the random output, x is the input, $\tilde{\omega}$ is the random weather index with $E\tilde{\omega} = \mu > 0$ and $\tilde{\varepsilon}$ denotes an aggregate production shock. The index $\tilde{\omega}$ is an observable weather event. For instance, it can measure the cumulative rainfall level or the cumulative degree-day heat level in a given geographical area and period. This component of the production risk is systemic because it affects all the farms located in the same geographical area. The $\tilde{\varepsilon}$ shock includes other sources of production risk that are not insurable. Thus it acts as an uninsurable background risk. From the decomposition of the production function in equation (1), $\tilde{\varepsilon}$ can be farm-specific or systemic. The functions $g(\cdot)$ and $k(\cdot)$ are assumed non-negative. Hence, smaller realizations of $\tilde{\omega}$ and $\tilde{\varepsilon}$ correspond to relatively more severe disasters.[1] This functional form can be viewed as a Taylor's expansion of a more general production function $f(x, \tilde{\omega}, \tilde{\varepsilon})$ around $(E\tilde{\omega}, E\tilde{\varepsilon})$. The Just and Pope production function is obtained when $\tilde{\varepsilon}$ equals zero almost surely. The cumulative distribution function of $(\tilde{\omega}, \tilde{\varepsilon})$ is denoted $T(\omega, \varepsilon)$ and it is defined over the support $[\omega_{\min}, \omega_{\max}] \times [\varepsilon_{\min}, \varepsilon_{\max}]$ with $0 < \omega_{\min} < \omega_{\max}$ and $\varepsilon_{\min} < 0 < \varepsilon_{\max}$. The marginal distribution function of $\tilde{\varepsilon}$ is denoted $\Phi(\omega)$.

The second feature of this stochastic production function is that random variables can be independent or correlated. The stochastic dependence between both sources of risk is characterized by stochastic dominance. More precisely, we assume that a decrease in ω induces a riskier conditional distribution of $\tilde{\varepsilon}$ either by first-order stochastic dominance (FSD)

$$(2) \qquad \Psi_\omega(\varepsilon/\tilde{\omega} = \omega) \leq 0 \text{ for all } \varepsilon \text{ and } \omega$$

or by second-order stochastic dominance (SSD)

$$(3) \qquad \int_{\varepsilon_{\min}}^{\varepsilon} \Psi_\omega(s/\tilde{\omega} = \omega)\, ds \leq 0 \quad \text{for all } \varepsilon \text{ and } \omega$$

where $\Psi(\varepsilon/\tilde{\omega} = \omega)$ is the cumulative distribution function of $\tilde{\varepsilon}$ conditional on $\tilde{\omega} = \omega$, with $\Psi_\omega \equiv \partial\Psi/\partial\omega$. Notice that the SSD criterion is satisfied when the FSD criterion holds. For instance, if the weather variable $\tilde{\omega}$ denotes the cumulative rainfall level or the cumulative degree-day heat unit reported by a regional weather station and $\tilde{\varepsilon}$ is an aggregate of all other sources of yield risk, such a correlation indicates that drought or excessive heat increases other sources of production uncertainty in the sense of FSD or SSD. From the production function expressed in (1), the marginal productivity of the weather index can be decomposed as

$$(4) \qquad \frac{dy}{d\omega} = g(x) + k(x)\frac{d\varepsilon}{d\omega}.$$

The impact of the weather index on output is thus the sum of two effects. The first right-hand-side (RHS) term is the direct marginal productivity. The second term, called indirect marginal productivity, stems from the correlation between the weather index and the aggregate production variable. It is zero if the two sources of production risk are stochastically independent.

The decomposition of individual yield proposed in (1) differs from that of Miranda, and Mahul (1999), in which the individual

[1] This linear form prevents us from considering simultaneously the negative effects of droughts and floods on individual yields, but they can be examined separately through an appropriate definition of the realizations of the weather index.

yield is the sum of a systemic component and an independent farm-specific component. It also differs from a decomposition where both sources of risk are stochastically independent and interact in a non-linear manner (Ramaswami and Roe; Mahul 2000).

The Design of a First-Best Insurance Contract against a Climatic Risk

The risk-averse producer is endowed with a non-random initial wealth π_0 and the stochastic production function equation (1).[2] Input and output prices are assumed to be non-random. The price of one unit of input is denoted p and the output price is normalized to unity without loss of generality. Therefore, the producer faces only production uncertainty. He can purchase an insurance contract that provides protection against a specific climatic experience, such as droughts or frosts. Nevertheless, other sources of production risk are uninsurable and he has to bear their consequences. We focus in this section on the design of an optimal insurance policy against a weather index in this context of incomplete markets, assuming the input level of the producer is fixed. The impact of the insurance contract on the level of input use will be examined in a subsequent section.

The insurance contract against a specific weather event is described by a couple $[I(\cdot), P]$, where P is the insurance premium and $I(\omega)$ is the indemnity payments received by the policyholder if the insurer observes the realized weather index ω. A feasible coverage function satisfies

$$(5) \qquad I(\omega) \geq 0 \text{ for all } \omega.$$

The risk-averse producer with utility function u, where $u' > 0$ and $u'' < 0$, maximizes the expected utility of final wealth π. He purchases the insurance contract $[I(\cdot), P]$ if his expected utility level is greater with this policy than without it

$$(6) \qquad Eu\big(\pi_0' + \tilde{y} + I(\tilde{\omega}) - P\big) \geq Eu(\pi_0' + \tilde{y})$$

where $\pi_0' = \pi_0 - px$.

A risk-averse insurance company with a non-random initial wealth w_0 maximizes the expected value of its utility function ν, with $\nu' > 0$ and $\nu'' \leq 0$. It faces firm-specific costs of risk bearing such as convex tax functions and/or transaction costs associated with bankruptcy, agency costs caused by conflicts between shareholders or information asymmetries between managers and providers of capital (Doherty and Dionne). The transaction cost function $c(\cdot)$ is thus assumed increasing and convex with indemnity payments

$$(7) \qquad c(0) = 0, c'(I) \geq 0 \quad \text{and} \quad c''(I) \geq 0 \quad \text{for all } I.$$

The insurer offers the insurance contract $[I(\cdot), P]$ if and only if

$$(8) \qquad E\nu[w_0 + P - I(\tilde{\omega}) - c\big(I(\tilde{\omega})\big)] \geq \nu(w_0).$$

The participation constraints (6) and (8) define the set of insurance contracts which are acceptable to both parties. This set is assumed not to be empty and therefore that climatic risk $\tilde{\omega}$ is assumed insurable. This hypothesis seems realistic if the weather event is hail. Nevertheless, this insurance scheme may not be viable for other unfavorable weather events, like droughts or extreme temperatures, which affect a large number of farms simultaneously. The high correlation among individual farm-level yields may force the insurer to charge a high risk premium which makes insurance unattractive. Hence, the presence of a strong systemic component in the climatic risk may be responsible for its uninsurability. The presence of systemic risk as a main obstacle of insurability was developed by Quiggin (1994) in the case of rainfall insurance and more recently by Miranda and Glauber. Weather derivatives provided by financial markets should contribute to overcoming this obstacle.

The first-best insurance contract against a specific weather event is a couple $[I(\cdot), P]$ that maximizes the producer's expected utility of final wealth subject to the constraint that indemnity payments are non-negative and that the insurer's expected utility is greater than or equal to a constant:

$$(9) \qquad \max_{I(\cdot),\, P} Eu\big(\pi_0' + \tilde{y} + I(\tilde{\omega}) - P\big)$$

subject to conditions (1), (5) and (8).

The insurance premium P is taken as given and problem (9) is solved via optimal control theory (Raviv). The following proposition states that the optimal insurance contract

[2] An equivalent approach is to assume that the producer is risk neutral but is motivated to insure by market imperfections (Greenwald and Stiglitz). He thus behaves "as if" he were risk averse.

design depends on the stochastic dependence between insurable and uninsurable risks and on the producer's attitude towards risk.

PROPOSITION 1. *If one of the following assumptions are satisfied*:

(i) *the producer is risk averse and a decrease in the insurable weather index makes the uninsurable aggregate production variable riskier according to equation* (2):
(ii) *the risk-averse producer is prudent* $[u''' > 0]$ *and a decrease in the insurable weather index makes the uninsurable aggregate production variable riskier according to* (3);

then a trigger weather index $\hat{\omega} \in [\omega_{\min}, \omega_{\max}]$ *exists such that the optimal indemnity, when the premium and the level of input use are fixed, takes the form*

$$(10)\quad I^*(\omega)\begin{cases} = 0 & \text{if } \omega \geq \hat{\omega} \\ > 0 & \text{if } \omega < \hat{\omega}. \end{cases}$$

When $I^*(\omega) > 0$, *the marginal coverage satisfies*

$$(11)\quad I^{*\prime}(\omega) = \left\{ g(x)\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi)d\Psi\big(\varepsilon/\tilde{\omega}=\omega\big) + \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'(\pi)d\Psi_{\omega}\big(\varepsilon/\tilde{\omega}=\omega\big)\right\}\frac{1}{\Sigma} < 0,$$

with

$$\Sigma \equiv -\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi)d\Psi(\varepsilon/\tilde{\omega}=\omega) + \left[\frac{c''}{1+c'} + (1+c')A_{\nu}(w)\right] \times \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'(\pi)d\Psi(\varepsilon/\tilde{\omega}=\omega) > 0.$$

where $\pi = \pi_0' + g(x)\omega + k(x)\varepsilon + h(x) + I(\omega) - P$, $w = w_0 + P - I(\omega) - c\big(I(\omega)\big)$, c' *and* c'' *are evaluated at* $I^*(\omega)$, *and* $A_{\nu} \equiv -v''/v'$ *is the insurer's index of absolute risk aversion.*

The proof of this proposition is in the appendix. If the insurable climatic risk and the uninsurable aggregate production shock are independent, i.e., $\Psi_{\omega}(\varepsilon/\tilde{\omega} = \omega) = 0$ for all ε and ω, then equation (2) holds. Therefore, the design of an optimal insurance contract against a climatic experience in the presence of an independent background risk contains a trigger level such that indemnity payments are made if the realized weather index falls below this trigger level, as shown by Mahul (1999). When the two sources of risk are correlated, however, the indemnity schedule can take basically any form without further restrictions on stochastic dependence and on the producer's behavior. If a decrease in ω induces a riskier conditional distribution of $\tilde{\varepsilon}$ in the sense of FSD, then the first-best insurance design contains a trigger level under which indemnity payments are made. Risk aversion is not sufficient to characterize the first-best indemnity schedule if the cumulative distribution of $\tilde{\varepsilon}$ conditional on $\tilde{\omega} = \omega$ becomes riskier in the sense of SSD as ω decreases. The producer must also exhibit a convex marginal utility function, i.e., $u''' > 0$, which is a well-known condition introduced by Leland. Kimball uses "prudent" to characterize agents who behave this way and the following economic interpretation can be offered: prudence is the propensity to prepare and forearm oneself in the face of uncertainty, in contrast to risk aversion which is how much one dislikes uncertainty and would turn away from uncertainty if one could. Hence, in the intertemporal model of saving under uncertainty, prudence represents the intensity of the precautionary saving motive. It is also a necessary condition for decreasing absolute risk aversion. The concept of prudence has been recently stressed by Gollier to examine optimal insurance contracts when the indemnity is contingent only on an imperfect signal of the final wealth of the policyholder.

The optimal marginal coverage expressed in equation (11) is the sum of two terms weighted by a third. The first RHS term in curly brackets is the direct effect of the weather index on output. The second one is the indirect effect of the weather index on production through the stochastic dependence between sources of production risk. Finally, the denominator Σ is positive and greater than $\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi)d\Psi(\varepsilon/\tilde{\omega} = \omega)$ because of the insurer's risk aversion and its convex transaction cost function. The second RHS term in curly brackets is equal to zero when the insurable weather index and the uninsurable aggregate production shock are stochastically independent and thus the optimal marginal coverage under the trigger index is proportional to the direct marginal productivity of the weather index $g(x)$.

Two special cases of first- and second-order stochastic dependence are examined. First, if

$\tilde{\varepsilon}$ is a positive linear function of $\tilde{\omega}$:

$$(12) \quad \tilde{\varepsilon} = \alpha + \beta\tilde{\omega} + \tilde{e} \text{ with } \beta > 0$$

where $\tilde{e}$ and $\tilde{\omega}$ are stochastically independent and $E\tilde{e} = 0$, then the cumulative distribution of $\tilde{e}$ conditional on $\tilde{\omega} = \omega$, $\Psi(\varepsilon/\tilde{\omega} = \omega)$, increases as ω decreases. This positive linear relationship between an insurable risk and an uninsurable background risk, which has been recently analyzed by Mahul (1999) in the context of area yield insurance, is a particular case of an increase in risk in the sense of FSD. Second, if the aggregate production shock is inversely proportional to the weather index

$$(13) \quad \tilde{\varepsilon} = \frac{\tilde{e}}{\tilde{\omega}}$$

where $\tilde{e}$ and $\tilde{\omega}$ are stochastically independent and $E\tilde{e} = 0$, then $\Psi_\omega(\varepsilon/\tilde{\omega} = \omega)$ is negative for all $\varepsilon \leq 0$ and positive otherwise. Because $E[\tilde{\varepsilon}/\tilde{\omega} = \omega] = [E\tilde{e}]/w = 0$, this stochastic relationship is a particular case of an increase in risk according to equation (3).

The following proposition, proven in the appendix, defines the effect of transaction costs on the trigger weather index selected by the producer.

Proposition 2. *Under the same assumptions as in proposition 1, the optimal trigger weather index satisfies* $\hat{\omega} = \omega_{\max}$ *if insurance is sold at an actuarially fair price* $[c'(I) = 0$ *for all* $I]$, *and it satisfies* $\hat{\omega} < \omega_{\max}$ *otherwise* $[c'(I) > 0$ *for some* $I]$.

Rational Weather Insurance Purchasing Decisions

Area yield crop insurance (AYCI) contracts have been recently proposed to U.S. farmers. These are insurance programs where the indemnity schedule is not based on a producer's individual yield but rather on an index that is not affected by individual decisions. As a consequence, moral hazard and adverse selection are essentially eliminated and administration costs are substantially reduced (Miranda; Mahul 1999). Another alternative contract is an insurance policy in which the indemnity depends on a weather index. It is called a weather insurance contract and is described by a couple $[I^c(\cdot), P^c]$ where

$$(14) \quad I^c(\omega) = \phi \max[\hat{\omega} - \omega, 0]$$

is the indemnity and P^c is the premium. The producer selects a trigger weather index $\hat{\omega}$ such that indemnity payments are made if the realized weather index falls below it, and a coverage level $\phi \geq 0$.

Throughout this section, we assume that the insurer in the competitive insurance market is risk neutral and his administrative cost function is linear. The insurance premium is thus proportional to the expected indemnity. Under this common assumption of proportional loading, the optimal marginal indemnity function expressed in equation (11) becomes

$$(15) \quad I^{*\prime}(\omega) = -g(x) + \frac{\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'(\pi) d\Psi_\omega(\varepsilon/\tilde{\omega} = \omega)}{-\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi) d\Psi(\varepsilon/\tilde{\omega} = \omega)}$$

for all $\omega < \hat{\omega}$. The slope of the first-best insurance policy is thus equal to the negative of the direct marginal productivity of the insurable weather index plus a ratio that depends on the stochastic dependence between both sources of uncertainty and on the producer's preferences.

The first-best insurance contract expressed by proposition 1 with equation (11) replaced by equation (15) is used to investigate the producer's rational weather insurance purchasing decisions, and especially the optimal coverage level. This allows us to examine how the producer's optimal coverage when the random weather index and the background risk are stochastically dependent differs from the optimal decision under independent risks.

The indemnity schedule I^c of the weather insurance contract is restricted to be piecewise linear in the insurable weather index ω while the marginal coverage of the first-best solution (15) is usually non-linear in ω. An exception is when the random weather variable and the aggregate production shock are stochastically independent. The first-best solution can thus be perfectly replicated with the weather insurance contract: the optimal coverage level ϕ^* is equal to the direct marginal productivity $g(x)$. This specific case corresponds to the result derived by Mahul (1999). Under stochastic dependence between both sources of production risk, the non-linearity of the optimal coverage (15) precludes replication of the first-best solution with the weather insurance contract. This creates a second source of incompleteness in the crop insurance market, in addition to the presence of an uninsurable background risk.

Integrating the numerator of the RHS ratio in equation (15) by parts, once and twice, yields for all $\omega < \hat{\omega}$, respectively:

$$\text{(16a)} \quad I^{*\prime}(\omega) = -g(x) + k(x) \times \frac{\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi)\Psi_\omega(\varepsilon/\tilde{\omega} = \omega)\,d\varepsilon}{\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi)\,d\Psi(\varepsilon/\tilde{\omega} = \omega)}$$

and

$$\text{(16b)} \quad I^{*\prime}(\omega) = -g(x) + k(x)u''(\pi_{\max}) \times \frac{\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \Psi_\omega(s/\tilde{\omega} = \omega)\,ds}{\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi)\,d\Psi(s/\tilde{\omega} = \omega)\,ds} + k^2(x) \times \frac{\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'''(\pi)\left[\int_{\varepsilon_{\min}}^{\varepsilon} \Psi_\omega(s/\tilde{\omega} = \omega)\,ds\right]d\varepsilon}{-\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi)\,d\Psi(s/\tilde{\omega} = \omega)\,ds}$$

where $\pi_{\max} = \pi_0' + g(x)\omega + k(x)\varepsilon_{\max} + h(x) + I^*(\omega) - P$. The ratio in equation (16a) is negative if the producer is risk averse and if a decrease in ω induces a riskier conditional distribution of $\tilde{\varepsilon}$ in the sense of FSD. Both RHS ratios in equation (16b) are also negative if the risk-averse producer is prudent and if a decrease in ω induces a riskier conditional distribution of $\tilde{\varepsilon}$ in the sense of SSD. This implies that the first-best marginal coverage is, in absolute value, higher than the direct marginal productivity $g(x)$ for all $\omega < \hat{\omega}$. However, the producer is not able to replicate this first-best solution with a piecewise linear indemnity schedule I^c. Because $I^{*\prime}(x) < -g(x)$ for all $\omega < \hat{\omega}$, he will select $-\phi = I^{c\prime}(\omega) < -g(x)$ for all $\omega < \hat{\omega}$ in order to replicate as close as possible the first-best solution. Comparing the slope of the first-best indemnity schedule and of the weather insurance contract yields the following corollary.

COROLLARY 1. *If the insurance premium is proportional to the expected indemnity and one of the following assumptions is satisfied*:

(i) *the producer is risk averse and a decrease in the insurable weather index makes the uninsurable aggregate production variable riskier according to equation* (2);
(ii) *the risk-averse producer is prudent and a decrease in the insurable weather index makes the uninsurable aggregate production variable riskier according to equation* (3);

then the optimal coverage level of the weather insurance contract is higher than the direct marginal productivity of the insurable weather variable: $\phi^* > g(x)$.

When both sources of production uncertainty are correlated, the risk-averse and prudent producer hedges against the uninsurable aggregate production shock by selecting a coverage level higher than the direct marginal productivity of the weather index. Furthermore, if $\tilde{\varepsilon}$ depends on $\tilde{\omega}$ according to the positive linear relationship (12), then one can show that the coverage level selected by the risk-averse producer is $\phi^* = g(x) + \beta k(x)$, with $\beta > 0$. It is thus equal to the sum of the direct and indirect marginal productivity of the weather index. The existence of stochastic dependence between both sources of production risk expressed by equation (2) or (3) thus induces the risk-averse and prudent producer to select a coverage level higher than the one chosen under stochastic independence.

Until now, we have assumed that the aggregate production shock $\tilde{\varepsilon}$ becomes riskier as the realized weather variable ω decreases, according to equation (2) or (3). Nevertheless, it could also be realistic to assume that $\tilde{\varepsilon}$ becomes riskier as ω increases. For example, an increase in the cumulative rainfall level could be in favor of the development of disease and insect infestation. Formally, we define the following increases in risk according to the first-order stochastic dominance:

$$\text{(17)} \quad \Psi_\omega(\varepsilon/\tilde{\omega} = \omega) \geq 0 \quad \text{for all } \varepsilon \text{ and } \omega,$$

and according to the second-order stochastic dominance:

$$\text{(18)} \quad \int_{\varepsilon_{\min}}^{\varepsilon} \Psi_\omega(s/\tilde{\omega} = \omega)\,ds \geq 0 \text{ for all } \varepsilon \text{ and } \omega.$$

Without additional restrictions, the indemnity function can basically take any form. This is due to the fact that the direct and indirect marginal productivity of the weather index ω have opposite effects on output: a decrease in ω reduces the direct productivity of ω but, at the same time, increases the expected productivity of $\tilde{\varepsilon}$ conditional on ω and decreases its riskiness. This negative correlation between sources of risk tends to lessen the impact of a low weather index on production. The design of a first-best indemnity schedule is thus indeterminate.[3] This differs from the previous case where a decrease in ω decreases its direct productivity while it decreases the conditional expectation of the productivity of $\tilde{\varepsilon}$ and increases its riskiness.

[3] Following the proof of proposition 1, the first RHS term of K' expressed in equation (A5) is negative, whereas its second term is positive under equation (17) or (18).

To overcome this indeterminacy, we examine specific increases in risk according to equations (17) and (18). First, if $\tilde{\varepsilon}$ is a negative linear function of $\tilde{\omega}$:

(19) $\tilde{\varepsilon}=\alpha+\beta\tilde{\omega}+\tilde{e}$ with $\beta<0$

where $\tilde{e}$ and $\tilde{\omega}$ are stochastically independent and $E\tilde{e}=0$, then this increase in risk satisfies equation (17). It is straightforward in this case to show that $\phi^*=g(x)+\beta k(x)<g(x)$ if $0>\beta>-k(x)/g(x)$. If $\beta\leq-k(x)/g(x)$, the producer does not purchase the weather insurance policy, i.e., $\phi^*=0$. Second, assume that the aggregate production shock is proportional to the weather index

(20a) $\tilde{\varepsilon}=\tilde{\omega}\tilde{e}$

where $\tilde{e}$ and $\tilde{\omega}$ are stochastically independent and $E\tilde{e}=0$, and

(20b) $g(x)+k(x)e\geq0$

for all e. This means that yield always increases with the insurable weather index whatever the realization of the aggregate production variable. One can easily show that this increase in risk satisfies equation (18). The stochastic production function then becomes

(21) $\tilde{y}=[g(x)+k(x)\tilde{e}]\tilde{\omega}+h(x)$.

Using the same arguments as those developed in the proof of proposition 1, one can show that $I^{*\prime}(\omega)>-g(x)$ for all $\omega<\hat{\omega}$.[4] This implies that $I^{c\prime}(\omega)=-\phi>-g(x)$ for all $\omega<\hat{\omega}$. This discussion on the effect of stochastic dependence on the optimal coverage level is summarized in the following corollary.

COROLLARY 2. *If the insurance premium is proportional to the expected indemnity and one of the following assumptions is satisfied*:

(i) *the producer is risk averse and an increase in the insurable weather index makes the uninsurable aggregate production variable riskier according to equation* (19);
(ii) *the risk-averse producer is prudent and an increase in the insurable weather index makes the uninsurable aggregate production variable riskier according to equation* (20);

then the optimal coverage level of the weather insurance contract is lower than the direct marginal productivity of the insurable weather index, $0\leq\phi^*<g(x)$.

Therefore, the risk-averse and prudent producer responds to the presence of an uninsurable and dependent background risk, where the stochastic dependence is expressed in equation (19) or (20), by choosing a coverage level lower than the optimal one under stochastic independence. This corollary is not exactly the reverse of corollary 1 because the former is derived under specific forms of first- and second-order stochastic dominance. As explained previously, inverting the inequalities in equations (2) and (3) leads to an ambiguous form of the first-best solution and thus of the optimal coverage level. This can be overcome with additional assumptions such as those expressed in equation (19) or (20).

Input Response to Weather Insurance Contract

Until now, input use has been assumed fixed. In this section, we examine how buying insurance alters the input level. We focus on the simplest case where the input use affects the output level only through the insurable weather index.[5] The stochastic production function is thus assumed to be

(22) $\tilde{y}=g(x)\tilde{\omega}+h(x)+\tilde{\varepsilon}$,

where $\tilde{\omega}$ and $\tilde{\varepsilon}$ are independent.[6] The input is either risk-decreasing like pesticides, or risk-increasing like fertilizers, depending on whether the productivity is higher or lower in more adverse states of nature. This implies that the marginal productivity of the input $g'(\cdot)$ is negative or positive, respectively.

For sufficiently highly fixed values of the trigger weather index $\hat{\omega}$, the optimal coverage

[4] Under assumptions (20), the function K' expressed in equation (A5) is negative for all ω. Therefore, the optimal insurance contract is such that indemnity payments are made whenever the realized index is lower than a trigger index.

[5] It must be recognized that a more general formulation in which the input level would also affect the uninsurable background risk leads to ambiguous results.

[6] If $\tilde{\omega}$ and $\tilde{\varepsilon}$ are correlated, one can project orthogonally the aggregate production risk $\tilde{\varepsilon}$ onto the random weather variable $\tilde{\omega}$:

$$\tilde{\varepsilon}=\beta(\tilde{\omega}-\mu)+\tilde{e}$$

where $\beta=\mathrm{cov}(\tilde{\varepsilon},\tilde{\omega})/\mathrm{var}(\tilde{\omega}), E\tilde{e}=0, \tilde{\omega}$ and $\tilde{e}$ are assumed to be independent. The stochastic production function can thus be rewritten as $\tilde{y}=[g(x)+\beta]\tilde{\omega}+[h(x)-\beta\mu]+\tilde{e}$.

ϕ^* is equal to $g(x)$.[7] The indemnity schedule of the weather insurance contract is thus

$$(23)\quad I^c(\omega) = g(x)J(\omega) \text{ with } J(\omega) = \max(\hat{\omega} - \omega, 0).$$

The insurance contract is assumed actuarially fair, i.e., $P^c = EI^c(\tilde{\omega}) = g(x)Q$ with $Q = EJ(\tilde{\omega})$. Notice that the insurance premium P^c depends on input use of the producer, contrary to the premium associated with the standard insurance contract based on individual yield (Ramaswami).

If weather insurance is not available, the optimal choice of x is given by

$$(24)\quad \max_x U(x,0,0) = E\hat{u}\big(\tilde{\pi}_{ni}(x)\big),$$

where $\pi_{ni}(x) = \pi_0 + g(x)\omega + h(x) - px$ and the indirect utility function is defined as

$$(25)\quad \hat{u}(\pi) = Eu(\pi + \tilde{\varepsilon}) \text{ for all } \pi.$$

The optimal level of input use x^{ni} satisfies the first-order necessary condition:

$$(26)\quad U_x\big(x^{ni},0,0\big) = g'(x^{ni})\mu + h'(x^{ni}) - p - \left\{ -g'\big(x^{ni}\big)\frac{\text{cov}\Big[\tilde{\omega}, \hat{u}'\big(\tilde{\pi}_{ni}\big(x^{ni}\big)\big)\Big]}{E\hat{u}'\big(\tilde{\pi}_{ni}\big(x^{ni}\big)\big)} \right\} = 0,$$

and the second-order condition is satisfied if $g(\cdot)$ and $h(\cdot)$ are concave in x and the producer is risk averse. The RHS term in curly brackets in the above equation is the marginal risk premium of the input when there is no insurance contract. It is zero if the producer is risk neutral. Under risk aversion it is positive (negative) if the input is risk-increasing (risk-decreasing).

When the weather insurance program $[I^c, P^c]$ is available, the producer maximizes the following objective function

$$(27)\quad U\big(x, I^c, P^c\big) = E\hat{u}\big(\tilde{\pi}_i(x)\big),$$

where $\pi_i(x) = \pi_{ni}(x) + I^c(\omega) - P^c = \pi_{ni}(x) + g(x)[J(\omega) - Q]$.

The optimal level of input use x^i satisfies

$$(28)\quad U_x\big(x^i, I, P\big) = g'(x^i)\mu + h'(x^i) - p + g'(x^i)\frac{\text{cov}[\tilde{\omega}, \hat{u}'\big(\tilde{\pi}_i\big(x^i\big)\big)]}{E\hat{u}'\big(\tilde{\pi}_i\big(x^i\big)\big)} - g'(x^i)\frac{\text{cov}[J(\tilde{\omega}), \hat{u}'\big(\tilde{\pi}_i\big(x^i\big)\big)]}{E\hat{u}'\big(\tilde{\pi}_i\big(x^i\big)\big)} = 0,$$

where $E[(J(\tilde{\omega}) - Q)\hat{u}'(\tilde{\pi}_i(x^i))] = \text{cov}[J(\tilde{\omega}), \hat{u}'(\tilde{\pi}_i(x^i))]$ under actuarially fair insurance. Compared with equation (26), there is an additional term represented by the last term in equation (28), characterizing the impact of the actuarially fair weather insurance policy on the producer's input decision. Because the objective function $U(x,0,0)$ is concave in x, then $x^i \leq (\geq) x^{ni}$ as $U_x(x^i,0,0) \geq (\leq) 0$. From equations (26) and (28), this inequality is rewritten as

$$(29a)\quad g'(x^i)\text{cov}[\tilde{\omega}, T\big(x^i, \tilde{\omega}\big)] + g'(x^i)\frac{\text{cov}[J(\tilde{\omega}), \hat{u}'\big(\tilde{\pi}_i\big(x^i\big)\big)]}{-E\hat{u}'\big(\tilde{\pi}_i\big(x^i\big)\big)} \geq (\leq) 0,$$

where

$$(29b)\quad T(x,\omega) = \frac{\hat{u}'(\tilde{\pi}_{ni}(x))}{E\hat{u}'(\tilde{\pi}_{ni}(x))} - \frac{\hat{u}'(\tilde{\pi}_i(x))}{E\hat{u}'(\tilde{\pi}_i(x))}.$$

The first term in the above inequality represents the difference between the marginal risk premiums of the input with and without the weather insurance contract. It can be shown (see appendix) that $\text{cov}[\tilde{\omega}, T(x,\tilde{\omega})]$ is non-positive for all x if the insurance premium is actuarially fair and if the risk-averse producer exhibits prudence. The first term in equation (29a) is thus positive or negative depending on whether the input is risk-decreasing or risk-increasing. The second covariance term in equation (29a) depends on the shape of J and $\hat{u}'(\tilde{\pi}_i)$ with respect to ω. The indemnity function J is non-increasing with ω and the profit function π_i is non-decreasing with ω. Because the producer's risk aversion implies that the indirect utility function $\hat{u}$ is concave (Kihlstrom, Romer and Williams), we deduce that $\text{cov}[J(\tilde{\omega}), \hat{u}'(\tilde{\pi}_i(x))]$ is positive. The second term in equation (29a) is thus positive (negative) if the input is risk-decreasing (risk-increasing).

It follows that the purchase of actuarially fair weather insurance induces the risk-averse and prudent producer to (i) increase his level

[7] This assumption on $\hat{\omega}$ is realistic because weather insurance does not generate moral hazard and therefore there is no reason to introduce a large deductible to encourage the producer to self-protect. If $\hat{\omega}$ was too low, the producer may be induced to choose a coverage level which could be different from $g(x)$.

of risk-increasing input use and (ii) decrease his level of risk-decreasing input use.

Ramaswami examines how the purchase of a multiple peril crop insurance contract, in which indemnity payments are based on the producer's individual yield, alters the input decision. He shows that under non-increasing absolute risk aversion, the impact of actuarially fair crop insurance on input use is to reduce usage if the input is risk-decreasing but the effect is indeterminate if the input is risk-increasing. This is the consequence of two effects. First, the risk-reduction effect induces the producer to increase his exposure towards risk by reducing (increasing) his level of risk-decreasing (risk-increasing) input use. The second effect is due to ex ante moral hazard: the purchase of insurance alters input decisions and thus the distribution of the individual yields. These changes cannot be observed by insurers and, consequently, they cannot be taken into account in the calculation of the insurance premium. This moral hazard effect leads the producer to reduce his use of both risk-increasing and risk-decreasing inputs. Under the weather insurance policy examined here, and more generally when the indemnity function is based on exogenous variables beyond the producer's control, changes in input use affect the insurance premium and therefore ex ante moral hazard does not exist. The impact of insurance purchase on the level of input use is thus unambiguous, even if the input is risk-increasing.

If the actuarially fair weather insurance contract provides complete coverage against the climatic risk, i.e., the trigger weather variable $\hat{\omega}$ is equal to $\omega_{\max}$ which implies that $J(\omega)-Q=\mu-\omega$, then both ratios in equation (28) equal zero. Therefore, the producer adopts a risk-neutral attitude towards the insurable weather event by selecting a level of input use so that the expected marginal productivity of the input equals its marginal cost. The insured risk-averse producer thus responds to the introduction of this insurance policy by increasing or decreasing input use depending on whether it is risk-increasing or risk-decreasing. The assumption of prudence is not necessary in this specific case of global risk reduction. This result differs from the impact of complete coverage offered by the multiple peril crop insurance contract where the moral hazard effect does not provide any incentive for positive level of input use (Ramaswami).

Buying the actuarially fair weather insurance contract generates a mean-preserving reduction in climatic risk. Leathers and Quiggin have shown that such a risk reduction induces the producer to increase his exposure towards risk if his utility function exhibits non-increasing absolute risk aversion. Our result can thus be viewed as slightly more general because the same finding is obtained under prudence. Following the same approach as the one used under a weather insurance contract, it is also straightforward to show that the result obtained by Ramaswami holds under slightly weaker assumptions of risk aversion and prudence.

Conclusion

Recent developments in the theory of insurance under incomplete markets have been used to design an optimal insurance contract against a specific weather event, like droughts or extreme temperatures, in the presence of an additive and dependent background risk. The first-best indemnity schedule of this insurance policy has been shown to depend on the stochastic dependence between the insurable climatic risk and the uninsurable aggregate production shock and on individual behavior towards risk. It contains a trigger weather index under which indemnity payments are made if (i) the producer is risk averse and the aggregate production variable becomes riskier according to the first-order stochastic dominance as the weather index decreases or (ii) the risk-averse producer exhibits prudence and the uninsurable aggregate production variable becomes riskier according to the second-order stochastic dominance as the weather index decreases. The optimal coverage level of the weather insurance program equals the direct marginal productivity of the weather index if sources of production uncertainty are stochastically independent. It is higher than this direct marginal productivity if the aggregate production variable becomes riskier according to second-order stochastic dominance as the weather index decreases and if the producer is prudent. When the aggregate production shock becomes riskier as the weather index increases, according to the first- or second-order stochastic dominance, the optimal form of the weather insurance contract is ambiguous because a decrease in the weather index increases the conditional expectation of the background risk and reduces its riskiness. Under some additional assumptions, however, such as a production function where both risks interact in a multiplicative manner, the optimal coverage level of the prudent producer is positive and lower than the direct marginal productivity. Therefore, the existence of a stochastic dependence between the insurable climatic risk and the uninsurable background risk induces the producer to choose a coverage level different from the optimal one under independence.

The level of input use has been shown to be altered by the presence of a weather insurance policy. When the input decision affects only the direct marginal productivity of the insurable climatic risk, the risk-averse and prudent producer responds to the introduction of an actuarially fair weather insurance contract by increasing his exposure towards risk, i.e., by reducing his level of risk-decreasing input and/or by increasing his level of risk-increasing input. Therefore, the behavioral concept of prudence has turned out to be sufficient to derive unambiguous effects of an exogenous decrease in production risk, holding expected production constant on the level of input use. Previous results obtained by Leathers and Quiggin and by Ramaswami are found to hold under the condition of prudence.

Due to the similarity between financial instruments and insurance contracts, the optimal weather insurance purchasing decisions can be easily re-interpreted to analyze the optimal hedging strategy with weather derivatives provided by financial markets.

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Appendix

Proof of Proposition 1

The objective function of equation (9) can be rewritten as

$$\text{(A1)} \quad \int_{(t)_{\min}}^{(t)_{\max}} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u(\pi_0' + g(x)\omega + k(x)\varepsilon + h(x) + I(\omega) - P)\, dT(\omega, \varepsilon),$$

where $dT(\omega,\varepsilon) = d\Psi(\varepsilon/\tilde{\omega}=\omega)d\Phi(\omega)$. The Hamiltonian of the maximization problem is

$$\text{(A2)} \quad H = \left\{ \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u(\pi)\, d\Psi(\varepsilon/\tilde{\omega}=\omega) + \zeta v(w) \right\} d\Phi(\omega),$$

where $\pi = \pi_0' + g(x)\omega + k(x)\varepsilon + h(x) + I(\omega) - P$ and $w = w_0 + P - I(\omega) - c(I(\omega))$. As already shown by Raviv, the multiplier function ζ is constant with respect to ω. The first-order necessary conditions are

$$\text{(A3)} \quad I^*(\omega) = 0 \text{ if } K(\omega) \equiv \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'(\pi_1) \times d\Psi(\varepsilon/\tilde{\omega}=\omega) - \zeta[1 + c'(0)] \times v'(w_0 + P) \leq 0,$$

where $\pi_1 = \pi_0' + g(x)\omega + h(x) - P + k(x)\varepsilon$ and

$$\text{(A4)} \quad I^*(\omega) > 0 \text{ if } \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'(\pi)\, d\Psi(\varepsilon/\tilde{\omega}=\omega) - \zeta[1 + c'(I^*(\omega))]v'(w) = 0.$$

The first derivative of K with respect to ω is:

$$\text{(A5)} \quad K'(\omega) = g(x)\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi_1)\, d\Psi(\varepsilon/\tilde{\omega}=\omega) + \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'(\pi_1)\, d\Psi_\omega(\varepsilon/\tilde{\omega}=\omega).$$

The first RHS term in (A5) is negative because the producer is risk averse and $g(\cdot)$ is a positive function. Integrating the second RHS term in (A5) yields:

$$\text{(A6)} \quad \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'(\pi_1)\, d\Psi_\omega(\varepsilon/\tilde{\omega}=\omega) = -k(x)\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi_1)\Psi_\omega(\varepsilon/\tilde{\omega}=\omega)\, d\varepsilon$$

with $\Psi_\omega(\varepsilon_{\min}/\tilde{\omega}=\omega) = \Psi(\varepsilon_{\max}/\tilde{\omega}=\omega) = 0$. Since we have $k(\cdot) > 0$, the above expression is negative if the producer is risk averse and if equation (2) is satisfied. Third, integrating again the RHS term of equation (A6) yields:

$$\text{(A7)} \quad \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'(\pi_1)\, d\Psi_\omega(\varepsilon/\tilde{\omega}=\omega) = -k(x)u''(\pi_{1\max}) \times \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \Psi_\omega(s/\tilde{\omega}=\omega)\, ds + k^2(x) \times \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'''(\pi_1)\left[\int_{\varepsilon_{\min}}^{\varepsilon} \Psi_\omega(s/\tilde{\omega}=\omega)\, ds\right] d\varepsilon$$

where $\pi_{1\max} = \pi_0' + g(x)\omega + h(x) - P + k(x)\varepsilon_{\max}$. The above expression is negative if $u'' < 0, u''' > 0$ and equation (3) is satisfied.

Consequently, K decreases with ω if the producer is risk averse, i.e., $u'' < 0$, and if a decrease in ω induces a riskier conditional distribution of $\tilde{\varepsilon}$ in the sense of FSD. It also decreases if the risk-averse producer is prudent, i.e., $u''' > 0$, and if a decrease in ω induces a riskier conditional distribution of $\tilde{\varepsilon}$ in the sense of SSD. If $K(\omega_{\min}) > 0 \geq K(\omega_{\max})$, then a unique value $\hat{\omega} \in [\omega_{\min}, \omega_{\max}]$ exists such that $K(\hat{\omega}) = 0$. The optimal form of the insurance contract expressed in equation (10) is thus derived. If $K(\omega_{\min}) \leq 0$, then the non-negativity constraint (5) is always binding. This implies that $I(\omega) = 0$ for all $\omega \in [\omega_{\min}, \omega_{\max}]$ or, equivalently, $\hat{\omega} = \omega_{\min}$ in equation (10). If $K(\omega_{\max}) > 0$, then constraint (5) is never binding and therefore $I(\omega) > 0$ for all $\omega[\omega_{\min}, \omega_{\max}]$. In other words, $\hat{\omega} = \omega_{\max}$ in equation (10).

Differentiating equation (A4) with respect to ω: $I^*(\omega) > 0$ yields:

$$\text{(A8)} \quad \left[g(x) + I'(\omega)\right]\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi)\, d\Psi(\epsilon/\tilde{\omega}=\omega) \times (\varepsilon/\tilde{\omega}=\omega) + \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'(\pi)\, d\Psi(\epsilon/\tilde{\omega}=\omega) - \zeta I'(\omega)c''v'(w) + \zeta I'(\omega)(1+c')^2 v''(w) = 0$$

where c' and c'' are evaluated at $I(\omega)$. Replacing the multiplier function ζ by its expression in (A4) and rearranging the terms yield equation (11).

Finally, notice that $I^*(x)$ also satisfies the sufficient condition because the Hamiltonian does not depend on the state variable (Kamien and Schwartz).

Proof of Proposition 2

Let the function $\lambda(\omega)$ be such that

$$\text{(A9)} \quad \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'(\pi)\, d\Psi(\varepsilon/\tilde{\omega}=\omega) - \zeta[1 + c'(I^*(\omega))]v'(w) + \lambda(\omega) = 0$$

for all $\omega \in [\omega_{\min}, \omega_{\max}]$. From first-order necessary conditions (A3) and (A4), $\lambda(\omega) = 0$ if $I^*(\omega) > 0$ and $\lambda(\omega) \geq 0$ otherwise. The maximization of problem

(9) with respect to the premium P yields

$$\text{(A10)} \qquad \zeta = \frac{E\{E[u'(\tilde{\pi})/\tilde{\omega} = \omega]\}}{Ev'(\tilde{w})}.$$

Introducing (A10) in equation (A9) and taking the expectation with respect to $\tilde{\omega}$ yields

$$\text{(A11)} \qquad E\lambda(\tilde{\omega}) = \frac{Eu'(\tilde{\pi})}{Ev'(\tilde{w})} E[c'(I^*(\tilde{\omega}))v'(\tilde{\omega})].$$

If $c'(I)=0$ for all I, then $E\lambda(\tilde{\omega})=0$. Since $\lambda(\cdot)$ is a non-negative function, this requires that $\lambda(\omega)=0$ for all $\omega \in [\omega_{\min}, \omega_{\max}]$ and, consequently, $I^*(\omega)>0$ for all $\omega \in [\omega_{\min}, \omega_{\max}]$. This means that $\hat{\omega}=\omega_{\max}$. If $c'(I)>0$ for some I, then we have $E\lambda(\tilde{\omega})=0$. This implies that $I^*(\omega)=0$ for some ω. Since $I^*(\cdot)$ is a decreasing function under risk aversion and a FSD increase in risk, or under risk aversion, prudence and a SSD increase in risk, the trigger weather index $\hat{\omega}$ is lower than $\omega_{\max}$.

Proof of the negativity of $\text{cov}[\tilde{\omega}, T(x, \tilde{\omega})]$

Using the same arguments as those developed by Ramaswami, one can prove the following lemma.

LEMMA. *If* $Q=EJ(\tilde{\omega})$ *and* $u''' \geq 0$, *then there exists* $\omega^* \in [\omega_{\min}, \omega_{\max}]$ *such that*

$$T(x,\omega)(\omega^* - \omega) \geq 0 \text{ for all } \omega \in [\omega_{\min}, \omega_{\max}]$$

where

$$T(x,\omega) = \frac{\hat{u}'(\pi_{ni}(x))}{E\hat{u}'(\tilde{\pi}_{ni}(x))} - \frac{\hat{u}'(\pi_i(x))}{E\hat{u}'(\tilde{\pi}_i(x))},$$
$$\pi_{ni}(x) = \pi_0 + g(x)\omega + h(x) - px \quad \text{and}$$
$$\pi_i(x) = \pi_{ni}(x) + I^c(\omega) - P^c.$$

Proof. The indirect utility function $\hat{u}$ is concave and its marginal utility is convex if the direct utility function u exhibits risk aversion and prudence. Under an actuarially fair weather insurance contract, we deduce from $\hat{u}'''>0$ that

$$\text{(A12)} \qquad E\hat{u}'(\tilde{\pi}_{ni}) \geq E\hat{u}'(\tilde{\pi}_i).$$

Since $J(\omega)=\max[\hat{\omega}-\omega, 0]$ decreases in ω, a trigger level ω_1 exists such that $[J(\omega)-Q](\omega_1-\omega)\geq 0$ where $Q=EJ(\tilde{\omega})$. Consequently, for $\omega \leq (\geq) \omega_1$, we have $\hat{u}'(\pi_{ni}) \geq (\leq)\ \hat{u}'(\pi_i)$ from the concavity of $\hat{u}$. Combining this inequality with (A12) yields

$$\text{(A13)} \qquad T(x,\omega) \leq 0 \text{ for all } \omega \geq \omega_1.$$

Differentiating T with respect to ω and rearranging the terms yield

$$\text{(A14)} \quad \frac{\partial T(x,\omega)}{\partial \omega} = g(x)\left[\frac{\hat{u}''(\pi_{ni})}{E\hat{u}'(\pi_{ni})} - \frac{\hat{u}''(\pi_i)}{E\hat{u}'(\pi_i)}\right] - g(x)J'(\omega)\frac{\hat{u}''(\pi_i)}{E\hat{u}'(\pi_i)}.$$

The second RHS term of (A14) is positive because $\hat{u}$ is concave and J is decreasing. From the assumption of prudence and inequality (A12), the first RHS term is negative for all $\omega \leq \omega_1$. This implies that

$$\text{(A15)} \qquad \frac{\partial T(x,\omega)}{\partial \omega} < 0 \text{ for all } \omega \leq \omega_1.$$

Combining (A13) and (A15) and using the fact that $ET(x,\tilde{\omega})=0$ yield that a trigger level $\omega^* \leq \omega_1$ exists such that $T(x,\omega)(\omega^*-\omega)\geq 0$ for all $\omega \in [\omega_{\min}, \omega_{\max}]$. This proves the lemma.

Ramaswami derives a similar result under the assumption of non-increasing absolute risk aversion. The above assertion is less restrictive because it is obtained under prudence which is a slightly weaker condition than non-increasing absolute risk aversion. We deduce from this lemma that

$$\text{(A16)} \quad \int_{\omega_{\min}}^{\omega^*} \omega T(x,\omega)\, d\Phi(\omega) \leq \omega^* \int_{\omega_{\min}}^{\omega^*} T(x,\omega)\, d\Phi(\omega)$$

and

$$\text{(A17)} \quad \int_{\omega^*}^{\omega_{\max}} \omega T(x,\omega)\, d\Phi(\omega) \leq \omega^* \int_{\omega^*}^{\omega_{\max}} T(x,\omega)\, d\Phi(\omega).$$

Consequently, we have

$$\text{(A18)} \quad \text{cov}(\tilde{\omega}, T(x,\tilde{\omega})) = \int_{\omega_{\min}}^{\omega^*} \omega T(x,\omega)\, d\Phi(\omega) + \int_{\omega^*}^{\omega_{\max}} \omega T(x,\omega)\, d\Phi(\omega) \leq \omega^* ET(x,\tilde{\omega}) = 0.$$