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RESEARCH ARTICLE

Bayesian ratemaking procedure of crop insurance contracts with skewed distribution

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Over the years, crop insurance program became the focus of agricultural policy in U.S., Spain, Mexico, and more recently in Brazil. Given the increasing interest in insurance, accurate calculation of the premium rate is of great importance. We address the crop-yield distribution issue and its implications to pricing an insurance contract considering the dynamic structure of the data and incorporating the spatial correlation in the Hierarchical Bayesian framework. Results show that empirical (insurers) rates are higher in low risk areas and lower in high risk areas. Such methodological improvement is primarily important in situations of limited data.

Keywords: crop insurance; Bayesian hierarchical model; premium rate; Skew-normal distribution; spatial correlation.

1. Introduction

In agricultural economics, crop-yield distributions have been an extensively explored issue. Over the years, the statistical aspects behind this variable have been a controversial point. In order to better reflect the innovation of the crop-yield, three main approaches have been proposed by economists: parametric, semiparametric [17] and nonparametric [12, 18, 22].

Considering the parametric approach, [16] suggested normality of yield distributions. However, other economists like [3], [27], [23], [24] and [1] found evidences against normality. Alternatively, some other parametric methods, with specific functional form and distributional assumptions, are considered in the literature: Beta distribution [4, 6, 7, 13, 20], inverse hyperbolic sine transformations, known as the SU family [19], and gamma distributions [9].

The issue of yield normality or non-normality is of paramount importance in the economic risk analysis and risk management tools. For example, the model selection problem could seriously affect the crop insurance results, and the assumption of non-normality would invalidate the approximations in the expected utility framework, such as those found in the mean-variance analysis.

In this paper we address the issue of crop-yield normality assuming an alternative distributional specification proposed by [2], known as the Skew-normal distribution. After estimating the conditional yield density as an intermediate step, our economic

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and final objective is pricing an insurance contract based on regional crop-yield (area-yield insurance).

The choice of a statistical model that better reflects the conditional density of yields is an important aspect in the actuarial calculation of the premium rate. When recovering the generating process of the data, we must often address issues related to the fact that crop-yield presents substantial trend over time and are significantly correlated over space due to the systemic nature of weather and soil conditions.

The crop-yield follows a spatio-temporal process, in the sense that if we take the average in a region conditional to the temporal process, one can recover the conditional density yield $f(y|\Omega_t)$ at certain moment in time and region, where Ω_t is the minimum σ-algebra generated by the information known at moment t [18].

In several empirical works, the only information known at time t is the time itself. Thus, in previous works the conditional density is based only on the temporal generating process of the data [21]. Our work incorporates to the dynamic structure of the generating process, the spatial effect, according to the concept of adjacent regions through the hierarchical structure of the model resulting in spatio-temporal models. Yet, we assume that the likelihood follows a Skew-normal distribution. All model parameters are estimated through a simulation-based method known as the Markov Chain Monte Carlo (MCMC) algorithm.

[26]considered a different approach based on the Bayes rule to estimate the moments of individual farm-level crop yield distributions. The yield densities are estimated using information theory and maximum entropy. The active learning rule developed reduced the bias and uncertainty of the premium rates. Unlike the approach of [26], our approach takes into account all modeling variability at once and the premium rate is directly estimated through MCMC. Another point of our likelihood distributional assumption is the possibility to estimate the skewness parameter which in the actuarial context is critical because we are interested in the left tail of the distribution. Our approach makes the premium rate calculation less ad hoc, in the sense that rates are derived from a predictive distribution obtained by the Bayesian approach. Moreover, when calculating the rate we are able to capture its model-based uncertainty through a standard error which is estimated within the model.

The paper is organized as follows: in Section 2 we briefly review the Skew-normal distribution and present a general hierarchical model that accounts for temporal, spatio and spatio-temporal autocorrelation of the data generating process. In Section 3 we discuss the problem of rating the crop insurance contract based on the optimal model specification. In Section 4 we describe the Brazilian yield data for corn, in Section 5 we present our empirical findings and discuss their implications, and in Section 6 we conclude the paper.

2. Statistical modelling

2.1. *Skew-normal distribution*

[2] proposed a method for constructing skewed distribution based on any symmetrical ones. Let f be a probability density function (p.d.f.) symmetric about 0, and G an absolute continuous cumulative distribution function (c.d.f.) such that $g = G'$ is symmetric about 0. Then

$$2f(x)G(\theta x), \qquad x \in \mathbb{R}, \tag{1}$$

is a p.d.f. for any $\theta \in \mathbb{R}$. From equation (1), the Skew-normal (SN) distribution, with location parameter μ, scale parameter σ and shape parameter θ, is defined by the following p.d.f.:

$$\phi(x|\theta,\mu,\sigma) = \frac{2}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)\Phi\left(\frac{\theta(x-\mu)}{\sigma}\right), \tag{2}$$

where ϕ and Φ are the p.d.f. and the c.d.f. of a standard normal random variable, respectively. We can measure the degree of skewness of the Skew-normal distribution given in equation (2) by

$$\gamma = \delta^3\left[\frac{4}{\pi}-1\right]\left[1-\frac{2\delta^2}{\pi}\right]^{-3/2}\sqrt{\frac{2}{\pi}} \tag{3}$$

where $\delta = \theta(1+\theta^2)^{-1/2}$ and $-0,99527 < \gamma < 0,99527$ with limiting cases ($\theta \to \pm\infty$) being half-normal distributions. In fact, we use the approach of [14], representing the Skew-normal distribution as a mean-variance mixture of a normal and a half-normal distributions[1].

2.2. *Bayesian hierarchical model*

Let y_{ij} be the agricultural yield in county i in year t, where $i = 1,\dots,S$ and $t = 1,\dots,T$. Conditional on a stochastic location, a scale and a shape parameters, we assume that the observed data follow a SN distribution, such that $y_{it} \sim \mathrm{SN}(\mu_{it},\sigma,\theta_i)$. The objective is to model the stochastic location component capturing possible skewness on the distribution of each county i, so that μ_{it} reflects the temporal effects, spatial variation and spatio-temporal relationships relevant to agricultural yield. Our general hierarchical structure is given by

$$y_{it} \sim \mathrm{SN}(\mu_{it},\sigma,\theta_i) \tag{4}$$

$$\mu_{it} = \sum_{\ell=0}^{m}\beta_{i\ell}t^{\ell} + \sum_{k=1}^{p}\phi_{ik}y_{i,t-k} \tag{5}$$

For each county i, $\beta_i = (\beta_{i1},\dots,\beta_{im})'$ consists of regression coefficients for the deterministic temporal trend, and $\phi_i = (\phi_{i1},\dots,\phi_{ip})'$ consists of autoregressive coefficients for the stochastic temporal trend. For the deterministic trend model, we center the variable t in order to improve the speed of convergence of our MCMC algorithm. Moreover, the spatial correlation can also be modelled in equation (5) considering a conditionally autoregressive (CAR) prior distribution for the parameters of the deterministic trend [5, 8] in the following way:

$$\beta_{i\ell} = \xi_{i\ell} + \omega_\ell,\ \ \xi_\ell = (\xi_{\ell 1},\dots,\xi_{\ell S})' \sim \mathrm{CAR}(\sigma^2_{\xi_\ell}) \text{ and } \omega_\ell \sim \mathrm{N}(\alpha_\ell,\tau^2_\ell). \tag{6}$$

Intuitively, we can think of the parameters as being correlated across space given time. Note that for $\ell = 0$ the spatial correlation is directly related to the location μ_{it}. To complete our model specification we need to define the prior distribution of the parameters. We assume that all parameters are independent *a priori* and we assign known proper distributions with specific hyperparameters to be given in detail in Section 5.

[1] [11] shows more details on multivariate Skew-normal distribution and other skewed distributions.

For selecting the best-fit model, we adopt a criteria based on the predictive density,

$$f(y_{new}|y_{obs}) = \int f(y_{new}|\Theta_M, M)p(\Theta_M|y_{obs}, M)d\Theta_M,$$

where Θ_M represents the set of all parameters in model M, formalized by [10] and known as the expected predictive deviance (EPD). The objective is to minimize the posterior predictive loss function. The penalty is already considered in the criteria regardless the model dimension.

3. Rating the crop insurance contract

The premium rate is a critical parameter of any insurance contract. An inaccurate premium rate results in distortions to the insurance pool and thus may result in losses as individuals adversely select against the insurance provider. In particular, low risk agents may be overcharged and high risk agents may be undercharged. This will distort participation in favor of the higher risks and thus premiums will not be sufficient to cover indemnity payments.

In the literature of insurance economics, this is often also referred to as the "hidden information problem" since agents tend to know more about their risks than does the insurance provider. This condition of adverse selection has been well documented for a number of insurance plans. The eventual failure of an insurance program as a result of such selection is often called the "death spiral of adverse selection."

Optimally, an insurance provider would prefer to calculate individual premium rates for each farmer on the basis of that farmer's risks and expected yields. However, individual yield data are usually rare and thus crop insurance plans are often based upon more aggregate data - such as data at the county level. Such index-based crop insurance plans were developed to overcome the problem of short or nonexistent individual crop yield series.

In Brazil the lack of crop-yield series at farm-level and the short number of observations at county-level challenge the insurance companies to price their insurance contracts. Traditionally, the methods commonly used to pricing crop insurance contracts are based on the relation of the average loss over liability, known as "empirical rates" (ER), and do not take into account any more advanced statistical analysis[1]. One of the main disadvantages of the ER method is its dependence on a large number of observations to accurately reflect the yield distribution. In order to overcome this problem we assume a flexible parametric probability distribution and incorporate the spatial and time correlation in the analysis (see Section 2).

Accurate pricing of crop insurance policies requires accurate estimation of the conditional yield densities. The insurance premium rate, from now on Bayesian Rates (BR), represents expected payouts as a proportion (or percentage) of total liability. In the simple case where a proportion λ $(0 \leqslant \lambda \leqslant 1)$, known as the level of coverage, of the expected crop yield y^e is used to form the basis of insurance, the premium rate is given by:

$$\mathrm{BR} = \frac{F_Y(\lambda y^e)E_Y[\lambda y^e - (Y|y < \lambda y^e)]}{\lambda y^e} = \frac{\int_0^{\lambda y^e}(\lambda y^e - y)P^G f(y)dy}{\lambda y^e P^G} \qquad (7)$$

[1]Information obtained during personal interview with several private insurance companies operating in Brazil.

where E is the expectation operator, P^G is the price at which losses will be paid at, f is the probability density for yields, and F is the cumulative distribution function of yields. In order to derive the premium rate directly from our Bayesian hierarchical model a slightly different derivation of the premium rate is convenient for our purposes. If we reparameterize y, such that, $y^* = y/(\lambda y^e)$, then (7) becomes:

$$\mathrm{BR} = P(y^* < 1)\mathrm{E}_{y^*}[1 - (y^*|y^* < 1)]. \tag{8}$$

Note that the support of the random variable y remains the same in this transformation. If we consider $w = 1 - y^*$, then (8) can be rewritten such that:

$$\mathrm{BR} = P(w > 0)\mathrm{E}_w[w|w > 0]. \tag{9}$$

After some simplification, the premium rate equation reduces to:

$$\mathrm{BR} = \int_0^1 wf(w)dw. \tag{10}$$

We can similarly write (10) as $\mathrm{BR} = E[wI(0 < w < 1)]$. Because of the change of variable, the support also changed such that w now lies between 0 and 1. In our model, we can easily computationally implement (9) using the predicted yields. This expression represents the posterior mean of w, which is the BR calculated for each county and for each level of coverage presented. Moreover, through the Bayesian approach we can derive standard error estimates of the premium rates.

4. Data description

The corn crop-yield data (kilograms per hectare) are provided by the IBGE (Statistical and Geography Brazilian Institute), in the state of Paraná (Brazil), in the period of 1990 trough 2002. The state of Paraná is located in the South Region and is the largest producer of corn in the country, with 9 797 816 tons produced in 2002, approximately 27% of all Brazilian production.

There are 399 counties, where only 290 counties have 13 years of observations (complete series). Consequently, we included only those counties with the largest number of observations. The five largest counties in terms of average yield are Castro (6 142 kg/ha), Ponta Grossa (5 629 kg/ha), Marilândia do Sul (5 488 kg/ha), Tibagi (5 346 kg/ha) and Catanduvas (4 923 kg/ha).

5. Empirical application

As an initial data exploration technique, we use empirical plots and fit some regression models with autoregressive error structure to each county. Hence, we set the maximum value for m equal two and for p equal one. Moreover, we assume that the spatial structure are the same for each regression coefficient ℓ such that $\xi_{i\ell}$ conditional on $\xi_{j\ell}$, $(i \neq j)$, is proportional to:

$$\xi_{i\ell}|\xi_{j\ell} \sim \exp\left\{-\frac{1}{2\sigma^2_{\xi_\ell}}\left(\varphi_{i\ell}\xi_{i\ell} - \sum_{i\neq j}\eta_{i\ell j}\xi_j\right)^2\right\}$$

where $\varphi_{i\ell} \geqslant 0$ is a "sample size" associated with region i and $\eta_{i\ell j}$ is the weight reflecting the influence of $\xi_{j\ell}$ on the conditional mean of ξ_j. We let $\eta_{i\ell j} = 1$ if j is a neighbor of i and 0 otherwise, and set $\varphi_{i\ell}$ equal to the number of neighbors of i.

We fitted the model given by equations (4), (5) and (6) and its particular cases with $m \in \{1, 2\}$ and $p \in \{0, 1\}$, and chose the best-fit model based on the EPD. For each model, we ran three chains to check the mixing of the Markov sequence and to properly analyze the convergence. We fitted, in total, 15 models - including a very simple model with only the deterministic trend and more sophisticated models with $m = 2$ and $p = 1$ in equation 5. From among all the alternatives, the three best-fit competing models, related to the skewness of the distribution of y_{it} and based on EPD, are the following:

- Model 1: $\theta_i = 0$ for $i = 1, \ldots, S$ such that $y_{it} \sim \mathrm{N}(\mu_{it}, \sigma^2)$;
- Model 2: $\theta_i \sim \mathrm{N}(a_\theta, b_\theta)$; and
- Model 3: $\theta_i|\mu_\theta, \sigma_\theta \sim \mathrm{N}(\mu_\theta, \sigma^2_\theta)$, $\mu_\theta \sim \mathrm{N}(a_\theta, b_\theta)$ and $\sigma^2_\theta \sim \mathrm{IG}(c_\theta, d_\theta)$.

Table 1 compares the best-fit model based on EPD for the three models above. The best-fit is Model 3, which with specified prior and hyperparameters is given by

$$y_{it} \sim \mathrm{SN}(\mu_{it}, \sigma, \theta_i), \;\; \mu_{it} = \beta_{0i} + \beta_{1i}(t - 7) + \phi_i y_{i,t-1},$$

$\beta_{0i} \sim \mathrm{N}(300, 10^6)$, $\beta_{1i} = \xi_i + \omega$, $\xi \sim \mathrm{CAR}(\sigma^2_{\beta_1})$, $\sigma^2_{\beta_1} \sim \mathrm{IG}(100/9, 1/90)$, $\omega \sim \mathrm{N}(0, 10^{10})$, $\phi_i \sim \mathrm{N}(\mu_\phi, \sigma^2_\phi)$, $\mu_\phi \sim \mathrm{N}(0, 10)$, $\sigma^2_\phi \sim \mathrm{IG}(0.1, 0.1)$, $\theta_i \sim \mathrm{N}(\mu_\theta, \sigma^2_\theta)$, $\mu_\theta \sim \mathrm{N}(0, 10)$, $\sigma^2_\theta \sim \mathrm{IG}(6.25, 1.25)$ and $\sigma^2 \sim \mathrm{IG}(100/9, 1/90)$.

[PLEASE INSERT TABLE 1 AROUND HERE]

In Table 2 we present the summary statistics of the posterior distribution of some important parameters. The posterior distribution of γ_μ is calculated through equation (3) for each sampled value of μ_θ. Since the mean of γ_μ is negative and its credible interval do not include zero, we can conclude that, on average of the counties, the corn crop-yield distribution is slightly skewed to the left. The posterior distribution of the autoregressive coefficients, ϕ_i, for each county shows that most of them is statistically significant with some of them in a positive region while other in the negative regions. It is important to note that in previous works, in the traditional crop insurance literature, agricultural yield was modeled using ARIMA process [17, 18]. According to [18] the use of serial correlation on agricultural yields is based on the idea that an adverse climatic event could affect the yield in the next growing season. In their work, these effects are represented in the innovations (because weather is not a conditioning variate) and, thus, suggesting the existence of an MA component. A problem with estimating the IMA (d, q) process is the need to employ non-linear least squares in small samples. In this case, convergence and parameter stability become issues. To address these problems, they replaced the MA (1) process with its AR (4) representation. In our case, we cannot afford modeling the data set using an AR (4) process, because of the data limitation. Instead, we capture the serial correlation using only an AR (1) process.

[PLEASE INSERT TABLE 2 AROUND HERE]

Table 3 compares the ER and BR for the five most important counties, named Castro, Catanduvas, Marilândia do Sul, Ponta Grossa and Tibagi. As one can note, all BR equal zero for the 70% level of coverage and are quite low for 75%.

These results reflect the fact that those five counties present low variability in terms of yields. At the minimum level of coverage there is no loss in the historical yield series. In other words, at any point in time the observed yield is lesser than the guaranteed yield.

[PLEASE INSERT TABLE 3 AROUND HERE]

The comparison between the BR and ER shows great differences. In all cases the ER overestimate the BR. These results have critical practical implications. In those counties the climate and soil conditions are ideal for corn. The historical data of corn yield is quite stable over time in terms of variability suggesting that these counties have low risk. Charging an inaccurate (overestimate) premium rate in low risk counties might considerably reduce the demand for such contracts. Despite the fact that the ER is greater than the BR in Table 3, for all levels of coverage, there are cases in which the opposite is true. Figure 1 shows the state of Paraná divided in 10 macro-regions, named MESO.

[PLEASE INSERT FIGURE 1 AROUND HERE]

In Table 4 we show the average premium rates for MESOs 4, 5, 7, 8, 9 and 10, in which the ER is greater than the BR for almost all levels of coverage. As one can observe, in MESOs 4, 5, 7, 8 and 9, the ER is much higher than the BR, except for MESO 10 where rates are quite similar. The ER and BR, at 90% level of coverage, have almost equal values for MESOs 7 and 10.

[PLEASE INSERT TABLE 4 AROUND HERE]

In riskier regions, such as 1, 2, 3 and 6, the ER underestimate the BR and overestimate in low risk regions. The northwestern region of the state presents inappropriate climate and soil conditions for planting corn. In that region farmers are undercharged according to the BR calculation. On the opposite, farmers are overcharged in low risk regions. This fact tends to reduce the demand for such contracts by the high expected yield-low risk producers and increase the demand for those who really need the insurance contract - the low expected yield-high risk producers. This fact strengthens the adverse selection problem in the insurance market.

For illustrative purpose, we will show a comparison between the premium rate charged by a private insurance company[1] and rates calculated in this research. Rates for Castro, Catanduvas, Marilândia do Sul, Ponta Grossa and Tibagi counties charged by this company are equal to 4.5%. We found rates completely different in our empirical analysis and, respectively, equal to 0.183%, 1.679%, 0.178%, 0.314% and 0.885%, at 90% level of coverage. From the insurer point of view, the probability of loss is much higher than this research suggests and consequently an inaccurate premium rate is calculated. At this point, we stress two points when insurance providers charge 4.5%, in a low risk area: i) producers might opt to manage their own risk without an insurance mechanism, and ii) the insurance company will negatively select among the insured pool.

[1] Rates charged are based only on five years of observations using the empirical rates method.

6. Concluding remarks

In this study we address the crop yield distribution issue and its empirical application of pricing crop insurance contracts using an alternative statistical method based on Bayesian hierarchical models under the Skew-Normal assumption. When modelling the generating process of yield data we consider the temporal and spatial effects and its interactions resulting in spatio-temporal models.

Most of the crop insurance programs in U.S. use a deviation from a linear trend - two-stage model as in [12], [26] and more recently in [22] in the Brazilian context. However, in this article we simultaneously model the time trend and temporal and spatial autocorrelation and obtain premium rate estimates directly (within the model) in contrast to two-stage methods. A typical two-stage method will first detrend the time series and then treat the detrended yield data (often referred to as "normalized yields") as "observed" data to estimate the premium rate. Thus, this method fails to adequately capture the uncertainty of the premium rate estimate. Our approach makes the premium rate calculation less ad hoc, in the sense that rates are derived directly from a predictive distribution obtained by using the Markov Chain Monte Carlo algorithm. Moreover, when calculating the rate we are able to capture its model-based uncertainty through a standard error that is estimated within the model.

Empirical rates method is commonly used by most of the crop insurance companies in Brazil when pricing agricultural contracts, which is based on the relation of the average loss over liability. This method does not take into account any more advanced statistical analysis. One of the main disadvantages of the empirical rates method is its dependence on a large number of observations to accurately reflect the probability distribution [12, 22]. Our work refines the actuarial methodology used to price crop insurance contracts based on Bayesian spatio-temporal models and comparing the results with the "ER" used by the Brazilian insurers.

Looking more carefully at the results, we can observe that ER is higher than BR for low risk areas and lower in high risk areas. It means that the insurance companies are underpricing the insurance contract for high risk areas and overpricing for low risk areas. The pure premium rate calculated in our model is actually higher or lower than the premium rate charged, depending on the situation. The consequence for the insurance provider is the financial loss because high risk producers may find this situation attractable to demand the insurance contract and low risk producers might bear the remaining risk themselves.

Our findings have important practical implications. For illustrative purposes consider a situation in which the insurance company underprice an insured located in Santa Isabel do Ivaí county. Instead of charging a premium rate equal to 10% (Bayesian premium rate), the insurance provider decided to calculate the premium rate using the empirical rates method, charging 6.5%. Suppose, for instance, the average liability is equal US$ 1 mi in a pool (5 thousand producers). The average premium charged is US$ 65 000 instead of US$ 100 000. The average loss is equal to US$ 35 000 but the total loss is approximately US$ 175 mi.

In a market where historically the total loss ratio (indemnity paid divided by total premium collected) is greater than one, better actuarial methods (the spatio-temporal approach proposed in this paper) should be taking into account by insurance companies to more effectively address the probability of loss and the accurateness of the premium rate. Despite empirical results can only be applied to county-level group risk plan, Brazilian insurers use county-level data set as a proxy to the individual farm-level risks. The Brazilian government just started a national program to collect farm-level information to solve this problem and correctly price and subscribe individual risks.

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References

[1] J. Atwood, S. Shaik and M. Watts, *Are crop yields normally distributed? A reexamination*, Amer. J. Agric. Econ. 85 (2003) pp. 888–901.
[2] A. Azzalini, *A class of distributions wich includes the normal ones*, Scand. J. Statist. 12 (1985), pp. 171–178.
[3] R.H. Day, *Probability distributions of field crop yields*, J. Farm Econ. 47 (1965) 713–741.
[4] B.A. Babcock and D.A. Hennessy, *Input demand under yield and revenue insurance*, Amer. J. Agric. Econ. 78 (1996), pp. 416–427.
[5] J. Besag, *Spatial interaction and the statistical analysis of lattice systems*, J. Royal Statist. Soc. Ser. B 36 (1974), pp. 192–236.
[6] R. Borges and W. Thurman, *Marketing quotas and subsidies in peanuts*, Amer. J. Agric. Econ. 76 (1994), pp. 801–817.
[7] K.H. Coble et. al., *Modeling farm-level crop insurance demand with panel data*, Amer. J. Agric. Econ. 78 (1996) 439–447.
[8] N.A.C. Cressie and N.H. Chan, *Spatial modeling of regional variables*, J. Amer. Statist. Assoc. 84 (1989), pp. 393–401.
[9] P. Gallagher, *U.S. Soybean yields: estimation and forecasting with nonsymmetric disturbances*, Amer. J. Agric. Econ. 69 (1997), pp. 796–803.
[10] A. Gelfand and S. Ghosh, *Model choice: a minimum posterior predictive loss approach*, Biometrika 85 (1998), pp. 1–11.
[11] M. Genton, *Skew-elliptical distributions and their applications*, Chapman & Hall/CRC, London, 2004.
[12] B.K. Goodwin and A.P. Ker, *Nonparametric estimation of crop yield distributions: implications for rating group-risk crop insurance contracts*, Amer. J. Agric. Econ. 80 (1998), pp. 139-153.
[13] D.A. Hennessy, B.A Babcock and D.J. Hayes, *Budgetary and producer welfare effects of revenue insurance*, Amer. J. Agric. Econ. 79 (1997), 1024–1034.
[14] H. Henze, *A probabilistic representation of the skew-normal distribution*, Scand. J. Statist. 13 (1986), pp. 271–275.
[15] IBGE 2004. *Statistical and Geography Brazilian Institute*, Available at http://www.ibge.gov.br/english, (Access 2004, January).
[16] R.E. Just and Q. Weninger, *Q. 1999. Are crop yields normally distributed?*, Amer. J. Agric. Econ. 81 (1999), pp. 287–304.
[17] A.P. Ker and K. Coble, *Modeling conditional yield densities*, Amer. J. Agric. Econ. 85 (2003), pp. 291–304.
[18] A.P. Ker and B.K. Goodwin, *Nonparametric estimation of crop insurance rates revisited*, Amer. J. Agric. Econ. 83 (2000), pp. 463–478.
[19] C.B. Moss and J.S. Shonkwiler, *Estimating yield distributions with a stochastic trend and nonnormal errors*, Amer. J. Agric. Econ. 75 (1993), pp. 1056–1062.
[20] C.H. Nelson and P.V. Preckel, *The conditional beta distribution as a stochastic production function*, Amer. J. Agric. Econ. 71 (1989), pp. 370–378.
[21] V.A. Ozaki, S.K. Ghosh, B.K. Goodwin and R. Shirota, *Spatio-temporal modeling of agricultural yield data with an application to pricing crop insurance contracts*, Amer. J. Agric. Econ. (2008), to appear.
[22] V.A. Ozaki, B.K. Goodwin and R. Shirota, *Parametric and nonparametric statistical modeling of crop yield: implications for pricing crop insurance contracts*, App. Econ. 40 (2008), pp. 1151–1164.
[23] O.A. Ramirez, *Estimation and use of a multivariate parametric model for simulating heteroskedastic, correlated, nonNormal random variables: the case of corn belt corn, soybean and wheat yields*, Amer. J. Agric. Econ. 79 (1997), pp. 191–205.
[24] O.A. Ramirez, S. Misra and J. Field, *Crop-yield distributions revisited*, Amer. J. Agric. Econ. 85 (2000), pp. 108–120.
[25] B.J. Sherrick at al., *Crop insurance valuation under alternative yield distributions*, Amer. J. Agric. Econ. 86 (2004), pp. 406–419.
[26] S.M. Stohs and J.T. LaFrance, *A learning rule for inferring local distributions over space and time*, Adv. Economet. 18 (2004), pp. 295–331.
[27] C.R. Taylor, *Two practical procedures for estimating multivariate nonnormal probability density functions*, Amer. J. Agric. Econ. 72 (1990), pp. 210-217.

Table 1. Expected Predictive Deviance (EPD) for three best-fit competing models for the corn crop-yield in the state of Paraná, Brazil.

	EPD	EPD-Goodness	EPD-Penalty
Model 1	268.3E+10	268.1E+10	284.6E+7
Model 2	318.6E+10	318.3E+10	337.9E+7
Model 3	236.0E+10	235.7E+10	337.4E+7

Table 2. Summary statistics of the posterior distribution of the best-fit hierarchical model based on a sample size of 1 500. S.D., 2.5% and 97.5% are the standard deviation, 2.5 and 97.5 percentiles, respectively.

Parameter	Mean	S.D.	2.5%	Median	97.5%
σ	803.1	20.79	763.8	802.8	844.8
ω	107.9	3.335	101.3	107.8	114.5
σ_{β_1}	74.57	6.207	62.85	74.60	87.48
μ_θ	-0.875	0.121	-1.119	-0.876	-0.625
σ_θ	1.158	0.097	0.993	1.154	1.367
γ_θ	-0.103	0.032	-0.174	-0.101	-0.044
μ_ϕ	0.083	0.017	0.048	0.083	0.117
σ_ϕ	0.124	0.014	0.098	0.124	0.153

Table 3. Bayesian (BR) and empirical rates (ER) for different levels of coverage (LC) for Castro, Catanduvas, Marilândia do Sul, Ponta Grossa and Tibagi counties.

County	LC(%)	BR(%)	ER(%)
	70	0.000	0.379
	75	0.000	0.730
Castro	80	0.003	1.444
	85	0.048	2.464
	90	0.183	3.695
	70	0.000	0.010
	75	0.000	0.019
Catanduvas	80	0.001	0.030
	85	0.006	0.043
	90	0.017	0.059
	70	0.000	0.659
	75	0.000	1.226
Marilândia do Sul	80	0.008	2.205
	85	0.034	3.528
	90	0.178	5.076
	70	0.000	1.506
	75	0.000	2.202
Ponta Grossa	80	0.002	3.100
	85	0.074	4.425
	90	0.314	5.981
	70	0.000	4.136
	75	0.003	5.254
Tibagi	80	0.051	6.373
	85	0.241	7.589
	90	0.885	9.090

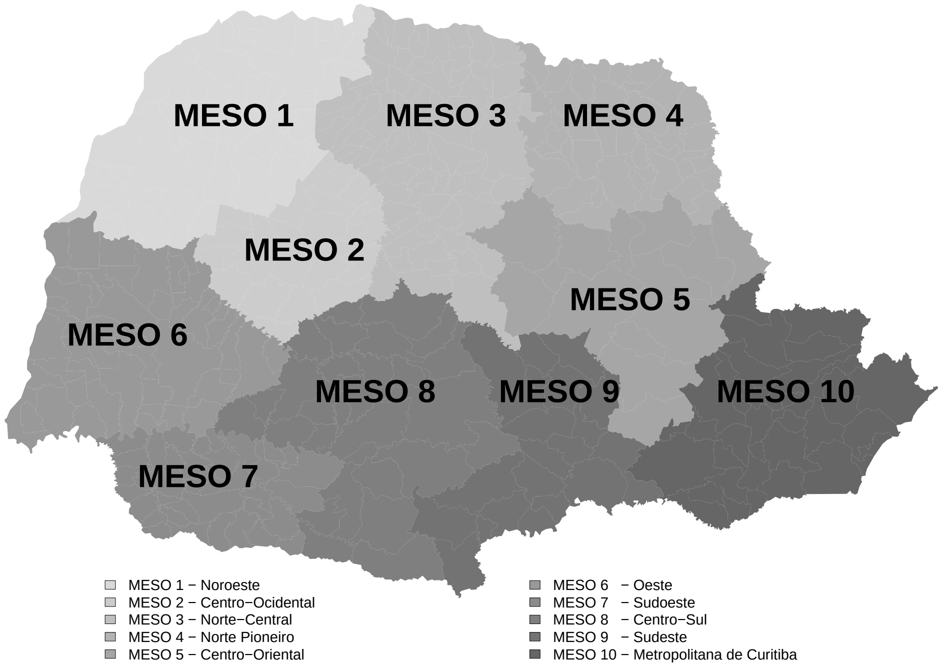

Figure 1. State of Paraná divided into 10 macro-regions (MESO).

Table 4. Bayesian (BR) and empirical rates (ER) for all levels of coverage (LC) by macro-regions (MESO).

MESO	70%		75%		80%		85%		90%	
	BR	ER	BR	ER	BR	ER	BR	ER	BR	ER
1	5.93	2.25	8.17	3.04	10.68	4.02	13.40	5.21	16.22	6.56
2	5.42	1.69	7.81	2.49	10.51	3.49	13.40	4.73	16.35	6.24
3	2.12	1.56	3.23	2.25	4.65	3.13	6.45	4.22	8.65	5.57
6	3.15	1.63	4.28	2.34	5.59	3.24	7.11	4.37	8.87	5.75
4	0.38	2.26	0.64	2.95	1.05	3.82	1.66	4.91	2.54	6.32
5	0.06	1.47	0.17	2.17	0.43	3.15	0.90	4.59	1.74	6.39
7	0.08	1.33	0.32	1.71	0.93	2.26	2.16	3.05	4.15	4.14
8	0.02	1.62	0.09	2.19	0.32	3.03	0.97	3.99	2.33	5.17
9	0.16	1.81	0.42	2.45	0.99	3.23	1.99	4.22	3.52	5.49
10	0.05	0.05	0.15	0.20	0.44	0.53	1.15	1.21	2.51	2.42