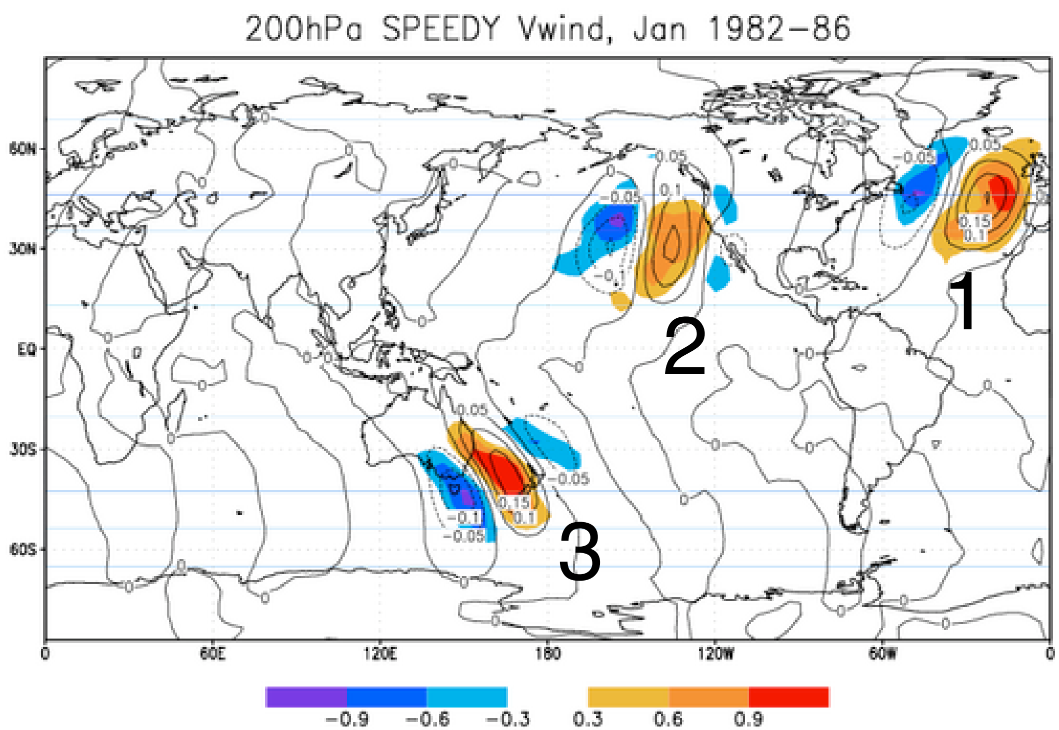
Estimating and Correcting Global Weather Model Error



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Problem

- Inaccuracies in initial conditions and model deficiencies interact non-linearly, causing numerical weather forecast errors to grow.
- With recent progress in data assimilation, the accuracy of initial conditions has improved dramatically.
- Accounting for model deficiencies has become relatively more important for data assimilation and ensemble forecasting.

Outline

- Brief review of model error correction
- SPEEDY model
- Generation of 6-hour forecasts and errors with NCEP reanalysis
- Separation of errors into monthly, diurnal, and state-dependent components
- Estimation and correction of model errors
- Results: our method is effective and computationally feasible
- Conclusions

Background

Schemm et. al. (81, 86)

- introduced procedures for statistical correction of numerical predictions when verification data are only available at discrete times
- applying corrections only when verification data were available, they were successful in correcting artificial model errors
- reduced the small scale 12-hr errors of the NMC model
- errors at the larger scales grew due to randomization of the residual errors by the regression equations

Background

Saha (1992)

- *nudged* a low-resolution version of the NMC operational forecast model to estimate systematic errors
- reduced systematic errors (measured against hi-res model) by adding artificial sources and sinks in heat, momentum, and mass
- correction *during* the integration and *a posteriori* correction were seen to give equivalent improvement in forecasts

Background

Klinker and Sardeshmukh (1992)

- estimated average 6-hour forecast error from analysis increments of ECMWF operational model
- switched off each individual parameterization
- found that the model's gravity wave parameterization dominated the 1-day forecast error

Background

D'Andrea and Vautard (2000)

- using a simple QG model, estimated tendency errors by solving for the model forcing which minimized 6-hour forecast errors
- during forecasts, found the closest analogues to the model state from the reference time series
- used the tendency errors corresponding to these analogues to correct forecasts online
- improved forecasts in Euro-Atlantic region
- computationally prohibitive

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- the tendency error $\mathbf{g}$ of the improved model is given by

$$\mathbf{g} = \dot{\mathbf{x}}_{\mathrm{t}} - \mathrm{M}\left(\mathbf{x}_{\mathrm{t}}\right) - \mathrm{L}\mathbf{x}_{\mathrm{t}} - \mathbf{b}$$

where $\mathbf{x}_{\mathrm{t}}$ is the state taken as truth (e.g. reanalysis)

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- $\mathbf{b}_{\mathrm{L}}$ is a state-independent bias estimate
- $\mathrm{L}_{\mathrm{L}}\mathbf{x}$ is a state-dependent estimate of the model error
- computationally prohibitive for operational models

Background

DelSole and Hou (1999)

- applied Leith's procedure to a 2-layer QG model on an 8 x 10 grid (N=160 degrees of freedom)
- perturbed the model parameters to generate 'nature'
- resulting model errors were strongly state-dependent by design
- Leith's state-dependent error correction extended forecast skill
- computationally prohibitive for operational use

Our Method

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b. by a new low-dimensional method based on regression.

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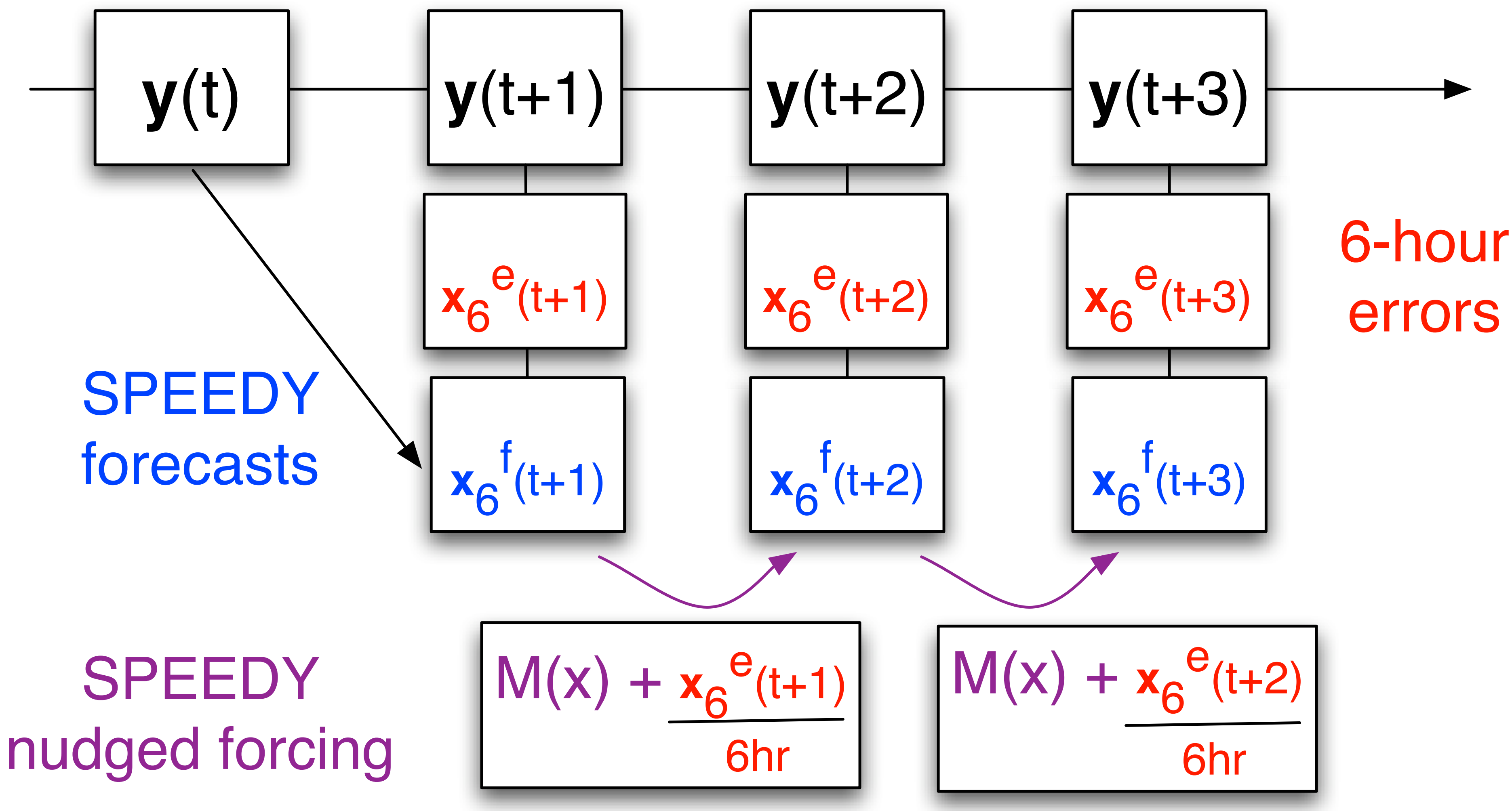
IV. Correct the state-dependent errors.

SPEEDY Model, Molteni (2003)

- primitive equations, global spectral model
- contains parameterizations of condensation, convection, clouds, radiation, surface fluxes, and vertical diffusion
- T30 horizontal resolution, 7 sigma levels
- integrates vorticity, divergence, temperature, specific humidity, and surface pressure
- post-processed into horizontal wind, temperature, specific humidity, geopotential height, and surface pressure on 96x48 grid, 7 pressure levels
- dissipation and time-dependent forcing determined by climatological SST, surface moisture, albedo, land-surface vegetation, etc.

Generating Time Series of Model Forecasts and Errors

1982-1986 NCEP Reanalysis



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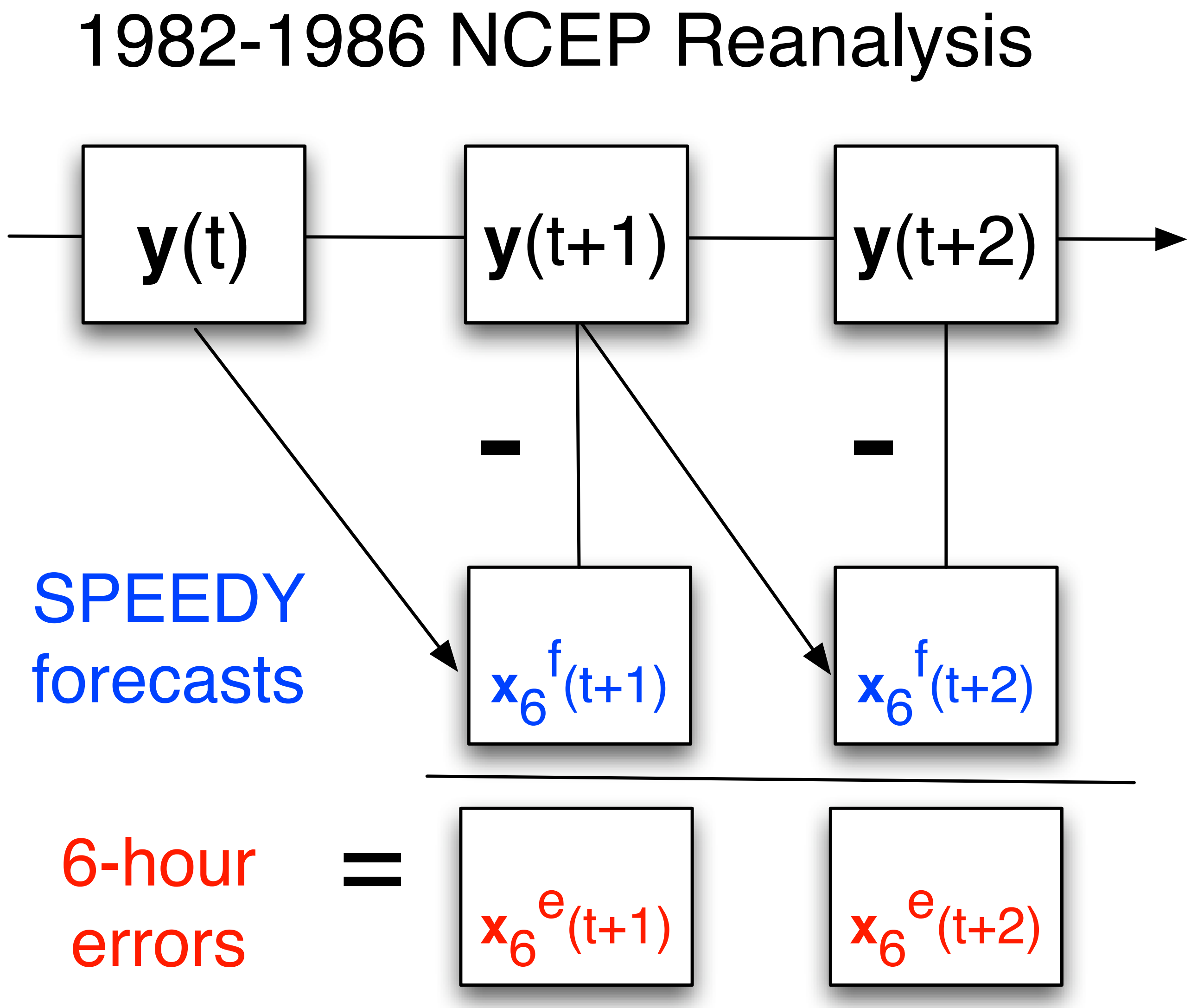

Time Series and 5-year Climatology

- $\mathbf{x}_6^f(t)$ = time series of model states
- $\mathbf{x}_6^e(t)$ = corresponding 6-hour errors
- 5-year SPEEDY 6-hour climatology given by monthly mean $\overline{\mathbf{x}_6^f}$
- 5-year reanalysis climatology given by monthly mean $\overline{\mathbf{y}}$
- Bias given by monthly mean $\overline{\mathbf{x}_6^e}$

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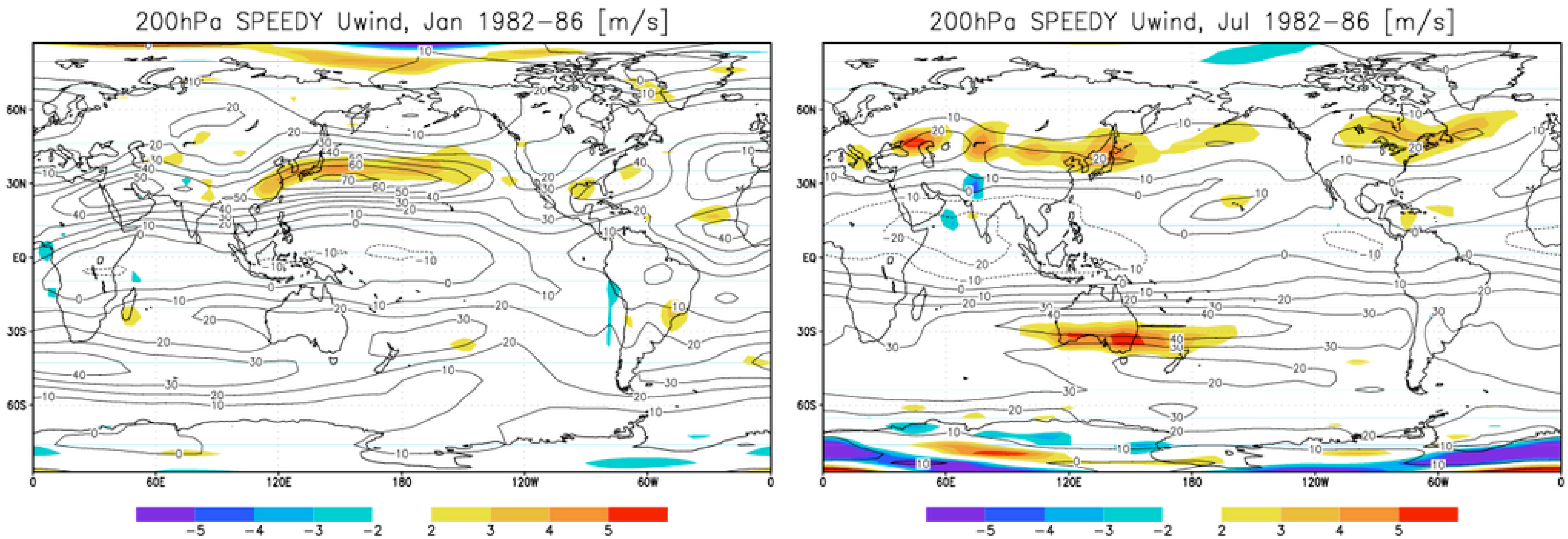
200hPa Zonal Wind Monthly Bias

5-year Reanalysis Climatology $\overline{\mathbf{y}}$ (contour), Bias $\overline{\mathbf{x}_6^e}$ (color)

January July

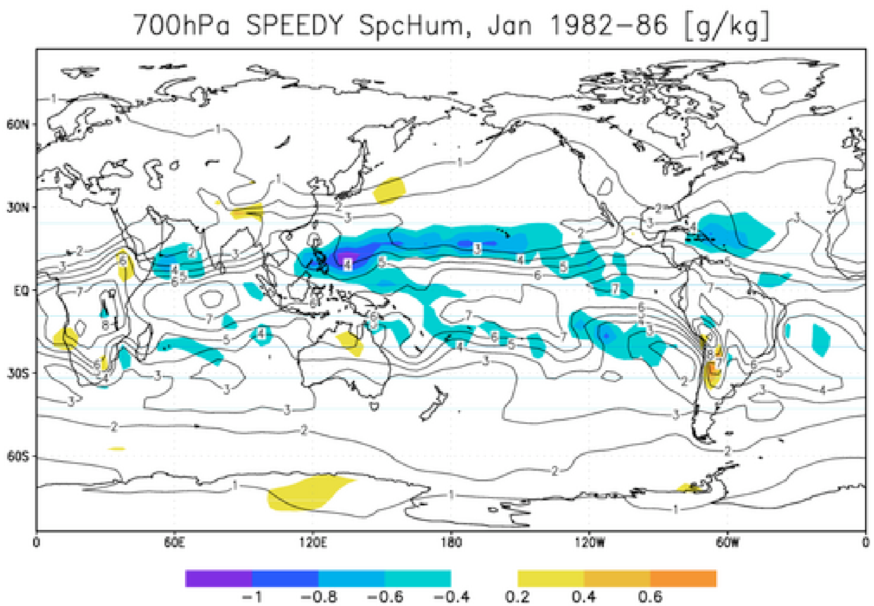


- SPEEDY underestimates zonal wind on the poleward side of the winter hemisphere jet.
- Exhibits large polar bias.

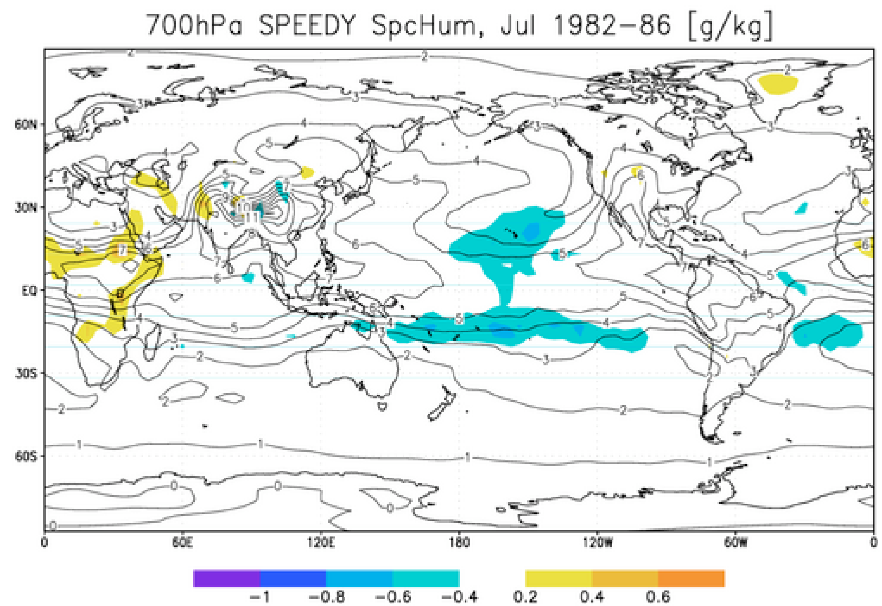
700hPa Specific Humidity Monthly Bias

5-year Reanalysis Climatology $\overline{\mathbf{y}}$ (contour), Bias $\overline{\mathbf{x}_6^e}$ (color)

January



July

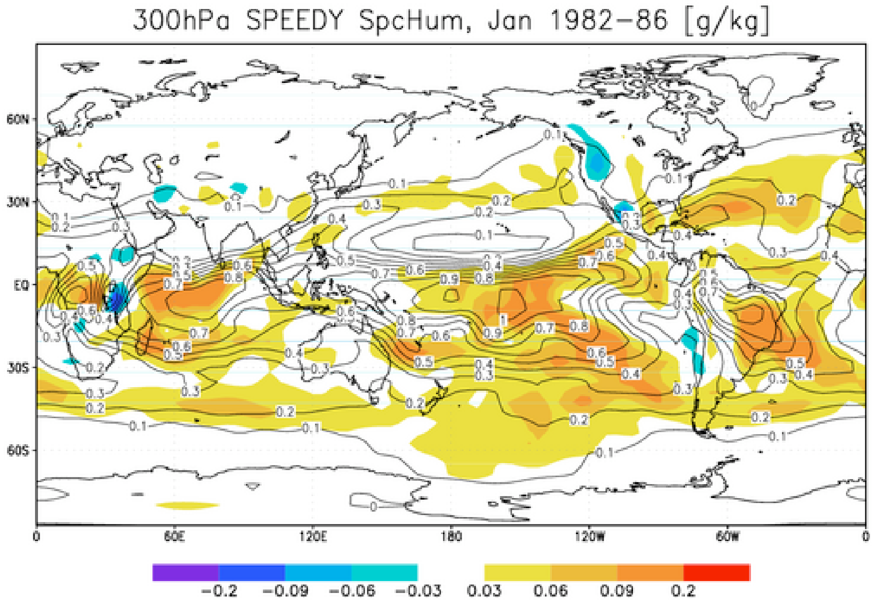


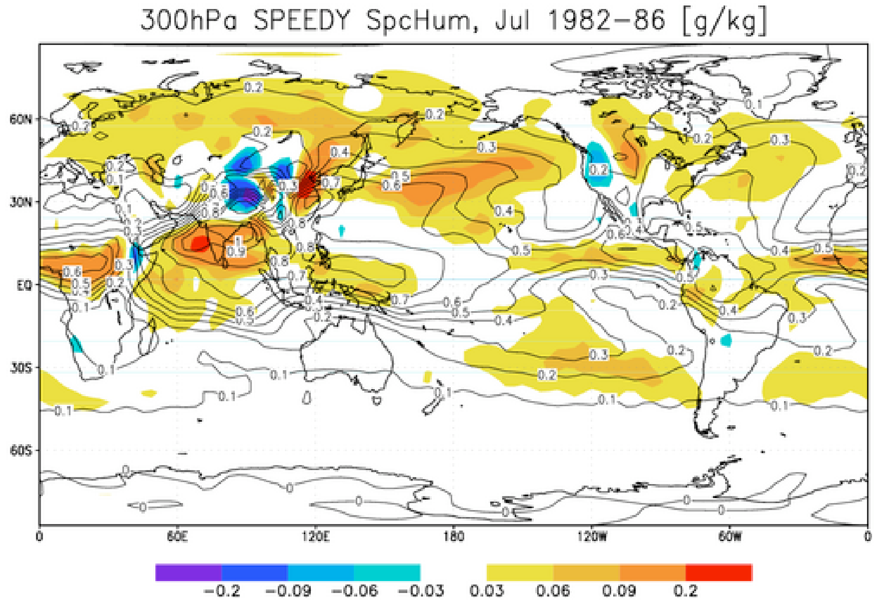
- SPEEDY overestimates specific humidity at lower levels in the tropics, especially in the winter hemisphere over the oceans.

300hPa Specific Humidity Monthly Bias

5-year Reanalysis Climatology $\bar{\mathbf{y}}$ (contour), Bias $\overline{\mathbf{x}_6^e}$ (color)

January July





- SPEEDY underestimates specific humidity at upper levels, especially in the summer hemisphere.

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2. Corrected *a posteriori*: Correct control forecast by 6-hour bias $\overline{\mathbf{x}_6^{\mathrm{e}}}$ at 6 hours, 12-hour bias $\overline{\mathbf{x}_{12}^{\mathrm{e}}}$ at 12 hours, etc.

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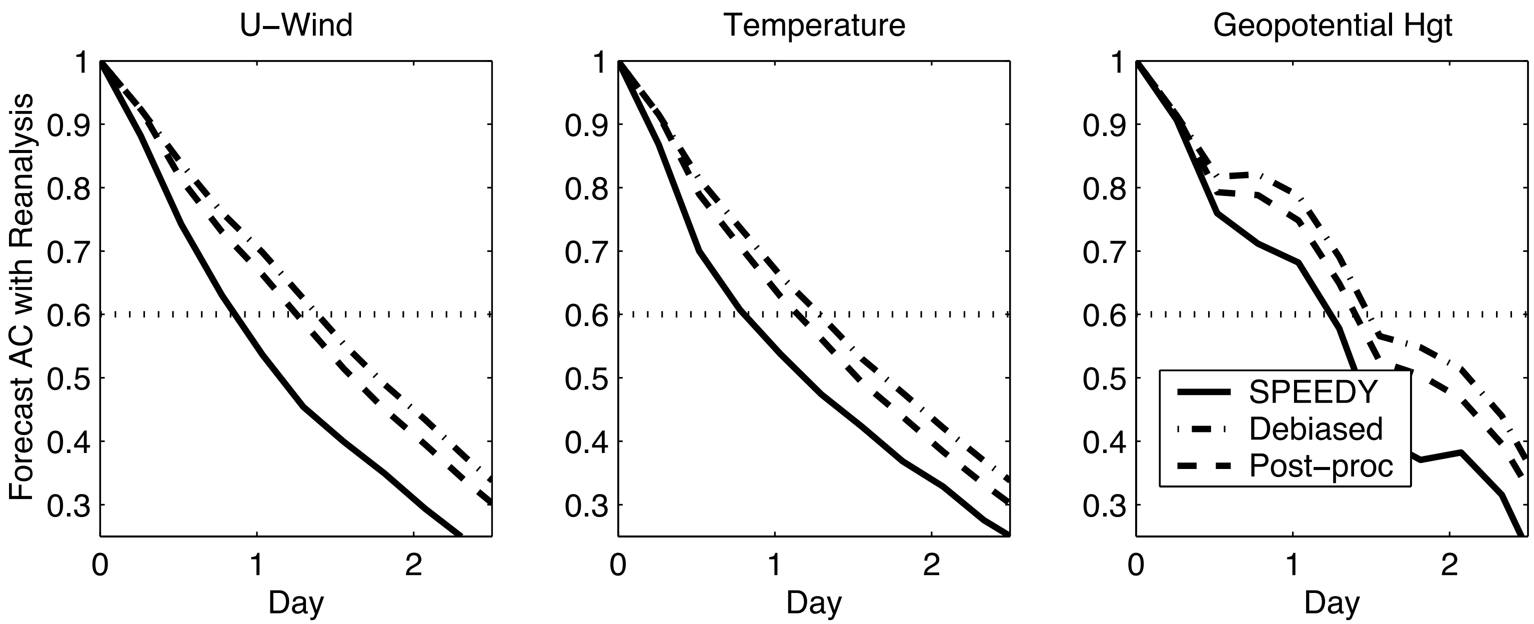
3. Corrected *online*: Integrate model, $\dot{\mathbf{x}} = \mathrm{M}(\mathbf{x}) + \frac{\overline{\mathbf{x}_6^{\mathrm{e}}}}{\Delta \mathrm{t}}$,

$\overline{\mathbf{x}_6^{\mathrm{e}}}$ is a daily linear interpolation (e.g. on July 1, $\overline{\mathbf{x}_6^{\mathrm{e}}} = \frac{\overline{\mathbf{x}_6^{\mathrm{e}}(\mathrm{Jun})} + \overline{\mathbf{x}_6^{\mathrm{e}}(\mathrm{Jul})}}{2}$)

I. Monthly Bias Correction

850hPa February 1987 Global Mean Anomaly Correlation



- Monthly bias correction gives substantial forecast improvement.

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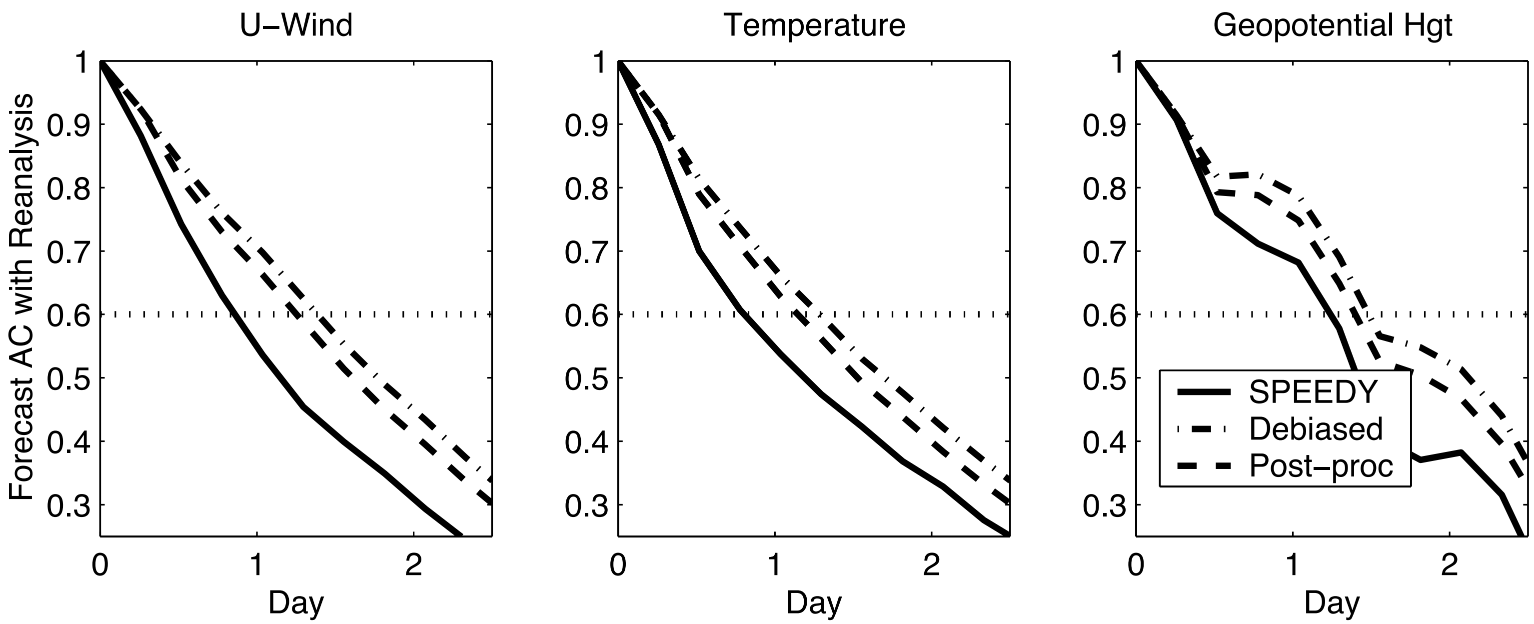
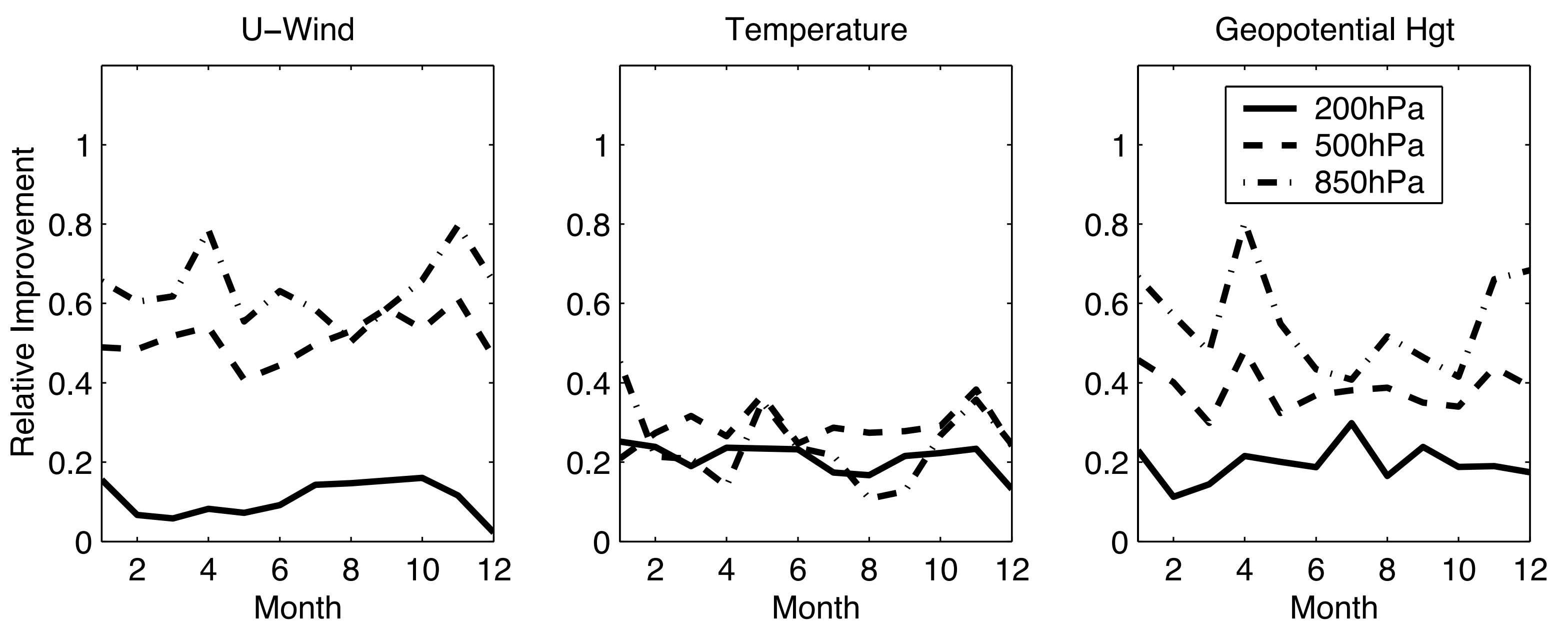


- Monthly bias correction gives substantial forecast improvement.
- Online correction performs better than a posteriori correction.

I. Monthly Bias Correction

Improvement of Online Correction Relative to Control



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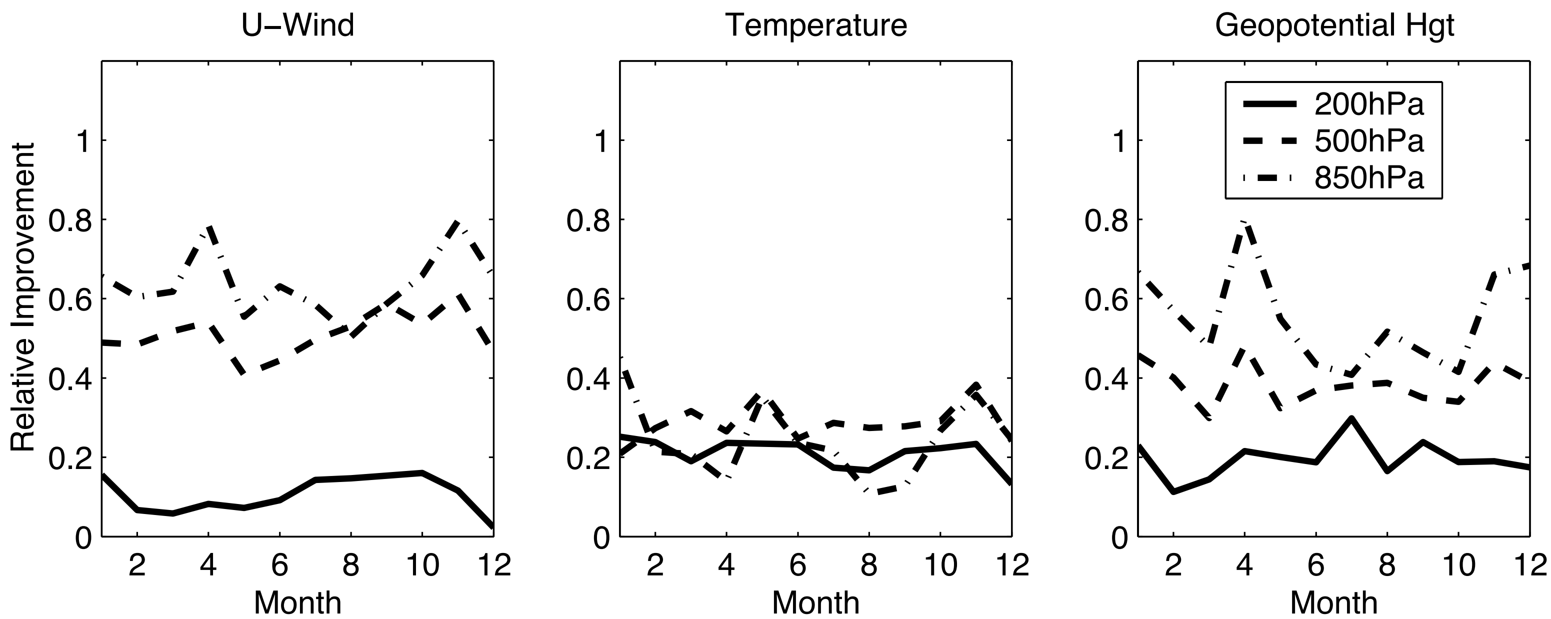
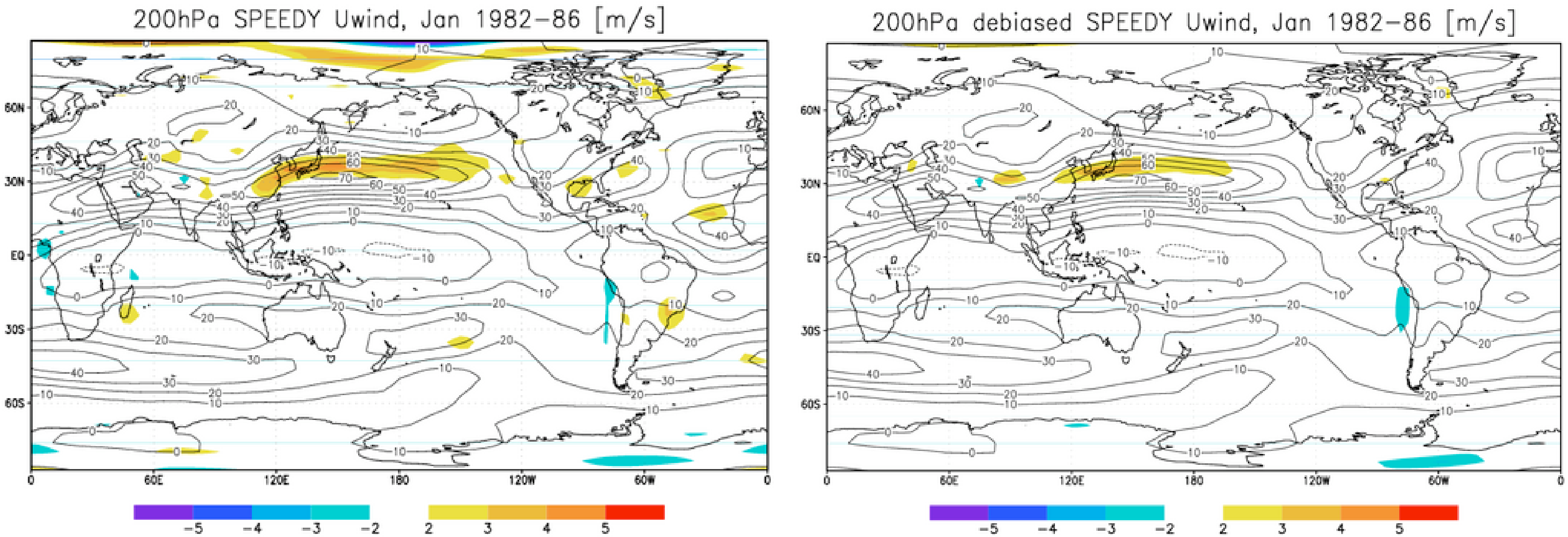


- Online correction is most effective at lower levels.
- Improvements are uniform across levels in T, across seasons by level.

I. Monthly Bias Correction

200hPa January 1982-86 Zonal Wind Bias

Control | Online Corrected Forecasts

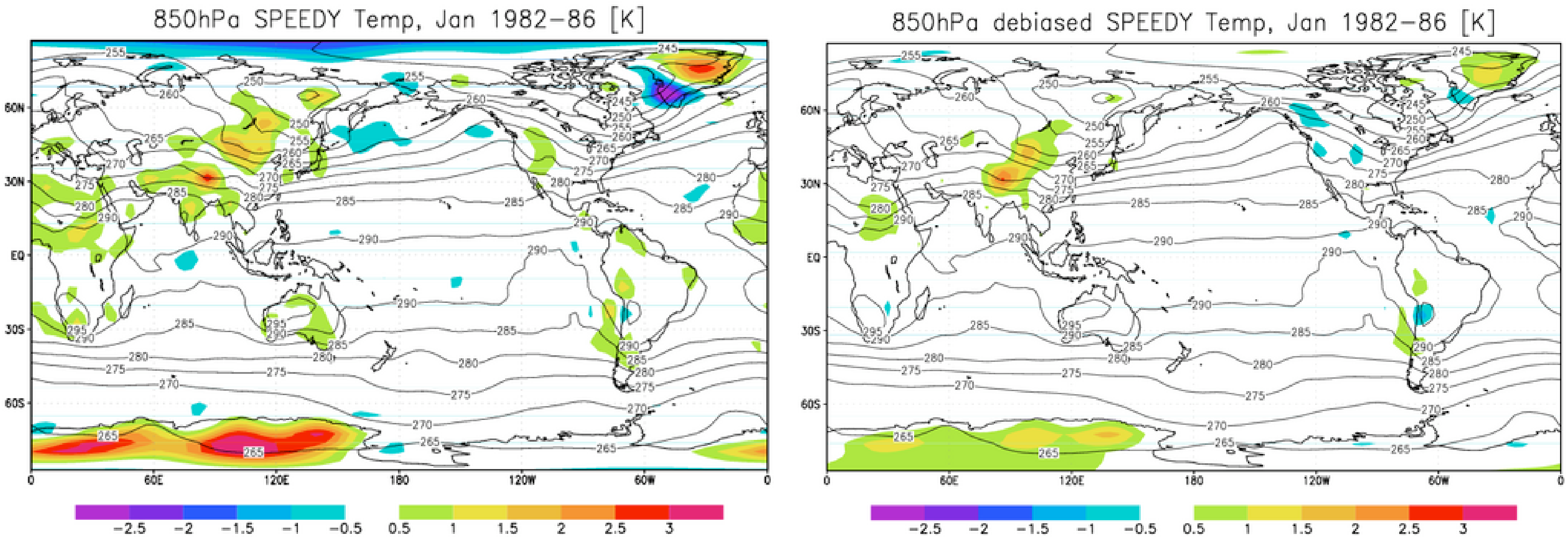


- Most of the bias is removed by the online correction.

I. Monthly Bias Correction

850hPa January 1982-86 Temperature Bias

Control / Online Corrected Forecasts



- Most of the bias is removed by the online correction.

Our Method: I. Monthly Bias Correction

Generate time series of 6-hour model forecasts and errors relative to the NCEP reanalysis using a simple but realistic GCM.

Separate the 6-hour errors into monthly, diurnal, and state-dependent components.

I. Estimate the monthly bias.

a. Compare the impact of correcting the model integration with statistical corrections performed *a posteriori*.

Our Method: II. Diurnal Bias Correction

Generate time series of 6-hour model forecasts and errors relative to the NCEP reanalysis using a simple but realistic GCM.

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II. Estimate and correct the diurnal errors.

II. Diurnal Bias Correction

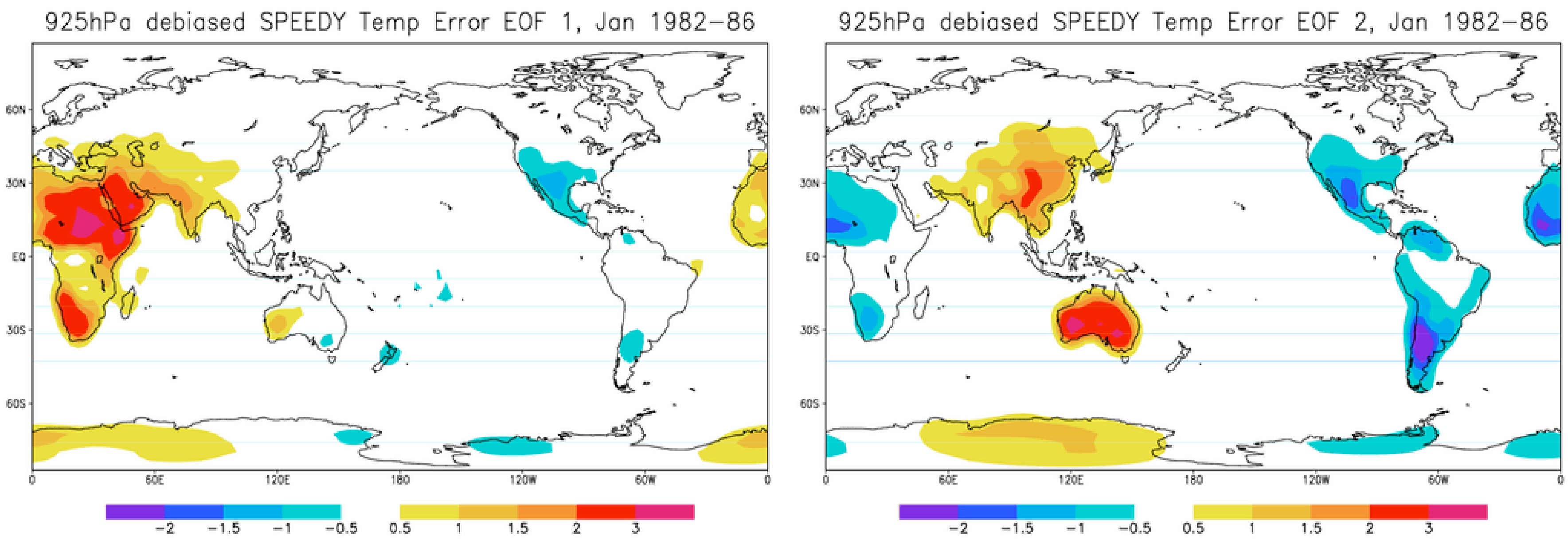
- Anomalous model error time series: $\mathbf{x}_6^{e\prime}(i) = \mathbf{x}_6^e(i) - \overline{\mathbf{x}_6^e}$

- Anomalous model error matrix: $D = [\mathbf{x}_6^{e\prime}(1) \;\; \mathbf{x}_6^{e\prime}(2) \; \ldots \; \mathbf{x}_6^{e\prime}(N)]$

- The leading EOFs of $DD^\top$ represent patterns of diurnal variability which are poorly represented by SPEEDY.

II. Diurnal Bias Correction

Leading EOFs of $DD^\top$, T at 925hPa, Jan 1982-1986

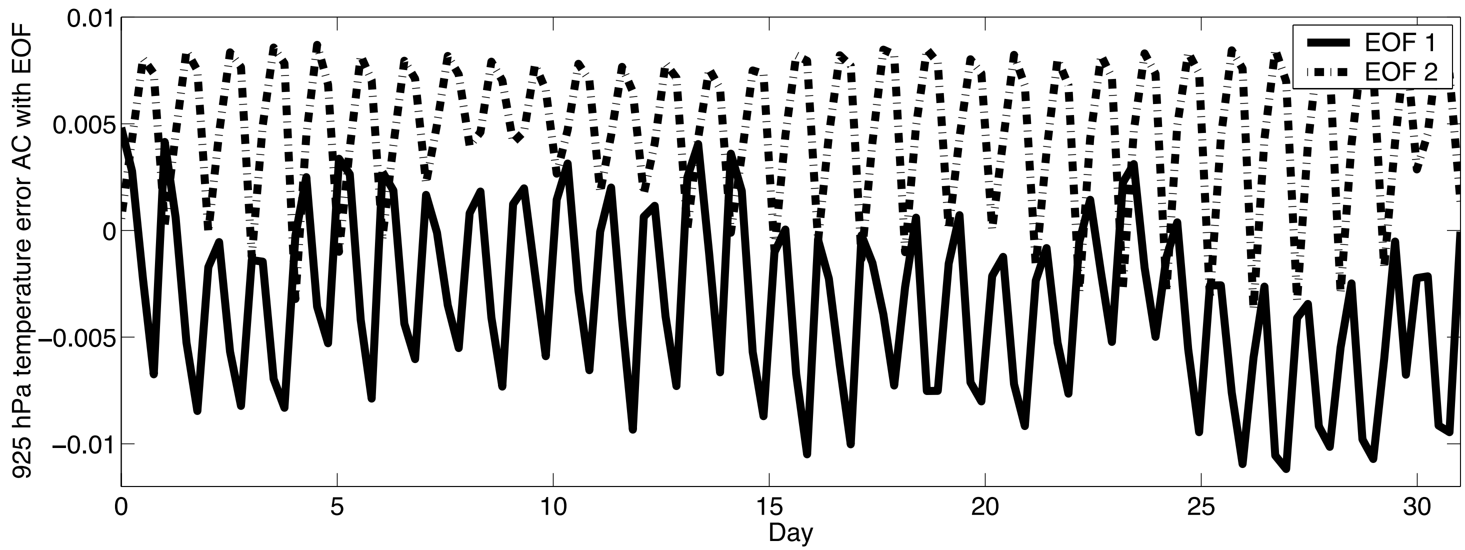


- Lack of diurnal forcing results in wavenumber 1 structure in the errors
- SPEEDY underestimates (overestimates) near surface daytime (night-time) temperatures, more prominent over land

II. Diurnal Bias Correction

Principal Components

- Project leading EOFs onto anomalous errors (January, 1982)



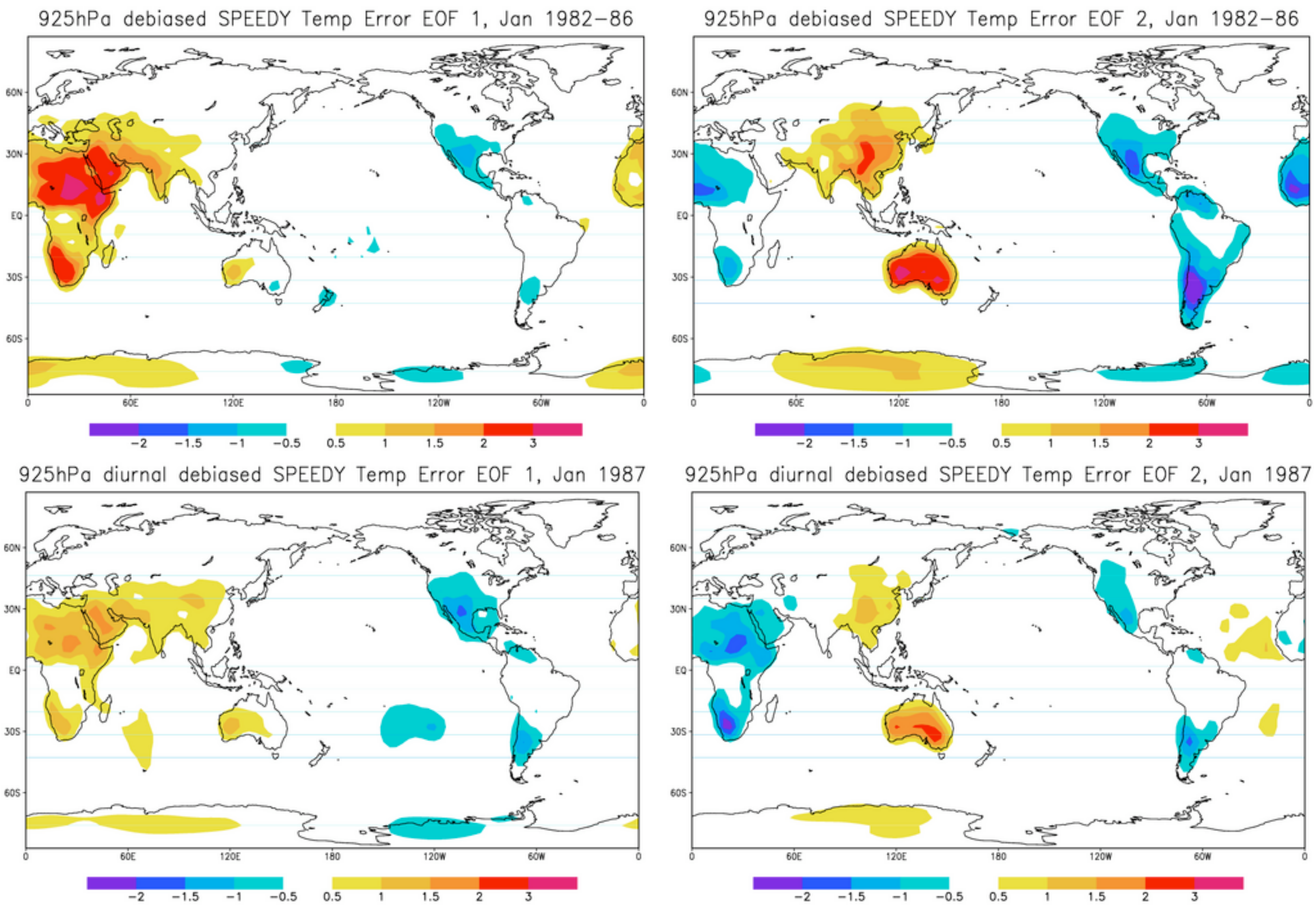
- Leading pair of EOFs out of phase by 12 hours
- Find average strength of daily cycle over Jan 1982-86
- Compute diurnal correction as a function of the time of day

II. Diurnal Bias Correction

EOFs of $DD^\top$

January
1982-1986

Diurnally
Corrected
1987



- Diurnal correction substantially reduces error amplitude

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III. Estimate the state-dependent errors.

III. State-Dependent Error Estimation

Leith (1978) Empirical Correction Operator

- Anomalous analysis state time series: $\mathbf{y}'(t) = \mathbf{y}(t) - \overline{\mathbf{y}}$
- Anomalous 6-hour error time series: $\mathbf{x}_6^{e\prime}(t) = \mathbf{x}_6^{e}(t) - \overline{\mathbf{x}_6^{e}}$

III. State-Dependent Error Estimation

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- Anomalous 6-hour error time series: $\mathbf{x}_6^{\mathrm{e}\prime}(\mathrm{t}) = \mathbf{x}_6^{\mathrm{e}}(\mathrm{t}) - \overline{\mathbf{x}_6^{\mathrm{e}}}$
- Analysis state covariance: $\mathrm{B}_{\mathrm{yy}}(\mathrm{t}) = \mathbf{y}'(\mathrm{t})\ \mathbf{y}'^{\top}(\mathrm{t})$
- *Lagged* cross covariance: $\mathrm{B}_{\mathrm{x^e y}}(\mathrm{t}) = \mathbf{x}_6^{\mathrm{e}\prime}(\mathrm{t})\ \mathbf{y}'^{\top}(\mathrm{t}-1)$

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- *Lagged* cross covariance: $B_{x^e y}(t) = \mathbf{x}_6^{e\prime}(t)\ \mathbf{y}'^{\top}(t-1)$

Leith's correction operator, given by $L = \overline{B_{x^e y}}\ \overline{B_{yy}}^{-1}$, provides a state-dependent correction:

$$\dot{\mathbf{x}} = M(\mathbf{x}) + L\mathbf{x} + \mathbf{b}$$

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$$\dot{\mathbf{x}} = \mathrm{M}(\mathbf{x}) + \mathrm{L}\mathbf{x} + \mathbf{b}$$

Problem: Direct computation of $\mathrm{L}\mathbf{x}$ requires $O(\mathrm{N}^3)$ floating point operations *every* time step!

III. State-Dependent Error Estimation

Implementation

- DelSole and Hou (1999) used Leith's state-dependent empirical correction to extend forecast skill up to the limits imposed by observation error, for a very simple model (N = 160 degrees of freedom).

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- For the SPEEDY model, N = $O(10^5)$, so computation of $\mathrm{L}\mathbf{x}'$ requires $O(10^{15})$ floating point operations *every* time step.
- This operation would be prohibitive for operational forecast models where N $\approx O(10^7)$.

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I. Estimate the monthly bias.

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- univariate covariances generate block diagonal structure

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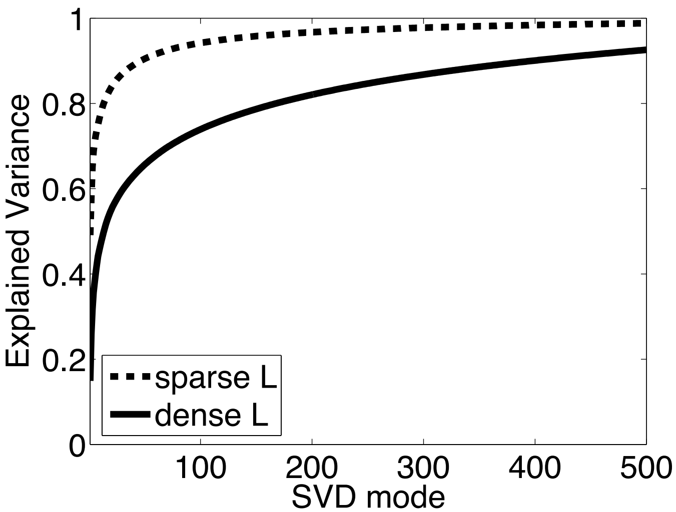

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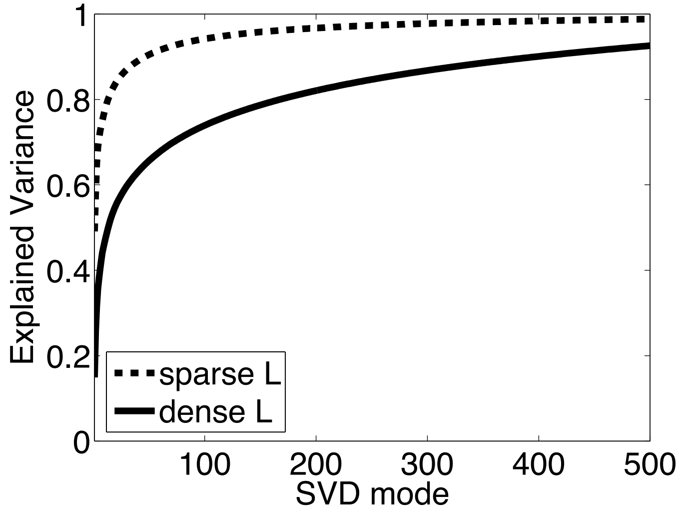

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- 400 modes required to explain 90% of variance in dense L
- 40 modes required to explain 90% of variance in sparse L

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Principal Components: project error and state anomalies onto EOFs

$$a_k(t) = \mathbf{x}^{e\prime}(t) \cdot \mathbf{u}_k$$
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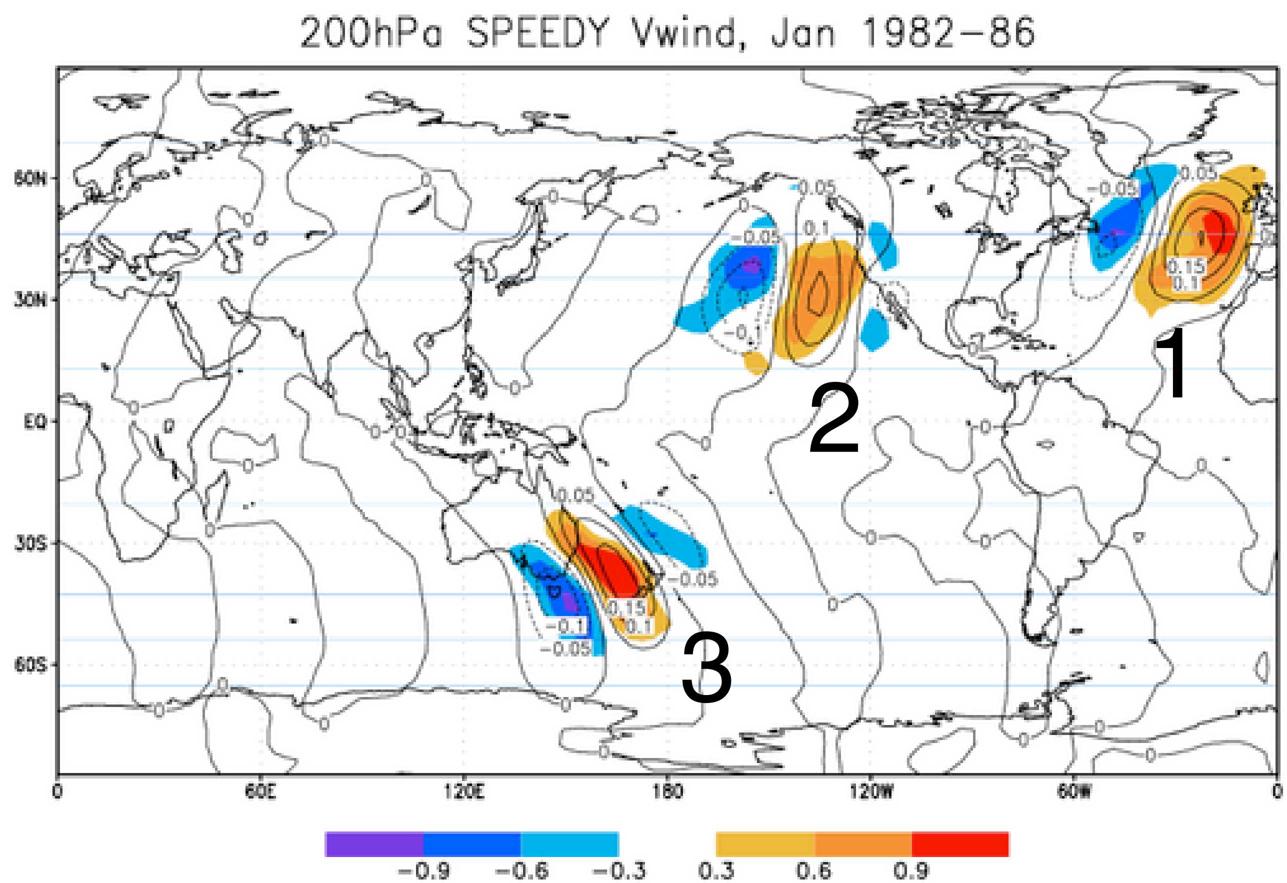
Heterogeneous correlation maps:

$$\rho[\mathbf{x}^{e\prime}, b_k] = \left(\frac{\sigma_k}{\sqrt{\overline{\overline{b_k^2(t)}}}}\right)\mathbf{u}_k$$

III. State-Dependent Error Estimation

1982-86 error (color) and state (contour) coupled signals



- signals exhibit clear correlation in local structure

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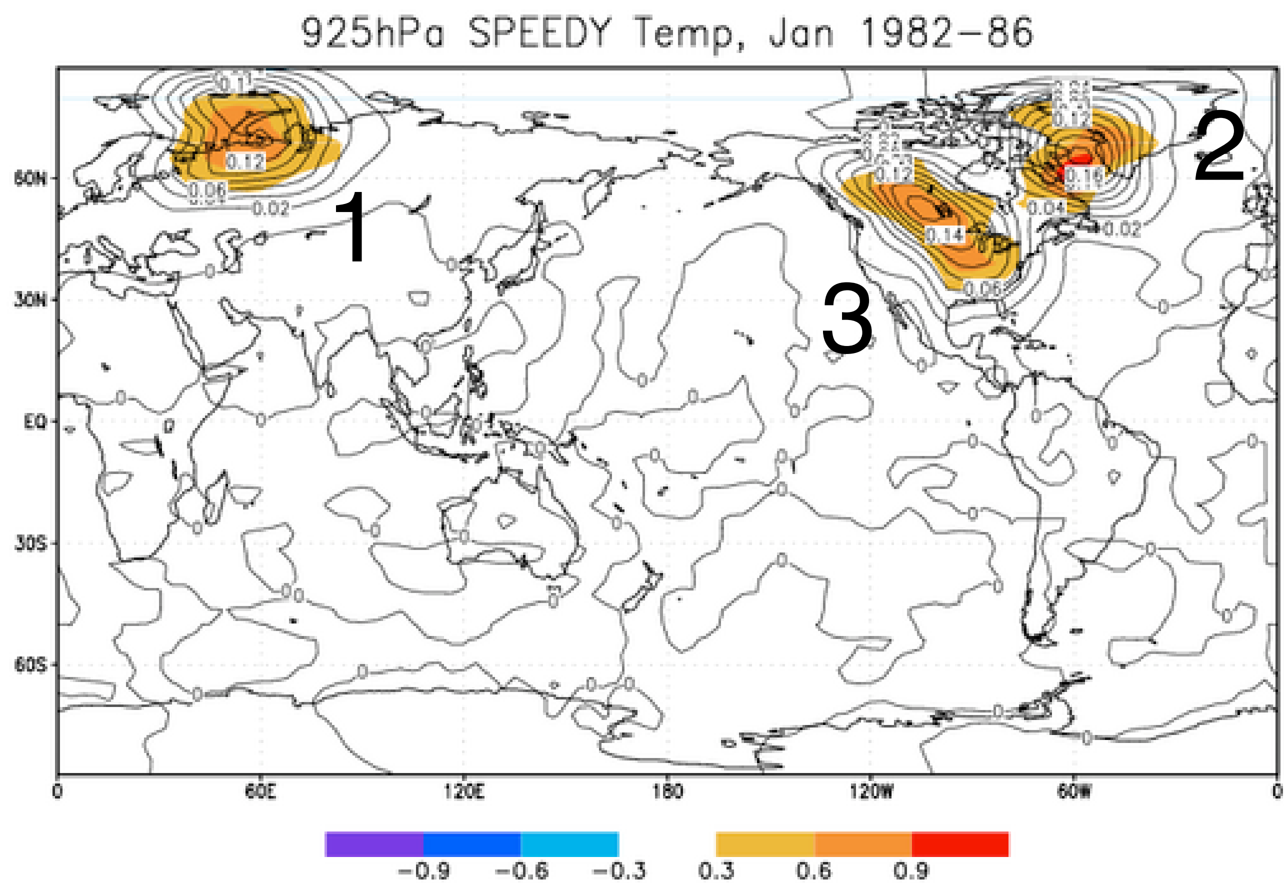

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Our Method: IV. State-Dependent Correction

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IV. Correct the state-dependent errors.

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$$\dot{\mathbf{x}} = \mathrm{M}\left(\mathbf{x}\right)$$

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$$\dot{\mathbf{x}} = \mathrm{M}\left(\mathbf{x}\right) + \frac{\overline{\mathbf{x}_6^{\mathrm{e}}}}{\Delta \mathrm{t}}$$

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Our low-dimensional state-dependent corrected model is given by:

$$\dot{\mathbf{x}} = \mathrm{M}(\mathbf{x}) + \left[\overline{\mathbf{x}_6^e} + \sum_{k=1}^{K} \tilde{\mathbf{u}}_k \left(\frac{\sigma_k}{\sqrt{\overline{b_k^2}}}\right) \mathbf{v}_k \cdot \mathbf{x}'\right] \frac{1}{\Delta t}$$

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During forecasts, the anomalous model state vector can be projected onto a few (K≈10) dominant anomalous model state signals $\mathbf{v}_k$.

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$$\sum_{k=1}^{K} \tilde{\mathbf{u}}_k \left(\frac{\sigma_k}{\sqrt{\overline{b_k^2}}}\right) \mathbf{v}_k \cdot \mathbf{x}'$$

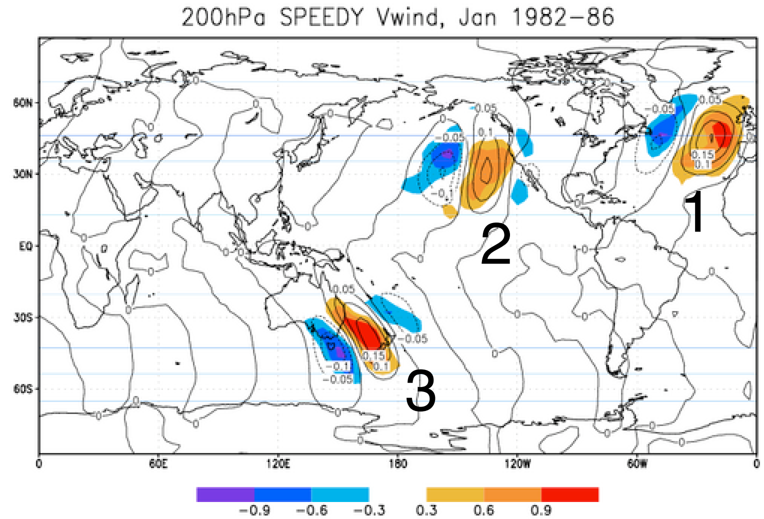
is the best representation of the original 6-hour forecast error anomalies $\mathbf{x}^{e\prime}$ in terms of the current anomalous forecast state $\mathbf{x}'$.

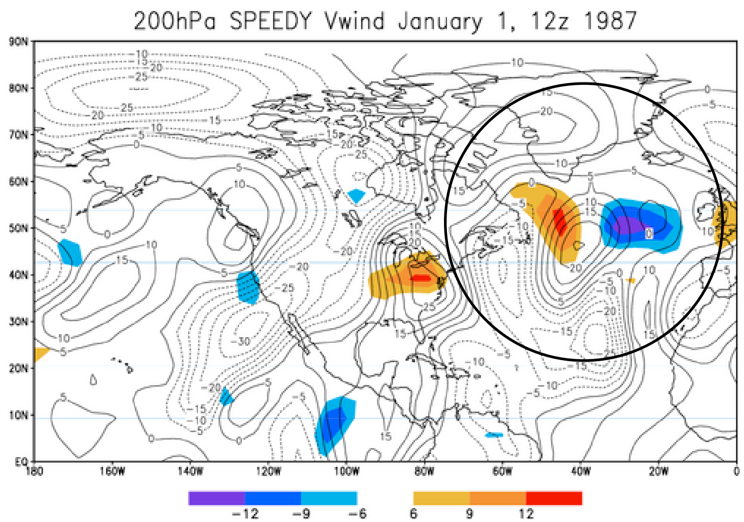
IV. State-Dependent Correction

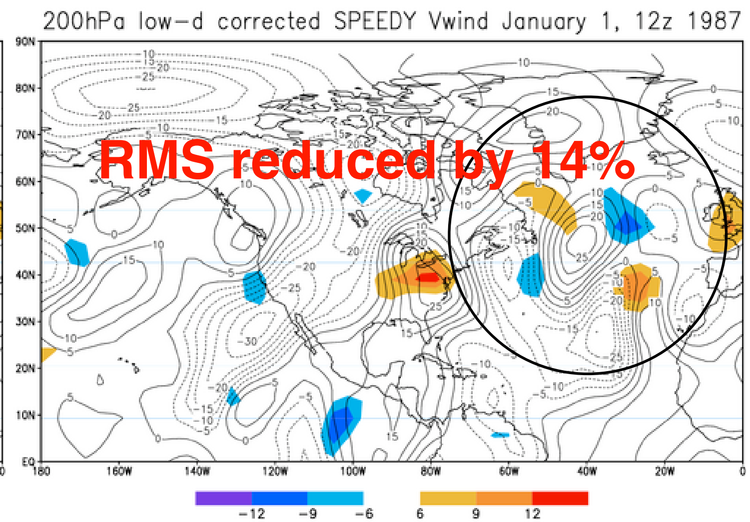
200hPa V-wind
Error (shades)
and State (contour)

12 hr forecasts

debiased low-d corrected





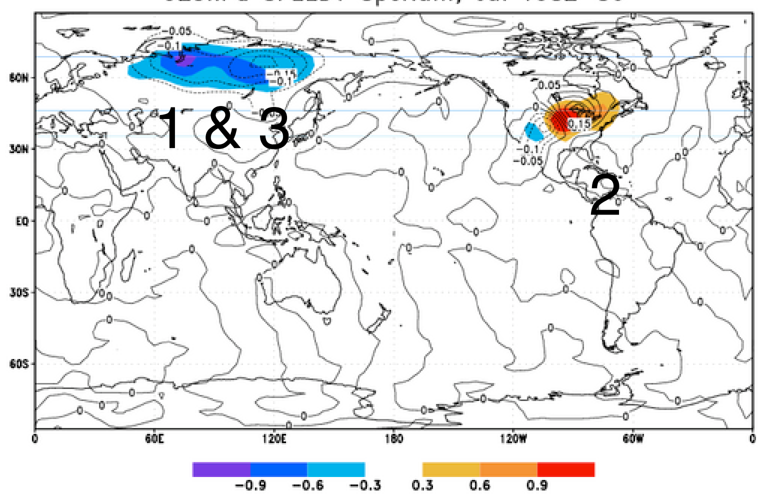


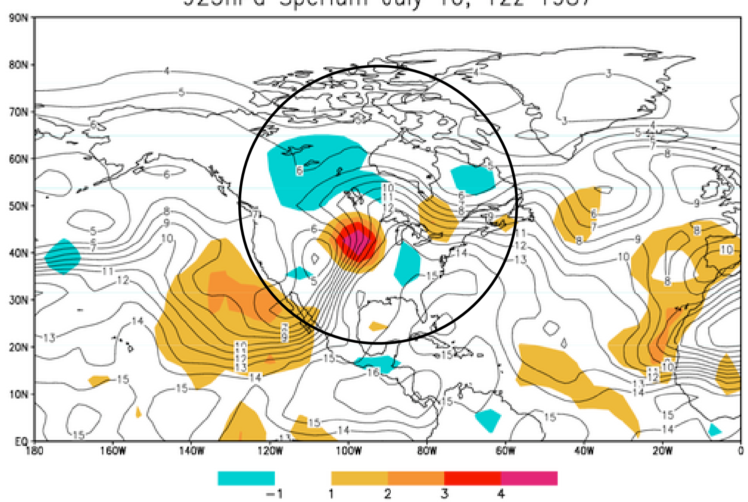
IV. State-Dependent Correction

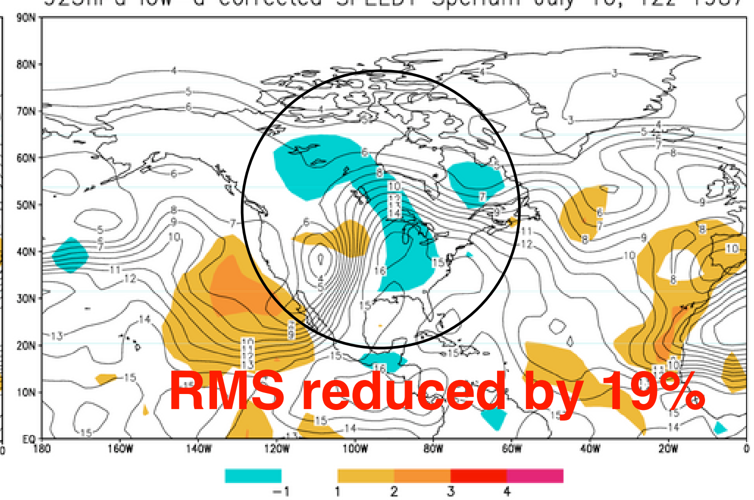
925hPa Specific Humidity
Error (shades)
and State (contour)

12 hr forecasts

debiased low-d corrected







IV. State-Dependent Correction

We measure the forecast improvement using Leith's (univariate) dense and sparse correction operators and our low-dimensional approximation.

	Dense Leith	Sparse Leith	Low-Dim
Flops per time step	$O(N_{gp}^3)$	$O(N_{gp}^2)$	$O(N_{gp})$
Global Improvement	-8% (-4hr)	2% (1hr)	4% (2hr)
N. American Improvement	-6% (-3hr)	4% (2hr)	6% (3hr)

Chart contains average January 1987 improvement over state-independent corrected forecasts. Correction is more effective in regions where the heterogeneous correlations ρ are large.

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- State-independent correction of SPEEDY monthly bias dramatically improves forecasts

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  - is 10 orders of magnitude cheaper! (SPEEDY implementation)
  - should work easily with existing data assimilation and ensemble schemes

Future

- Implement with data assimilation and ensemble schemes
- Test implementation on NCEP operational model (?)
- Reduce jumps in reanalysis climatology due to changes in observing system

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