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Adaptive Weather Forecasting using Local Meteorological Information

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In general, meteorological parameters such as temperature, rain and global radiation are important for agricultural systems. Anticipating on future conditions is most often needed in these systems. Weather forecasts then become of substantial importance. As weather forecasts are subject to uncertainties, there is a need in minimising the uncertainties. In this paper, a framework is presented in which local weather forecasts are updated using local measurements. Kalman filtering is used for this purpose as assimilation technique. This method is compared and combined with diurnal bias correction. It is shown that the standard deviation of the forecast error can be reduced up to 6 h ahead for temperature, up to 31 h ahead for wind speed, and up to 3 h for global radiation using local measurements. Combining the method with diurnal bias correction leads to a further increase in performance in terms of both bias and standard deviation.

1. Introduction

Weather forecasts are subject to uncertainties. The goodness of a weather forecast (Murphy, 1993), however, is not evident as objectives are different among users. Murphy identified three types of goodness: consistency, quality and value. Consistency, mainly concerns the meteorologist. Quality is related to the match between forecasts and observations. Value, defined by economic benefits, is mainly of interest to users. The types of goodness are related to one another. For instance, if an economic decision is based on a weather forecast, the relation between quality and value is defined by the user and depends on the type of problem (Buizza, 2001).

In general, meteorological parameters such as temperature, rain and global radiation are important for agricultural systems. Anticipating on future conditions is most often needed in these systems. Weather forecasts then become of substantial importance. The uncertainties of weather forecasts have a direct effect on the uncertainty of the system states (Atzema, 1995). If control strategies are used that anticipate to (changing) weather conditions (Chalabi *et al.*, 1996; Keesman *et al.*, 2003), the goodness of weather forecasting is related to value. The anticipating control strategies, however, are heavily based on the system model and as such on the quality of the forecast. Therefore, the quality (or accuracy) of the weather forecasts is examined in this paper.

When very short-term weather forecasts are needed, a simple method such as 'lazy man weather prediction' (Tap *et al.*, 1996) or a more sophisticated method based on neural networks (Coelho *et al.*, 2002) can be used. However, when forecasts up to a day ahead or more are needed, commercially available weather forecasts are more reliable.

Although work has been undertaken to improve meteorological models (Palmer *et al.*, 1997) local conditions are not covered by these models. Expert and historical knowledge of a specific location is needed to improve the local weather forecast. In addition, the forecast can be further improved by adaptive techniques using local observations. In previous work, it has been shown that biases of meteorological models can be reduced for short-term forecasts (up to 48 h) (Homleid, 1995). A comparable result has been obtained for maximum and minimum temperature forecasts (Galanis

Notation

$\boldsymbol{A}$	system matrix	$\boldsymbol{R}$	observation noise covariance matrix
$\boldsymbol{B}$	input matrix	$\boldsymbol{u}$	input vector
$\boldsymbol{C}$	observation matrix	$\boldsymbol{v}$	observation noise
E	expectation value	V	standard deviation of the observation noise
$\boldsymbol{G}$	noise matrix	$\boldsymbol{w}$	system or input noise
i	index number	$\boldsymbol{W}$	correlation matrix
$\boldsymbol{I}$	identity matrix	W	standard deviation of the system or input noise
k	discrete-time counter	$\boldsymbol{x}$	state vector
$\boldsymbol{K}$	Kalman gain matrix	$\boldsymbol{y}$	output vector
M	final forecast horizon	$\boldsymbol{0}$	null matrix
$\boldsymbol{p}$	length of output vector	$\boldsymbol{\Phi}$	system transition matrix
$\boldsymbol{P}$	covariance matrix of the estimated system states	$\mathbb{N}$	set of natural numbers
$\boldsymbol{Q}$	system or input noise covariance matrix	$\mathbb{R}$	set of real numbers

& Anadranistakis, 2002). Both methods are based on prediction of forecast errors. The errors obtained are then added to the available forecasts.

The main purpose of this paper is to show that local forecasts can be improved by using local meteorological information. This improvement is based on reduction of the standard deviation of the forecast error. The Kalman filter (Gelb, 1974) is used to update local weather forecasts, where the covariance matrices can be obtained from actual and historical data. In this paper, the temperature at research location 'De Bilt' in the Netherlands is chosen to demonstrate the procedure. In order to show the wide applicability of the procedure, results of wind speed and global radiation are included as well.

2. Background

2.1. *Weather data*

The weather data normally used for agriculture, are derived from local measurements and short-term forecasts. Results are presented for the temperature at 2 m, wind speed and global radiation at location 'De Bilt' in the Netherlands. Data used for analysis range from March 2001 until December 2003.

The local measurements are available with an hourly interval. The weather forecasts become available every 6 h at 0100, 0700, 1300 and 1900 coordinated universal time (UTC) and consist of forecasts from 0 to 31 h ahead with an hourly interval. These external data were extracted from the global forecast system (GFS) model formerly known as Aviation (AVN) model. The model was modified during the period of study several times. The forecasts were possibly adjusted by a meteorologist.

2.2. *Diurnal bias corrections*

Systematic errors from numerical weather prediction models can be filtered (Homleid, 1995; Bhattacharya, 2000; Galanis & Anadranistakis, 2002). In this case, the prediction errors are assumed to vary only in a 24 h context. The estimated prediction errors at forecast times are added to the forecasts independent from the forecast horizon. These corrected forecasts give good results for bias reduction. However, reductions in standard deviations of the forecasts error are, if any, low. Due to homogeneity of the weather in the Netherlands, *i.e.* in general no large bias is expected, our main focus is to obtain reduction of the standard deviation of the forecast error.

2.3. *Forecasting system and Kalman filter*

Local measurements are used to update the short-term forecasts. The updating algorithm uses a linear, time-varying system in state-space form that describes the evolution of the forecasts. Every stochastic l-steps ahead forecast at time instant k is treated as a state variable $x_l(k)$ where $l = 0, \ldots, M$ (final forecast horizon). Consequently, $x_0(k)$ represents the actual state, $x_1(k)$ the one-step ahead forecast, *etc*. The forecasts that become available at time instant k are treated as deterministic input $u(k)$. A discrete-time state-space notation is used to represent the 'forecasting' system

$$\boldsymbol{x}(k+1) = \boldsymbol{A}(k)\boldsymbol{x}(k) + \boldsymbol{B}(k)\boldsymbol{u}(k) + \boldsymbol{G}(k)\boldsymbol{w}(k) \quad (1)$$

$$\boldsymbol{y}(k) = \boldsymbol{C}\boldsymbol{x}(k) + \boldsymbol{v}(k) \quad (2)$$

where $\boldsymbol{x}(k) \in \mathbb{R}^{M+1}, u(k) \in \mathbb{R}^{M+1}$ and $\boldsymbol{y}(k) \in \mathbb{R}^p$ with p the number of measurements. Furthermore, it is assumed that the disturbance input $\boldsymbol{w}(k)$ (so called 'system noise') and measurement noise $\boldsymbol{v}(k)$ are zero-mean

Gaussian random sequences with

$$E[\boldsymbol{w}(k)] = 0, \quad E[\boldsymbol{w}(k)\boldsymbol{w}^{\mathrm{T}}(k)] = \boldsymbol{Q} \tag{3}$$

$$E[\boldsymbol{v}(k)] = 0, \quad E[\boldsymbol{v}(k)\boldsymbol{v}^{\mathrm{T}}(k)] = \boldsymbol{R} \tag{4}$$

where $\boldsymbol{w}(k)^{\mathrm{T}}$ denotes the transpose of $\boldsymbol{w}(k)$. The matrices $\boldsymbol{A}(k), \boldsymbol{B}(k), \boldsymbol{C}, \boldsymbol{G}(k), \boldsymbol{Q}$ and $\boldsymbol{R}$ are system dependent and will be defined in the subsequent sections. Furthermore, in what follows the matrices $\boldsymbol{C}$, $\boldsymbol{Q}$ and $\boldsymbol{R}$ are constant (time-invariant) matrices whereas $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{G}$ are time-varying.

In the next sections, the so-called Kalman filter is used to update local weather forecasts. Given a system in state-space form [Eqns (1) and (2)] and a measured output $\boldsymbol{y}(k)$, the Kalman filter estimates the states at time instance k with the smallest possible error covariance matrix $\boldsymbol{P}$ (Kalman, 1960). The following Kalman filter equations for the discrete-time system [Eqns (1) and (2)] are used to estimate the new states (*i.e.* updated l-steps ahead forecasts) when new observations become available

$$\hat{\boldsymbol{x}}(k+1|k) = \boldsymbol{A}(k)\hat{\boldsymbol{x}}(k|k) + \boldsymbol{B}(k)\boldsymbol{u}(k) \tag{5}$$

$$\boldsymbol{P}(k+1|k) = \boldsymbol{A}(k)\boldsymbol{P}(k|k)\boldsymbol{A}(k)^{\mathrm{T}} + \boldsymbol{G}(k)\boldsymbol{Q}\boldsymbol{G}(k)^{\mathrm{T}} \tag{6}$$

$$\boldsymbol{K}(k+1) = \boldsymbol{P}(k+1|k)\boldsymbol{C}^{\mathrm{T}}[\boldsymbol{C}\boldsymbol{P}(k+1|k)\boldsymbol{C}^{\mathrm{T}} + \boldsymbol{R}]^{-1} \tag{7}$$

$$\hat{\boldsymbol{x}}(k+1|k+1) = \hat{\boldsymbol{x}}(k+1|k) + \boldsymbol{K}(k+1)[\boldsymbol{y}(k+1) - \boldsymbol{C}\hat{\boldsymbol{x}}(k+1|k)] \tag{8}$$

$$\boldsymbol{P}(k+1|k+1) = \boldsymbol{P}(k+1|k) - \boldsymbol{K}(k+1)\boldsymbol{C}\boldsymbol{P}(k+1|k) \tag{9}$$

where $\hat{\boldsymbol{x}}(k+1|k)$ denotes the estimate of state $\boldsymbol{x}$ at time instant $k+1$ given the state at k, and $\boldsymbol{A}(k)^{\mathrm{T}}$ is the transpose of $\boldsymbol{A}(k)$. Furthermore, $\boldsymbol{K}(k+1)$, known as Kalman gain, denotes the weighting matrix related to the prediction error $[\boldsymbol{y}(k+1) - \boldsymbol{C}\hat{\boldsymbol{x}}(k+1|k)]$.

2.4. *Covariances*

In the Kalman filter covariance matrices $\boldsymbol{P}, \boldsymbol{Q}$ and $\boldsymbol{R}$ play an important role. As can be seen from Eqns (6), (7) and (9), given an initial covariance matrix $\boldsymbol{P}(0)$ (typically $\boldsymbol{P}(0) = 10^6\boldsymbol{I}$, with $\boldsymbol{I}$ the identity matrix) the corrected covariance matrix $\boldsymbol{P}(k+1|k+1)$ can be calculated. The matrix $\boldsymbol{R}$ is related to the measurement noise. It can mostly be determined from the sensor characteristics. The key problem here is how to choose $\boldsymbol{Q}$, the covariance matrix related to the disturbance inputs or system noise. This system noise covariance matrix of the short-term forecast can be determined from historical data by comparing forecasts to observations.

3. Local adaptive short-term forecasting

The general system (1)–(2) is further elaborated by defining the system variables and matrices. The dimension of both the state ($\boldsymbol{x}$) and input ($\boldsymbol{u}$) vectors is 32. The output ($\boldsymbol{y}$) is the observed meteorological parameter and is a scalar. The system matrix $\boldsymbol{A}$ is chosen such that at every time instance when a new observation becomes available, the states are moved up one place. As a consequence, the state vector x always represents the forecast horizon (from 0 to $31-k+1$ hours ahead). Subsequently, the effective system reduces as long as there are no new external forecasts available. Given the assumption that new external forecasts are better than updated old forecasts, the old states are reset and the new initial state is fully determined by the new forecast as soon as new one becomes available at time instance k^*. The system matrices are defined as follows:

$$\boldsymbol{A}(k) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}, \quad \forall k \neq k^*, \ \ k \in \mathbb{N} \tag{10}$$

$$\boldsymbol{A}(k) = \boldsymbol{0}, \quad \forall k = k^*, \ \ k \in \mathbb{N} \tag{11}$$

$$\boldsymbol{C} = [1 \ \ 0 \ \ \cdots \ \ 0] \tag{12}$$

The system noise or disturbance input is only relevant at time instant $k = k^*$, *i.e.* when a new external forecast becomes available. The time-varying system matrices $\boldsymbol{B}$ and $\boldsymbol{G}$ are defined as follows:

$$\boldsymbol{B}(k) = \boldsymbol{G}(k) = \boldsymbol{I} \quad \forall k = k^*, \ \ k \in \mathbb{N} \tag{13}$$

$$\boldsymbol{B}(k) = \boldsymbol{G}(k) = \boldsymbol{0} \quad \forall k \neq k^*, \ \ k \in \mathbb{N} \tag{14}$$

where $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{G} \in \mathbb{R}^{(M+1)\times(M+1)}$ and $\boldsymbol{C} \in \mathbb{R}^{M+1}$ $(M = 31)$. The Kalman filter can now be introduced to use local measurements for improving short-term weather forecasts. The stability of the filter is proven in Appendix A.

4. Results

From the forecast and observation files the average forecast error (*i.e.* forecasts-observations) of the predicted meteorological parameter was obtained and the covariance matrix $\boldsymbol{Q}$ of the forecast error was calculated. The data used to obtain this matrix covered the period from 1 March 2001 until 1 March 2002. As an example the covariance matrix of the 2 m temperature is given in

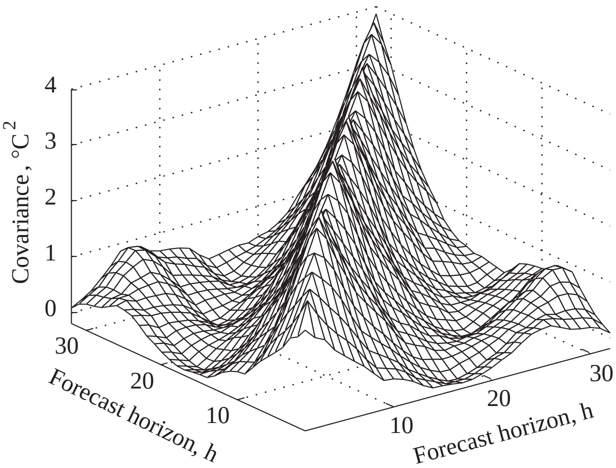

Fig. 1. Covariance matrix of the short-term forecast error

Fig. 1. On the diagonal the variance of the forecast error is given along the forecast horizon. Furthermore, $\boldsymbol{Q}$ is symmetrical, *i.e.* $\mathrm{cov}(x,y)=\mathrm{cov}(y,x)$.

For all meteorological parameters the same procedure was used. The Kalman filter was run over the period 1 December 2002 until 1 November 2003. The updated forecasts were compared with the original external forecasts after a new external input entered the system and a measurement was taken.

Furthermore a comparison is made between the previously described method and the method described in Homleid (1995) which is based on diurnal bias corrections.

4.1. *Air temperature at 2 m*

4.1.1. *Local adaptive forecasting*

The variance of the observational noise $\boldsymbol{R}$ [see Eqn (4)] was assumed to be 0·1 °C^2. Given $\boldsymbol{A},\boldsymbol{B},\boldsymbol{C},\boldsymbol{G}$ [Eqns (10)–(14)], $\boldsymbol{Q}$ and $\boldsymbol{R}$, the Kalman filter could then be implemented. The results are presented in *Fig. 2*. From this figure it can be seen that using a single observation, forecasts up to 6 h ahead can be positively adjusted. The periodicity that is observed is due to interpolation; the original model output, given at a 3 h interval, is interpolated by the weather agency to an hourly interval.

4.1.2. *Diurnal bias correction*

There are two important design criteria for this Kalman filter. First, the covariance matrix $\boldsymbol{Q}=W^2\boldsymbol{W}$ should be chosen where $\boldsymbol{W}$ is the correlation matrix and W^2 the variance. In this simulation experiment the correlation matrix is chosen similar as described by Homleid (1995), *i.e.* exponentially decaying until $k+12$ and then rising again until $k+24$. No difference in correlation is made between successive hours. The second design criterion is the value W/V where V denotes the standard deviation of the observation. The W/V-ratio appears to be crucial. When the chosen ratio is too large, the standard deviation of the forecast error increases compared to the original forecasts after a few forecast hours. When it is too small, standard deviations are hardly reduced. In this study, the optimum (in terms of minimum average standard deviation of the forecast horizons) is searched by a line search procedure. The results of diurnal bias correction with a W/V ratio of 0·01 are given in *Fig. 3*. It can be seen that a structural prediction error still remains and that standard deviations can be slightly reduced for the whole forecast range.

4.1.3. *Local adaptive forecasting on bias corrected forecasts*

While the local adaptive forecasting method works specifically on the short term (<6 h), the diurnal bias correction gains on the longer forecast horizons. A logical combination would be first to implement the diurnal bias correction and then use the local adaptive forecasting technique on the corrected forecasts. The results are shown in *Fig. 4*. It can be seen that local adaptive forecasting has the same behaviour when it is used on pre-filtered data as it has on the original data. In addition, the effect of decreasing standard deviation lasts longer when pre-filtered data are used (6 *versus* 9 h).

4.2. *Wind speed*

In comparison with the 2 m temperature the system noise covariance matrix of the diurnal bias correction is taken equal and the W/V was found to be optimal around 0·02. The variance of the observation noise of the wind speed in the local adaptive short-term system was assumed to be 0·1 m^2 s^{-2}. The results of all three procedures are summarised in *Fig. 5*. Again, both methods increase forecast performance when used separately. The bias reduction is in this case much more pronounced than for the temperature forecast. When the methods are combined, an extra increase in performance can be seen. The main part, of course, is found at the very short term (<6 h) but it remains on a small scale for the whole forecast horizon.

4.3. *Global radiation*

Diurnal bias correction was implemented for temperature and wind speed. However, for global radiation, the covariance matrix must be adjusted. At night, no radiation is available and so no correlation is present. The length of the nights also vary during the year. For simplicity, it is assumed that for the whole year no

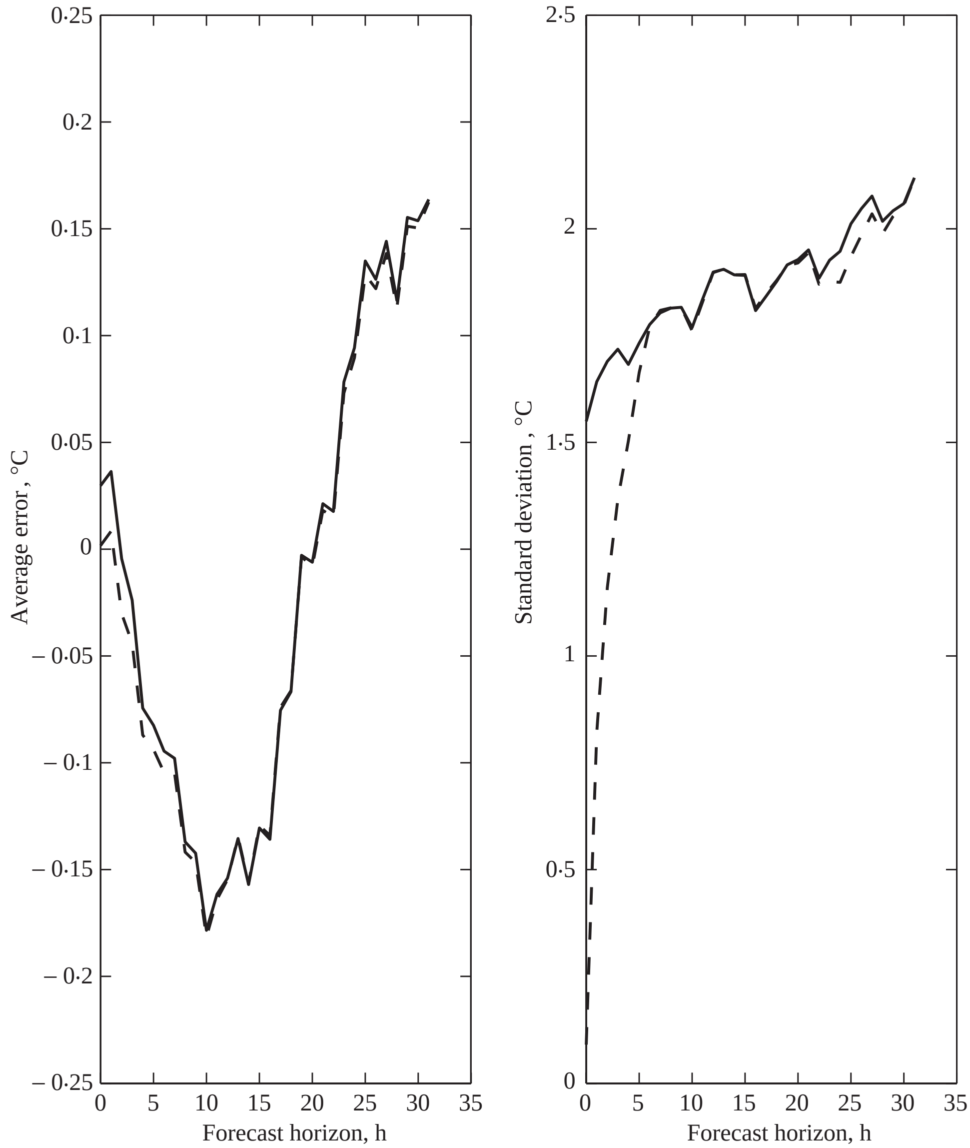

Fig. 2. Average forecast error and standard deviation of the original forecasts (—) and the filtered forecasts (---) for the 2 m temperature

correlation is needed between 1700 and 0500 UTC. The remaining correlations are kept the same as for the temperature case. The optimum W/V ratio appeared to be around 0·02. For local adaptive forecasting, no special covariance matrices were used. In *Fig. 6* the average error and standard deviation for all procedures are given. The 12 h dependency is clearly seen by the large periodic variations for the average error of the original forecast. The diurnal bias correction works well for the 0–12 h horizon. From this point the bias increases until 24 h forecast horizon where it again approaches zero. Furthermore, the same results are seen for global radiation as for temperature and wind speed. The standard deviation is reduced for both procedures. In this case the local adaptive forecasting procedure works only until a 3 h forecast horizon. However, when both procedures are combined, the standard deviation is reduced for more than 5 h horizon.

5. Discussion

The improvement of weather forecasts with the Kalman filter for the local adaptive forecasting procedure largely depends on the choice of covariance matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$. The period over which the covariance

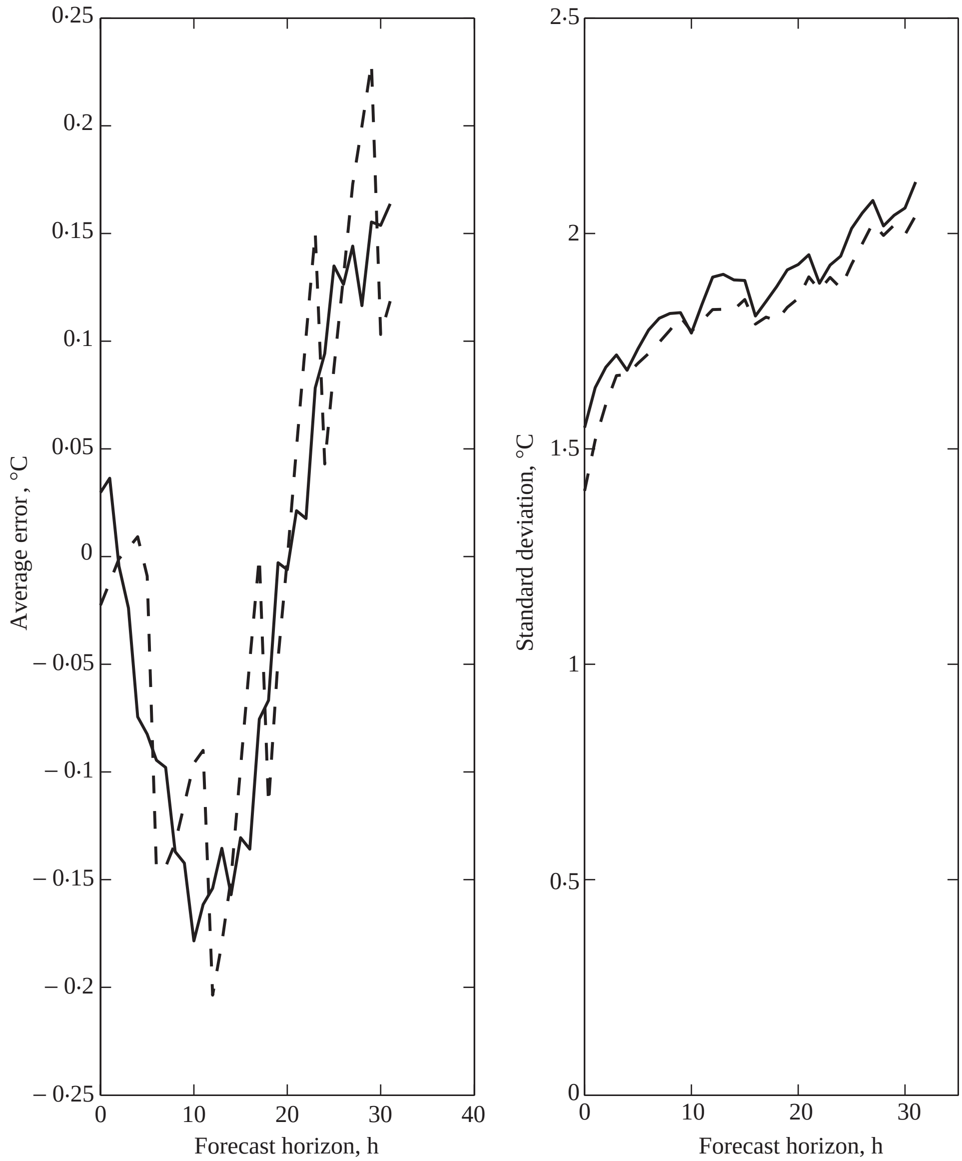

Fig. 3. Average forecast error and the standard deviation of the original forecasts (—) and of the diurnal bias corrected forecasts (- - -) of the 2 m temperature

matrices are determined plays an important role. In this study the covariance matrices are defined over a period of a year. Herewith seasonal effects are neglected. When seasonal effects on variability of the forecast error are expected $\boldsymbol{Q}$ and $\boldsymbol{R}$ must be determined from the seasons of a previous year. Apart from the seasonal effects, changes in weather forecasting models can effect the variability of the forecast error. Consequently, a 'window' for the covariances can be considered. The weather forecasting models were adjusted frequently during the period of research (NCEP, 2005). However, the Kalman filter still showed a good performance. As an alternative to the stochastic filtering approach one may also consider an unknown-but-bounded error approach (Schweppe, 1973; Keesman, 1997).

In Section 3, a low variance is assumed for the local observation. This might be true for the measurement device itself but the variance also depends on how and where the device is installed. The measurement represents the weather disturbance input of the real system under study. Weather forecasts are generally made only for a few meteorological stations. As both measurements and forecasts used in this paper were dedicated to the same specific meteorological location, larger deviations

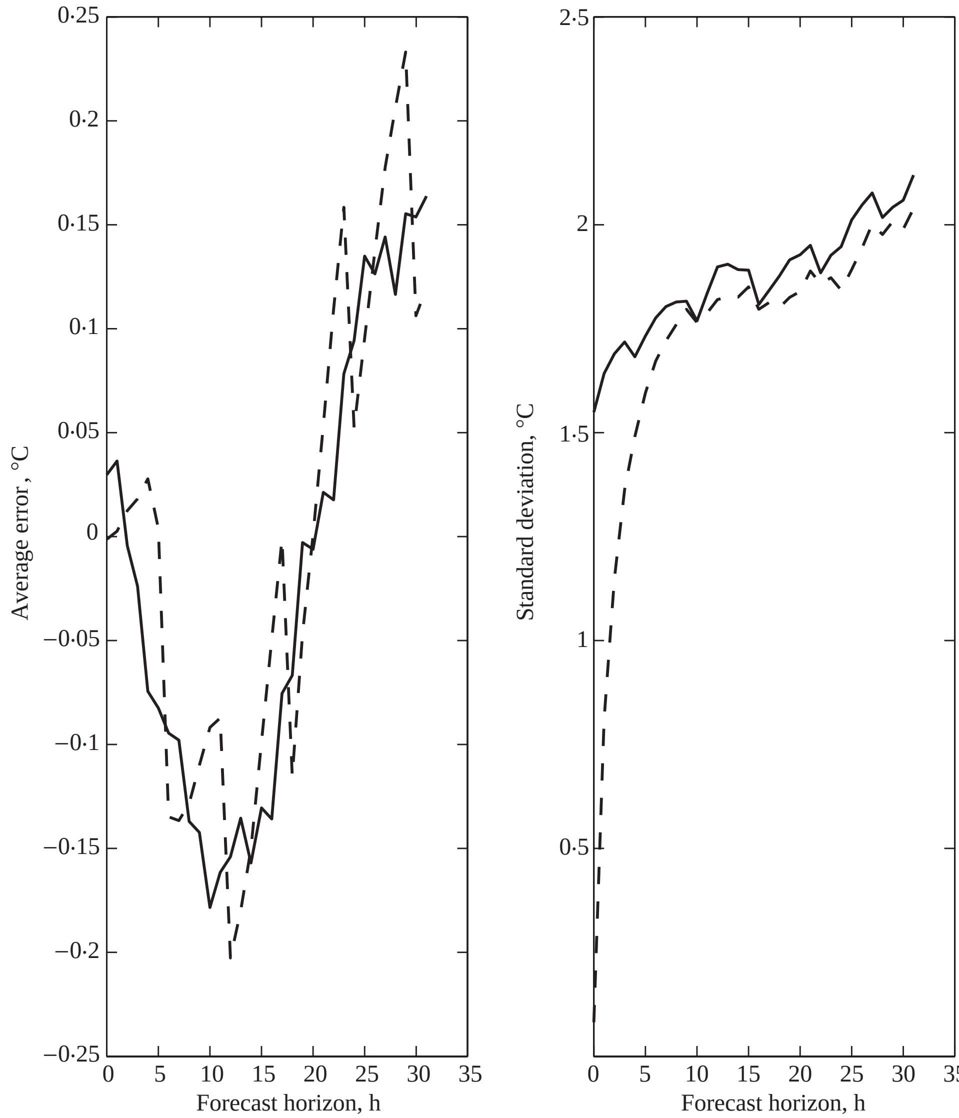

Fig. 4. Average forecast error and the standard deviation of the original forecasts (—) and of the combined diurnal bias corrected and local adaptive filtered forecasts (- - -) of the 2 m temperature

between forecasts and measurements are expected for specific agricultural systems. This is yet another reason why an extra improvement of the forecast is expected by using the Kalman filter in relation to the real system.

When considerable biases are present, the diurnal bias correction gives good results for bias reduction. When the W/V ratio is chosen correctly, also a reduction in standard deviation is obtained. This reduction can then last for the whole forecast range. However, when the ratio chosen is too large then a loss in performance is seen. When values are too low no change in forecasts is apparent. In this study a line search procedure was performed to find the optimum ratio. However, the optimum value for this ratio highly depends on the season. For temperature, an optimum value approaching 0 was found for the winter while during summer the optimum value increased to 0·03. A time-varying ratio based on past forecast errors gave satisfactory results for maximum and minimum daily temperatures (Galanis & Anadranistakis, 2002). However, the problem remains because the value is always obtained from past data. A time-varying ratio which is based on future uncertainties of the forecasts, *i.e.* uncertainties given by ensemble forecasts, or expected change in weather conditions is worthwhile to investigate.

The correlation matrix must be determined for every meteorological parameter. For global radiation, this matrix must also depend on the time of the

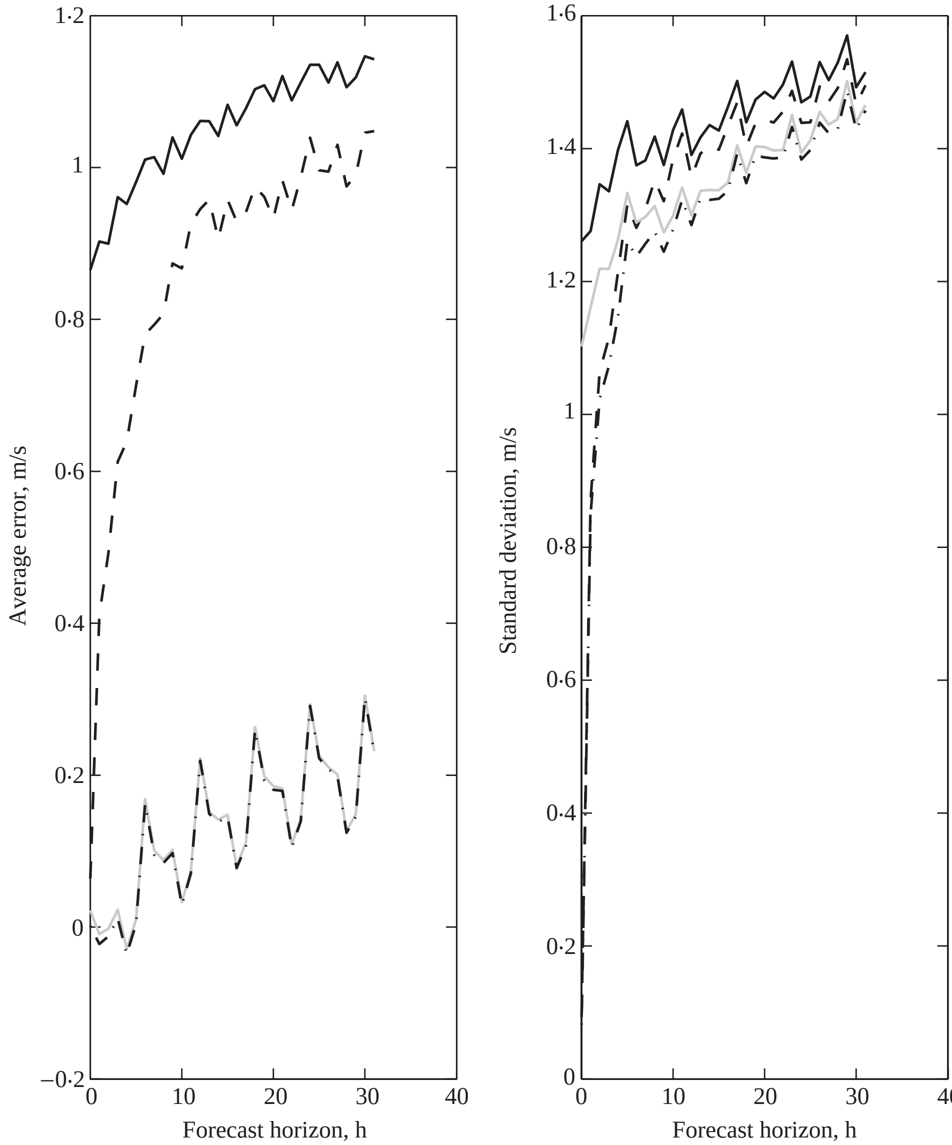

Fig. 5. Average forecast error and the standard deviation of the original forecasts (—), diurnal bias corrected forecasts (· · ·), local adaptive filtered forecasts (- - -) and of the combined diurnal bias corrected and local adaptive filtered forecasts (-·-) of the wind speed

year. This matrix can be found for every meteorological parameter at a specific location by time series analysis.

When both procedures are run in series, the results show that both bias correction as well as standard deviation reduction are obtained, *i.e.* the methods are complementary to each other. It is worthwhile to investigate whether both methods can be integrated into a single system. For instance, the diurnal bias correction supposes a full correlation of the diurnal pattern, *i.e.* a certain error obtained now will also be present tomorrow. A more valid assumption is that this relation is exponentially decaying. As a result the W/V ratio chosen can be larger.

6. Conclusions

The improvement of local weather forecasts using local measurements is demonstrated in this paper for both diurnal bias correction and local adaptive forecasting. If only very short-term forecasts (several hours ahead) are needed local adaptive forecasting is the proper method to use. If only bias correction is sufficient, the diurnal bias correction procedure is the best choice. Generally, first a diurnal bias correction followed by the local adaptive forecasting procedure is recommended because this gives the best results in both bias and standard deviation reduction.

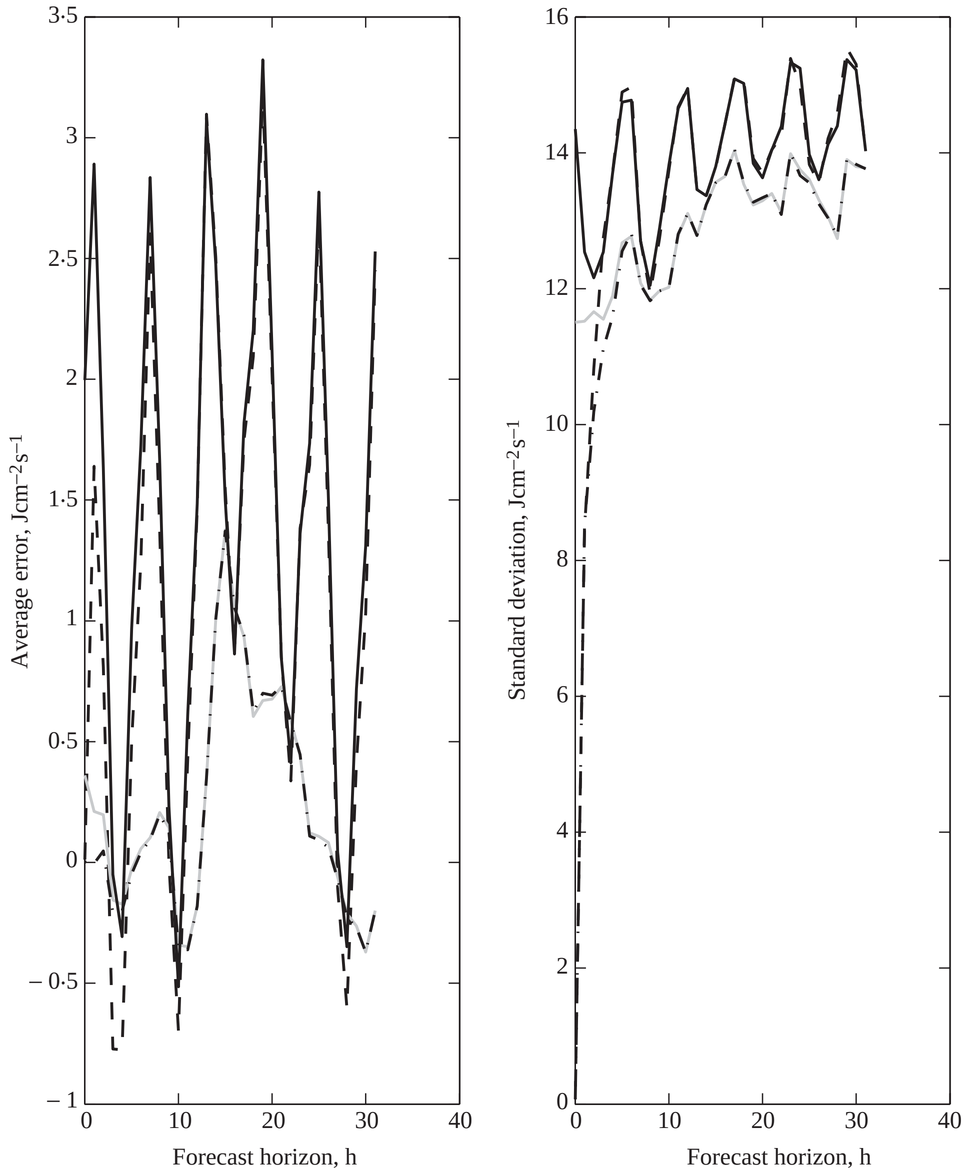

Fig. 6. Average forecast error and the standard deviation of the original forecasts (—), diurnal bias corrected forecasts (· · ·), local adaptive filtered forecasts (- - -) and of the combined diurnal bias corrected and local adaptive filtered forecasts (-·-) of the global radiation

Slow and fast dynamics of the weather are then incorporated.

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Appendix A: Stability of filter

The proposed forecasting system in Section 3 is checked on stability. Recall from systems theory that if $(\boldsymbol{A}, \boldsymbol{G})$ is stabilizable and $(\boldsymbol{C}, \boldsymbol{A})$ is detectable then $\lim_{t\to\infty} \boldsymbol{P}(t) = \boldsymbol{P}_1$, where $\boldsymbol{P}_1$ is the solution of an algebraic Riccati equation. A sufficient condition for detectability of $(\boldsymbol{C}, \boldsymbol{A})$ is that it is observable. A sufficient condition for stabilizability of $(\boldsymbol{A}, \boldsymbol{Q})$ is that it is controllable.

As the system noise matrix $\boldsymbol{G}$ from Eqn (1) is time-varying the following theorem should hold for complete controllability (Kwakernaak & Sivan, 1972): the system is controllable if and only if for every i_0 there exists an $i_1 \geqslant i_0 + 1$ such that the symmetric non-negative definite matrix:

$$\boldsymbol{W}(i_0, i_1) = \sum_{i=i_0}^{i_1-1} \boldsymbol{\Phi}(i_1, i+1)\boldsymbol{G}(i)\boldsymbol{G}^{\mathrm{T}}(i)\boldsymbol{\Phi}^{\mathrm{T}}(i_1, i+1) \qquad \text{(A1)}$$

is non-singular. Here, $\boldsymbol{\Phi}$ is the transition matrix of the system and is defined by

$$\boldsymbol{\Phi}(i+1, i_0) = \boldsymbol{A}(i)\boldsymbol{\Phi}(i, i_0) \quad i \geqslant i_0 \qquad \text{(A2)}$$

$$\boldsymbol{\Phi}(i_0, i_0) = \boldsymbol{I} \qquad \text{(A3)}$$

The system noise matrix $\boldsymbol{G}$ swaps between the values $\boldsymbol{I}$ and $\boldsymbol{0}$. If there exists a time instant k for which $\boldsymbol{G}(k) = \boldsymbol{I}$ then, for all $i_0 \leqslant k$ and $i_1 = k + 1$, Eqn (A1) can be rewritten as

$$\begin{aligned}
&\boldsymbol{W}(i_0, k+1) \\
&= \sum_{i=i_0}^{k} \boldsymbol{\Phi}(k+1, i+1)\boldsymbol{G}(i)\boldsymbol{G}^{\mathrm{T}}(i)\boldsymbol{\Phi}^{\mathrm{T}}(k+1, i+1) \\
&= \boldsymbol{\Phi}(k+1, k+1)\boldsymbol{G}(k)\boldsymbol{G}^{\mathrm{T}}(k)\boldsymbol{\Phi}^{\mathrm{T}}(k+1, k+1) \\
&\quad + \sum_{i=i_0}^{k-1} \boldsymbol{\Phi}(k+1, i+1)\boldsymbol{G}(i)\boldsymbol{G}^{\mathrm{T}}(i)\boldsymbol{\Phi}^{\mathrm{T}}(k+1, i+1) \\
&= \boldsymbol{I} + \sum_{i=i_0}^{k-1} \boldsymbol{\Phi}(k+1, i+1)\boldsymbol{G}(i)\boldsymbol{G}^{\mathrm{T}}(i)\boldsymbol{\Phi}^{\mathrm{T}} \\
&\quad \times (k+1, i+1) \qquad \text{(A4)}
\end{aligned}$$

The matrix $\boldsymbol{W}(i_0, k+1)$ is non-singular and therefore the system is controllable when future inputs are available.

As the matrix A is time-varying the following result should hold for complete observability (Kwakernaak & Sivan, 1972): the system is observable if and only if for every i_1 there exists an $i_0 \leqslant i_1 - 1$ for which the non-negative definite matrix:

$$\boldsymbol{M}(i_0, i_1) = \sum_{i=i_0+1}^{i_1} \boldsymbol{\Phi}^{\mathrm{T}}(i, i_0+1)\boldsymbol{C}^{\mathrm{T}}(i)\boldsymbol{C}(i)\boldsymbol{\Phi}(i, i_0+1) \qquad \text{(A5)}$$

is non-singular. Transition matrix $\boldsymbol{\Phi}$ is again defined by Eqns (A2) and (A3). Complete observability cannot be obtained through this theorem because of the term

$$\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C} = \left[\begin{array}{c|c} 1 & \boldsymbol{0} \\ \hline \boldsymbol{0} & \boldsymbol{0} \end{array}\right] \forall i.$$

The matrix $\boldsymbol{M}$ shall therefore always be singular. Hence, stability cannot be proven by these classical methods. However, the following theorem can be proven:

Theorem 1. *Given discrete-time state-space system* (1)–(2) *with matrices defined in Eqns* (10)–(14), *i.e. short-term forecasting system, and given no future inputs, so that* $k \neq k^*$ *the following holds*:

$$\lim_{k\to\infty} \boldsymbol{P}(k) = \boldsymbol{P}_\infty$$

where $\boldsymbol{P}_\infty = 0$, is the solution of the algebraic Riccati equation (Schweppe, 1973):

$$\boldsymbol{P}_\infty = \boldsymbol{P}_\infty(1|0) - \boldsymbol{P}_\infty(1|0)C^{\mathrm{T}}[\boldsymbol{C}\boldsymbol{P}_\infty(1|0)\boldsymbol{C}^{\mathrm{T}} + \boldsymbol{R}]^{-1} \times \boldsymbol{C}\boldsymbol{P}_\infty(1|0)$$

with

$$\boldsymbol{P}_\infty(1|0) = \boldsymbol{A}\boldsymbol{P}_\infty\boldsymbol{A}^{\mathrm{T}} + \boldsymbol{G}\boldsymbol{Q}\boldsymbol{G}^{\mathrm{T}}$$

Given future inputs at $k = k^*$, *the following holds*:

$$\boldsymbol{P}(k^* + 1|k^* + 1) = \boldsymbol{Q} - \boldsymbol{Q}\boldsymbol{C}^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{Q}\boldsymbol{C}^{\mathrm{T}} + \boldsymbol{R})^{-1}\boldsymbol{C}\boldsymbol{Q}.$$

Proof. Given $k \neq k^*\ \forall k$, matrix $\boldsymbol{A}$ is time-invariant of the form of Eqn (10) and $\boldsymbol{G} = \boldsymbol{0}$ [Eqn (14)]. Consequently, Eqn (6) reduces to $\boldsymbol{P}(k+1|k) = \boldsymbol{A}\boldsymbol{P}(k|k)\boldsymbol{A}^{\mathrm{T}}$. After substituting $\boldsymbol{P}_\infty(1|0) = \boldsymbol{A}\boldsymbol{P}_\infty\boldsymbol{A}^{\mathrm{T}}$, because $\boldsymbol{G} = \boldsymbol{0}$, the algebraic Riccati equation becomes:

$$\begin{aligned}\boldsymbol{P}_\infty &= \boldsymbol{A}\boldsymbol{P}_\infty\boldsymbol{A}^{\mathrm{T}} - \boldsymbol{A}\boldsymbol{P}_\infty\boldsymbol{A}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}} \\ &\quad \times(\boldsymbol{C}\boldsymbol{A}\boldsymbol{P}_\infty\boldsymbol{A}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}} + \boldsymbol{R})^{-1}\boldsymbol{C}\boldsymbol{A}\boldsymbol{P}_\infty\boldsymbol{A}^{\mathrm{T}} \\ &= \boldsymbol{A}\boldsymbol{P}_\infty\boldsymbol{A}^{\mathrm{T}}[\boldsymbol{I} - \boldsymbol{C}^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{A}\boldsymbol{P}_\infty\boldsymbol{A}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}} + \boldsymbol{R})^{-1} \\ &\quad \times\boldsymbol{C}\boldsymbol{A}\boldsymbol{P}_\infty\boldsymbol{A}^{\mathrm{T}}]\end{aligned} \tag{A6}$$

Consequently, the solution of (A6) is: $\boldsymbol{P}_\infty = \boldsymbol{0}$.

At $k = k^*$, $\boldsymbol{A} = \boldsymbol{0}$ (11) and $\boldsymbol{G} = \boldsymbol{I}$ (13). Hence, Eqn (6) reduces to $\boldsymbol{P}(k^* + 1|k^*) = \boldsymbol{Q}$. Consequently, the updated error covariance matrix is: $\boldsymbol{P}(k^* + 1|k^* + 1) = \boldsymbol{Q} - \boldsymbol{Q}\boldsymbol{C}^{\mathrm{T}}[\boldsymbol{C}\boldsymbol{Q}\boldsymbol{C}^{\mathrm{T}} + \boldsymbol{R}]^{-1}\boldsymbol{C}\boldsymbol{Q}$. Consequently, this covariance matrix only depends on the user-defined matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$. □