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A one-dimensional Kalman filter for the correction of near surface temperature forecasts

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A one-dimensional Kalman filter is proposed for the correction of maximum and minimum near surface (2 m) temperature forecasts obtained by a Numerical Weather Prediction model. In our study we used only one parameter (observed temperature), employing in this way scalar system and observation equations, and a limited time interval (7 days). As a result, our algorithm can be easily run on any PC with only minor technical support. The corresponding results are rather impressive, since the systematic error of our time series almost disappears, and they show the merit of post-processing even when only a simple method is applied.

1. Introduction

It is a common fact that Numerical Weather Prediction (NWP) models exhibit systematic errors in the forecasts of near surface weather parameters. This is a result not only of shortcomings in the physical parameterisation but also of the inability of these models to handle successfully sub-grid phenomena. Furthermore, predictions covering areas that are not close to grid points are usually based on interpolations of the results of the models, a fact which also increases the 'noise' in the final outputs. The 2m-temperature, for example, is one of the most commonly biased variables, where the magnitude of this bias depends, among other factors, on the geographical location and the season.

A first step towards the reduction of these types of error can be made using 'classical' statistical tools, such as linear regression or moving average correction methods. However, frequent changes in the versions of NWP models as well as seasonal alterations lead to analogous changes in the form of the results, a fact which, combined with the need for extended series of data, brings into question the effectiveness of traditional statistical methods.

One of the most convenient approaches to the above-mentioned problems is the use of Kalman filtering (see, for example, Brockwell & Davis 1987; Kalman 1960; Kalman and Bucy 1961). This technique gives excellent results in the correction of systematic errors in any type of prediction, based on the *recursive* combination of recent forecasts and observations. The main advantage is the easy adaptation to any changes in the data being used.

Here we present a simple (one-dimensional) Kalman filter, for the correction of systematic errors in the prediction of maximum and minimum 2m-temperatures of the NWP limited area model Skiron (Kallos et al. 1997). We used only observed temperatures over a restricted period of seven days. This makes our algorithm convenient enough to be used on any simple PC, even those employing MS Office Programs (Excel), with minimal technical support. Such simplicity in no way affects the credibility of the method. On the contrary, the systematic error of the time series almost vanishes, having, at the same time, acceptable variation intervals.

Applications of this filter to stations at different latitudes and with different topography are presented in the final section.

2. Preliminaries

For the convenience of the reader and to make the paper self-contained we present here some basic notions of the general Kalman filter theory. For details we refer to Brockwell & Davis (1987), Homleid (1995), Kalman (1960), Kalman & Bucy (1961) and Persson (1991).

Let $\mathbf{x_t}$ be a vector (the *state vector*) denoting an unknown process at time $\mathbf{t}$ and $\mathbf{y_t}$ a known (observable) relevant vector at the same time. We assume that the change of the process $\mathbf{x}$ from time $\mathbf{t}$-1 to $\mathbf{t}$ is given by *the system equation*:

$$\mathbf{x_t = F_t \cdot x_{t-1} + w_t} \quad (1)$$

and the relation between the observation vector and the unknown one is given by the *observation* (or *measurement*) *equation*:

$$\mathbf{y_t} = \mathbf{H_t} \cdot \mathbf{x_t} + \mathbf{v_t}. \tag{2}$$

Here the coefficient matrices $\mathbf{F_t}$ and $\mathbf{H_t}$, which are called the *system matrix* and the *observation* (*measurement*) *matrix* respectively, must be determined prior to the running of the filter. The same holds for the covariance matrices $\mathbf{W_t}$ of the random vector $\mathbf{w_t}$ and $\mathbf{V_t}$ of $\mathbf{v_t}$. Moreover, these vectors $\mathbf{w_t}$ and $\mathbf{v_t}$ have to follow the normal distribution with zero mean, must be independent, i.e. $E(\mathbf{w_s} \cdot \mathbf{v_t})=0$ for any $s,t \in \mathbf{N}$, and time independent in the sense that $E(\mathbf{w_s} \cdot \mathbf{w_t})=0$ and $E(\mathbf{v_s} \cdot \mathbf{v_t})=0$, for all $s \neq t$.

Kalman filter theory gives a method for the recursive estimation of the unknown state vector $\mathbf{x_t}$ utilising all the observation values $\mathbf{y}$ up to time $\mathbf{t}$. Roughly speaking, the algorithm can be described in the following way. Based on the vector $\mathbf{x_{t-1}}$ and its covariance matrix $\mathbf{P_{t-1}}$ at time t–1, the optimal estimate that we can give for their values at time $\mathbf{t}$ is:

$$\mathbf{x_{t/t-1}} = \mathbf{F_t} \cdot \mathbf{x_{t-1}}, \tag{3}$$

$$\mathbf{P_{t/t-1}} = \mathbf{F_t} \cdot \mathbf{P_{t-1}} \cdot \mathbf{F_t}^{\mathbf{T}} + \mathbf{W_t}. \tag{4}$$

Equations (3) and (4) are also referred to as the *prediction equations*. As soon as the new observation value $\mathbf{y_t}$ becomes known, we calculate the (new) value of $\mathbf{x}$ at time $\mathbf{t}$:

$$\mathbf{x_t} = \mathbf{x_{t/t-1}} + \mathbf{K_t} \cdot (\mathbf{y_t} - \mathbf{H_t} \cdot \mathbf{x_{t/t-1}}), \tag{5}$$

where

$$\mathbf{K_t} = \mathbf{P_{t/t-1}} \cdot \mathbf{H_t}^{\mathbf{T}} \cdot (\mathbf{H_t} \cdot \mathbf{P_{t/t-1}} \cdot \mathbf{H_t}^{\mathbf{T}} + \mathbf{V_t})^{-1} \tag{6}$$

is the most crucial parameter of the filter, the *Kalman gain*. This determines how easily the filter will adjust to any possible new conditions. Finally, the new value of the covariance matrix of the unknown state $\mathbf{x}$ is given by

$$\mathbf{P_t} = (\mathbf{I} - \mathbf{K_t} \cdot \mathbf{H_t}) \cdot \mathbf{P_{t/t-1}}. \tag{7}$$

Equations (5)–(7) are known also as *updating equations*. The initial values $\mathbf{x_0}$, $\mathbf{P_0}$ must be defined before the running of the filter but they do not seriously affect the results of the algorithm, since very soon $\mathbf{x_t}$ and $\mathbf{P_t}$ converge to their 'true' values. However, things are different with the covariant matrices $\mathbf{V_t}$ and $\mathbf{W_t}$. The way that they are calculated during the process crucially affects the final outcome. More precisely, the relation between them affects the Kalman gain and therefore the capability of the filter to fit fast at possible new conditions. In particular, large values for the quotients $\mathbf{w_{ij}}/\mathbf{v_{ij}}$ of the elements of $\mathbf{W}$ and $\mathbf{V}$ respectively, lead to increased adaptability of the filter.

3. A one-dimensional Kalman filter for correcting temperature values

In this section we present a one-dimensional Kalman filter for the correction of systematic errors of 2m-temperature forecasts by a NWP model. To this end, we define the measurement vector $\mathbf{y_t}$ as the difference between observation and the model forecast and the state vector $\mathbf{x_t}$ as the systematic part of this error. We work, therefore, in a one-dimensional state space. Concerning the change of $\mathbf{x}$ in time we have no solid evidence to rely on. As a result, we must assume this change to be random, setting the system coefficient (transition matrix) as $\mathbf{F_t=1}$. Hence, the system equation takes the form:

$$\mathbf{x_t} = \mathbf{x_{t-1}} + \mathbf{w_t}\ . \tag{8}$$

Analogously, the observation (measurement) equation is given by:

$$\mathbf{y_t} = \mathbf{x_t} + \mathbf{v_t}\ . \tag{9}$$

Obviously, $\mathbf{w_t}$, $\mathbf{v_t}$ are here scalar variables of zero mean, since in a different case their non zero part could also be included into the systematic error.

The initial value $\mathbf{x_0}$ of the systematic error can be assumed to be 0, unless of course we have different positive indications of its previous behaviour, and the initial variance $\mathbf{P_0}$ must, at the same time, have a considerably large value (here we propose $P_0=4$), which indicates that we do not really trust our first guess.

With the previous assumptions our Kalman filter algorithm has the following form:

State space	:	*The space of real numbers* $\boldsymbol{R}$
State vector	:	x_t *= the systematic part of the error of our NWP mode*
System equation	:	$x_t = x_{t-1} + w_t$
Observation (measurement) equation	:	$y_t = x_t + v_t$
Prediction equations	:	$x_{t/t-1} = x_{t-1}, \quad P_{t/t-1} = P_{t-1} + W_t$
Updating equations	:	$x_t = x_{t/t-1} + K_t(y_t - x_{t/t-1}),$
	:	$K_t = \dfrac{P_{t/t-1}}{P_{t/t-1} + V_t},$
	:	$P_t = (1-K_t)\, P_{t/t-1}$

One of the most serious difficulties in Kalman filtering models, as discussed in the previous section, concerns the way that the covariance matrices of $\mathbf{w_t}$ and $\mathbf{v_t}$ will be specified. Many authors (e.g. Homleid 1995; Simonsen 1991) consider them to be time independent, thereby losing the capability of making quicker adjustments to possible external changes. In our case, where these matrices reduce to the variances of the scalar variables $\mathbf{w_t}$ and $\mathbf{v_t}$, we propose the following procedure for their calculation:

We estimate the variance W_t of the system equation and V_t of the observation equation based on the sample of the last 7 values of $w_t=x_t-x_{t-1}$ and $v_t=y_t-x_t$ respectively.

More precisely, $\mathbf{W_t}$ is calculated as

$$\frac{1}{6}\cdot\sum_{i=0}^{6}\left((x_{t-i}-x_{t-i-1})-\left(\frac{\sum_{i=0}^{6}(x_{t-i}-x_{t-i-1})}{7}\right)\right)^2.$$

The latter is an objective estimator of $\mathbf{W_t}$, since the variable $\mathbf{w_t}$ follows the normal distribution. Analogously, $\mathbf{V_t}$ is assumed equal to

$$\frac{1}{6}\cdot\sum_{i=0}^{6}\left((y_{t-i}-x_{t-i})-\left(\frac{\sum_{i=0}^{6}(y_{t-i}-x_{t-i})}{7}\right)\right)^2.$$

This time period of seven days has proved to be the optimal choice in our study for successful correction and adaptability. However, it is possible to vary this for different geographic or climatological environments.

Based on the algorithm described above, we obtain as the outcome $\mathbf{x_t}$ the estimated systematic error at time t of the NWP model in use. When added to the prediction of the model for time t+1, this value gives the final/improved forecast for the parameter in study:

Improved forecast for time t+1 = (model outcome for time t+1) + (filter estimate at time t)

4. Applications

The Kalman filtering proposed in the previous section is used by the Hellenic National Meteorological Service for the correction of forecasts of maximum and minimum 2m-temperature obtained by the NWP model Skiron for 30 Greek cities. These forecasts are based on interpolations of the direct outputs of the model on the five nearest grid points. Since we use as initial data those of 00:00 UTC, the maximum forecasted temperature is the result of the 36th predicting hour and the minimum is the lowest forecast value of the 27th, 28th and 29th hour.

In Tables 1 and 2 below we present the results of the filter for the three largest cities in Greece: Athens (Station 16701), Thessaloniki (16622) and Heraklion (16754). These cover the full latitude range in Greece, and cover the period from July to December 2000. Because these cities are all close to the coast, some of the relevant interpolated grid points lie over the sea. This has resulted in increased biases, mainly for maximum temperature, which is, naturally, underestimated.

For each station the tables give the mean errors (biases), the absolute mean errors, the root mean square errors, the corresponding standard deviations as well as the score skills of the proposed Kalman filter with reference to Skiron's direct outputs. The latter variable is given by:

$$\textit{Score Skill} = 1-\frac{\textit{Absolute mean error of Kalman filter}}{\textit{Absolute mean error of SKIRON}}$$

Furthermore, the possibility of having a successful forecast is presented for each station, according to ECMWF's standards (successful forecast ⇔ absolute error less than 2° C).

Table 1. *Maximum temperatures*

	ATHENS		THESSALONIKI		HERAKLION	
	Skiron	Kalman	Skiron	Kalman	Skiron	Kalman
Mean Error (ME)	4.489	–0.106	5.271	–0.067	2.077	–0.086
Absolute Mean Error (AME)	4.499	1.169	5.271	1.345	2.200	1.116
Standard Deviation of Error	1.697	1.590	1.718	1.747	1.504	1.544
Standard Deviation of Abs Error	1.669	1.084	1.718	1.118	1.317	1.070
Root Mean Square Error	4.632	1.538	5.350	1.688	2.483	1.498
Possibility of successful forecast	0.048	0.719	0.034	0.678	0.483	0.793
Score Skill	0.740		0.745		0.493	

Table 2. *Minimum temperatures*

	ATHENS		THESSALONIKI		HERAKLION	
	Skiron	Kalman	Skiron	Kalman	Skiron	Kalman
Mean Error (ME)	2.333	–0.011	–0.635	0.047	0.740	–0.176
Absolute Mean Error (AME)	2.511	1.370	1.677	1.598	1.687	1.606
Standard Deviation of Error	1.753	1.728	2.499	2.550	2.055	1.945
Standard Deviation of Abs Error	1.487	1.052	1.959	1.988	1.388	1.112
Root Mean Square Error	2.817	1.668	2.497	2.470	2.108	1.885
Possibility of successful forecast	0.370	0.705	0.614	0.641	0.623	0.610
Score Skill	0.454		0.047		0.048	

Significant corrections are obtained mainly for maximum temperatures, with relevant score skills of 0.74 for Athens and Thessaloniki, and 0.49 for Heraklion (where the corresponding possibilities of successful forecasts are also satisfactory). The fact that the biases of the results of the proposed filter are very close to zero is an additional indication of the accuracy of our method.

By contrast, the performance of the filter is poorer for minimum 2m-temperatures. For example, at Thessaloniki and Heraklion, where the direct outputs of the Skiron model proved rather successful (mean errors close to zero), the filter did not significantly improve the absolute mean errors of the forecasts. However, the corresponding biases are well reduced and the possibility of an acceptable prediction remain in any case greater than 61 %.

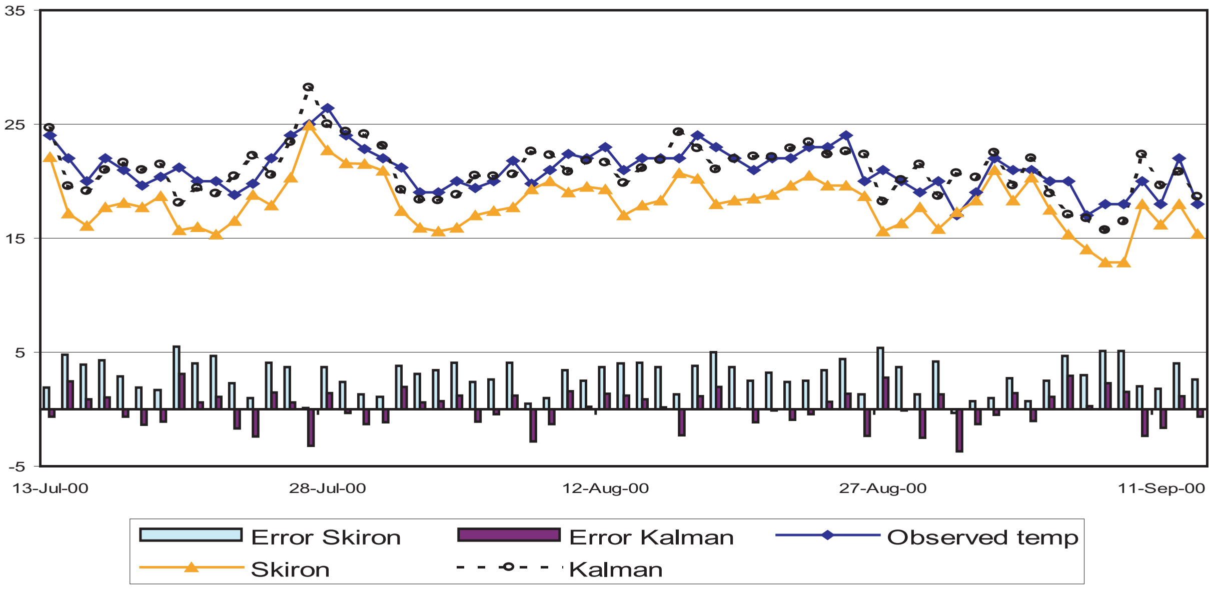

Figure 1: *Observed minimum 2m-temperature in Athens (Station 16701), the corresponding forecasts of the NWP model Skiron, the improved predictions using the Kalman filter as well as the relevant biases for the period 13 July–13 September 2000.*

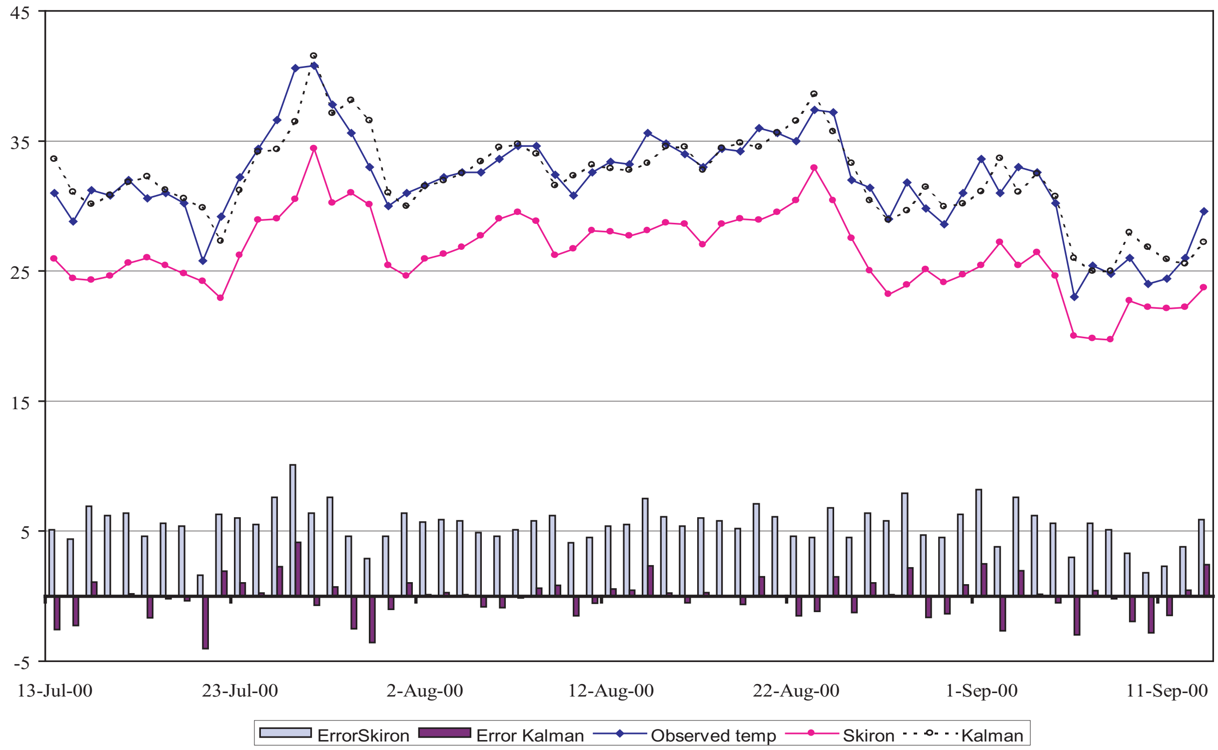

Figure 2: *Observed maximum 2m-temperature in Thessaloniki (Station 16622), the corresponding forecasts of the NWP model Skiron, the improved predictions using the Kalman filter, as well as the relevant biases for the period 13 July–13 September 2000.*

Figures 1 and 2 show the variation in time of the direct outputs of the model and the improved forecasts obtained by the proposed filter, against observations for the time 13 July to 13 September 2000. The corresponding biases are also presented.

It is worth noting here that although the main part of the biases of our NWP model has a standard form (underestimation of constant magnitude) the proposed Kalman filter proved better than a 7-day moving average correction. For example, in the case of the maximum 2m-temperature at Thessaloniki (station 16622), the Root Mean Square Error (RMSE) of Kalman was 1.6 when that of the moving average correction calculated was above 2.0. Similarly, at Athens (station 16701) the RMSE of Kalman for the minimum 2m-temperature was again 1.6 and that of the moving average correction 1.9. Moreover, in both cases the variation intervals of Kalman filtering errors proved significantly finer.

5. Conclusions

The results presented in the previous section clearly show that the performance of the proposed filter is very successful in cases where moderate to serious systematic errors are apparent in the direct output of NWP models. By contrast, if the results of a forecast model are very close to the observed values, our method makes only minor adjustments to the mean absolute error. However, this is to be expected since a possible absence of standard systematic error seriously restricts the frame in which any filter of this type could be applied.

Given the simplicity of the proposed Kalman filter, as well as the fact that it can be run on basic PCs, even in MS-Excel, with simple computer resources, we believe that it will prove to be a useful tool for forecasters in helping to eliminate the systematic errors of NWP models.

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