Método dos Momentos

1) Seja $X_1, \dots X_n$ a.a $X \sim U(-\theta, \theta)$. Encontre o estimador pelos método de momentos

R: Note que $\mathbb{E}(X) = 0$. Assim $\mu_1(\theta) = 0$, $M_1 = \bar{X}$ $\left\{ \bar{X} = 0 \right.$

Precisamos de mais equações do que o nº de parâmetros.

$$\begin{cases} \mu_2(\theta) = \mathbb{E}(X^2) = \mathbb{V}ar(X) + \mathbb{E}(X)^2 = \dfrac{(b-a)^2}{12} = \dfrac{(2\theta)^2}{12} = \dfrac{\theta^2}{3} \\ M_2 = \dfrac{\sum x_i^2}{n} \end{cases}$$

Igualando μ_2 com M_2 temos:

$$\frac{\sum x_i^2}{n} = \frac{\theta^2}{3} \Rightarrow \hat{\theta} = \sqrt{\frac{3 \sum x_i^2}{n}}$$

.) Seja $X_1 \dots X_n$ a.a com f.d.p. $f(x;\theta) = \frac{2}{\theta^2}(\theta - x)$, $0 < x < \theta$, $\theta \in \Omega = (0$
Encontre o estim. de momentos p/ θ.

$$\mathbb{E}(X) = \int_0^\theta x \cdot f(x;\theta) \cdot dx = \int_0 \frac{2x}{\theta^2}(\theta - x)\, dx = \frac{2}{\theta}\frac{x^2}{2} - \frac{2x^3}{\theta^2 3} \Big|_0^\theta$$

$$= \frac{\theta}{3}$$

$$M_1 = \frac{\sum x_i}{n}, \quad \mu_1 = \frac{\theta}{3} \quad \left\{ \hat{\theta}_{MM} = 3\bar{X} \right.$$

3) Seja $X_1, \dots X_n \sim Beta(\theta, 2)$. Encontre, pelo método dos momentos, o estimador de θ.

R: $\mu_1 = E(X) = \dfrac{\theta}{\theta+2}$

$M_1 = \dfrac{\sum X_i}{n}$

Igualando termos:

$$\frac{\theta}{\theta+2} = \bar{X} \Rightarrow \bar{X}\theta + 2\bar{X} - \theta = 0$$

$$\theta(\bar{X}-1) + 2\bar{X} = 0$$

$$\hat{\theta} = \frac{2\bar{X}}{1-\bar{X}}$$

4) Seja $X_1 \dots X_n$ a.a da f.d.p., $f(x|\theta) = \dfrac{(x+1)}{\theta(\theta+1)} e^{-x/\theta}$, $x > 0$, $\theta > 0$

Encontre o estimador de θ pelo método de momentos

R: $\mu_1 = E(X) = \displaystyle\int_0^\infty x \cdot \frac{(x+1)}{\theta(\theta+1)} e^{-x/\theta} \, dx = \int_0^\infty \frac{x^2 e^{-x/\theta}}{\theta(\theta+1)} dx + \int_0^\infty \frac{x e^{-x/\theta}}{\theta(\theta+1)} dx$

$$= \frac{\theta}{\theta+1} \left\{ \underbrace{\int_0^\infty x\left(\frac{1}{\theta^2} x e^{-x/\theta}\right) dx}_{E(X),\ \text{com } X \sim \Gamma(2, \frac{1}{\theta})} + \underbrace{\int_0^\infty \frac{1}{\theta^2} x e^{-x/\theta} \, dx}_{f(x;\theta)\ \text{da}\ \Gamma(2, \frac{1}{\theta})} \right\}$$

$$= \frac{\theta}{\theta+1} \left\{ 2\theta + 1 \right\} = \frac{\theta(2\theta+1)}{\theta+1}$$

$M_1 = \dfrac{\sum X_i}{n} \Rightarrow M_1 = \mu_1 : \dfrac{2\theta^2 + \theta}{\theta+1} = \bar{X} \Rightarrow 2\theta^2 + \theta(1-\bar{X}) - \bar{X} = 0$

$$a y^2 - y b - c = 0$$

$$\hat{\theta} = \frac{-b + \sqrt{b^2 - 4a.c}}{2a}$$

$$\hat{\theta} = \frac{\bar{X} - 1 + \sqrt{(\bar{X}-1)^2 + 8\bar{X}}}{4}$$