Lens Casella, p. 405

9.1-) $L(x), U(x) \mid P_\theta(L(X) \leq \theta) = 1-\alpha_1, P_\theta(U(X) \geq \theta) = 1-\alpha_2, L(x) \leq U(x) \ \forall x.$

$P_\theta(L(X) \leq \theta \leq U(X)) = 1-\alpha_1-\alpha_2$

$P_\theta(\theta \in [L(X), U(X)] = IC$

$P_\theta(L(X) \leq \theta) = 1-\alpha_1$

$P_\theta(U(X) \geq \theta) = 1-\alpha_2$

$P_\theta(L(X) \leq \theta \leq U(X)) = 1-\alpha_1(1-\alpha_2)$

$P_\theta(L(X) \leq \theta \leq U(X)) = 1-(\alpha_1-\alpha_2)$

9.12-) a.a. tamanho $n \sim N(\theta, \theta), \theta > 0$. Q p/ $1-\alpha$

$$Q_1 = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{\bar{X}-\theta}{\sqrt{\theta/n}}$$

$$Q_2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(n-1)s^2}{\theta}, \quad Q_1 \perp Q_2$$

$$P\left(-c \leq \frac{\bar{X}-\theta}{\sqrt{\theta/n}} \leq c\right) = \sqrt{1-\alpha}$$

$$P\left(a \leq \frac{(n-1)s^2}{\theta} \leq b\right) = \sqrt{1-\alpha}$$

$$P_{\theta,\theta}\left(-c \leq \frac{\bar{X}-\theta}{\sqrt{\theta/n}} \leq c \, ; \, a \leq \frac{(n-1)s^2}{\theta} \leq b\right) = 1-\alpha$$

$$P_\theta\left(-c \leq \frac{\bar{X}-\theta}{\sqrt{\theta/n}} \leq c\right) \cdot P_\theta\left(a \leq \frac{(n-1)s^2}{\theta} \leq b\right) = \sqrt{1-\alpha}\sqrt{1-\alpha}$$

$$P\left((\theta-\bar{X})^2 \leq \frac{c^2\theta}{n}\right) \cdot P\left(\frac{(n-1)s^2}{b} \leq \theta \leq \frac{(n-1)s^2}{a}\right) = 1-\alpha$$

$$P\left(\theta^2 - 2\theta\bar{X} + \bar{X}^2 \leq \frac{c^2\theta}{n}\right) \cdot P\left(\frac{(n-1)s^2}{b} \leq \theta \leq \frac{(n-1)s^2}{a}\right) = 1-\alpha$$

Exs. Bolfarine, p. 116

5.7-) X_1, X_n a a, $f(x|\theta) = \theta e^{-\theta x}$, $x>0$, $\theta>0$

$Q(X;\theta) = 2\theta \sum X_i$

$$f_Q(x) = \frac{x^{n-1} e^{-\frac{x}{2}}}{2^n \Gamma(n)} \sim \chi^2(2n)$$

$P(q_{\alpha/2} < Q < q_{1-\alpha/2}) = 1-\alpha$

$P(q_{\alpha/2} < 2\theta\sum X_i < q_{1-\alpha/2}) = 1-\alpha$

$$P\left(\frac{q_{\alpha/2}}{2\sum X_i} < \theta < \frac{q_{1-\alpha/2}}{2\sum X_i}\right) = 1-\alpha$$

Exs. Casella, p. 405

9.4. X_1, X_n, a.a. $\sim N(0, \sigma^2_x)$, Y_1, Y_m a.a. $\sim N(0, \sigma^2_y)$, independentes

$IC(\sigma_y^2/\sigma_x^2, 1-\alpha)$

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1)$$

$$Q = \frac{\dfrac{(n-1)s_x^2}{\sigma_x^2} \Big/ (n-1)}{\dfrac{(m-1)s_y^2}{\sigma_y^2} \Big/ (m-1)}$$

$$Q = \frac{s_x^2}{s_y^2}\frac{\sigma_y^2}{\sigma_x^2} \sim F$$

$$P\left(f_{\alpha/2} < \frac{s_x^2\sigma_y^2}{s_y^2\sigma_x^2} < f_{1-\alpha/2}\right) = 1-\alpha$$

$$f_{\alpha/2}\frac{s_x^2}{s_y^2} < \frac{\sigma_y^2}{\sigma_x^2} < f_{1-\alpha/2}\frac{s_x^2}{s_y^2}$$

$$IC\left(\sigma_y^2/\sigma_x^2, 1-\alpha\right) = \left[f_{\alpha/2}\frac{s_x^2}{s_y^2}, f_{1-\alpha/2}\frac{s_x^2}{s_y^2}\right]$$

Exs. Morettin, p. 308

14-) IC

tilibra

e-) $N(\mu, \sigma^2)$, $\bar{X} = 170$, $n = 100$, $\sigma = 15$, $1-\alpha = 95\%$

$P(-z_{1-\alpha/2} < Z < z_{1-\alpha/2}) = 1-\alpha$

$P(-z < Z < z) = 1-\alpha$

$P\left(-z < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < z\right) = 0,95$

$-\frac{z\sigma}{\sqrt{n}} < \bar{X}-\mu < \frac{z\sigma}{\sqrt{n}} = 0,95$

$-\frac{z\sigma}{\sqrt{n}} - \bar{X} < -\mu < \frac{z\sigma}{\sqrt{n}} - \bar{X}$

$\frac{z\sigma}{\sqrt{n}} + \bar{X} > \mu > -\frac{z\sigma}{\sqrt{n}} + \bar{X}$

$\bar{X} + \frac{z\sigma}{\sqrt{n}} > \mu > \bar{X} - \frac{z\sigma}{\sqrt{n}}$

$\bar{X} - \frac{z\sigma}{\sqrt{n}} < \mu < \bar{X} + \frac{z\sigma}{\sqrt{n}}$

$\frac{170 - 1,96(15)}{10} < \mu < \frac{170 + 1,96(15)}{10}$

$\frac{170 - 29,4}{10} < \mu < \frac{170 + 29,4}{10}$

$170 - 2,94 < \mu < 170 + 2,94$

$167,06 < \mu < 172,94$

18-) $n = 625$, $\hat{p} = 70\%$; $0,7$; $\alpha = 90\%$

$Z = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \approx N(0,1)$

$P(-z < Z < z) = 1-\alpha$

$P\left(-z < \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} < z\right) = 0,9$

$\hat{p} - z\sqrt{\frac{p_0(1-p_0)}{n}} < p < \hat{p} + z\sqrt{\frac{p_0(1-p_0)}{n}}$

* abordagem otimista: $p_0(1-p_0) = \hat{p}(1-\hat{p})$

$0,7 - 1,65\sqrt{\frac{0,7(0,3)}{625}} < p < 0,7 + 1,65\sqrt{\frac{0,7(0,3)}{625}}$

$0,7 - 1,65(0,0916) < p < 0,7 + 165(0,0916)$

$0,7 - 0,1511 < p < 0,7 + 0,1511$

$0,5489 < p < 0,8511$

* abordagem conservadora: $p_0(1-p_0) = 1/2(1/2) = 1/4$

$0,7 - 1,65\sqrt{\frac{0,25}{625}} < p < 0,7 + 1,65\sqrt{\frac{0,25}{625}}$

$0,7 - 1,65(0,1) < p < 0,7 + 1,65(0,1)$

$0,535 < p < 0,865$