Measurement Techniques, Vol. 47, No. 4, 2004

DISCRETE FOURIER TRANSFORM FOR SHORT SAMPLES IN THE PRESENCE OF NOISE

E. I. Gol'dshtein and D. V. Li

UDC 621.317.389

We consider the dependences of the error of a discrete Fourier transform on the length of the realization of the considered signal. The introduction of a procedure of making a preliminary determination of the period makes it possible to consider signals in short samples and this significantly reduces the computing and apparatus costs.

Key words: *discrete Fourier transform, realization length, signal period, short sample.*

The universal transition to digital methods of measurement increases the interest in algorithms for processing signals in short samples, and this corresponds to a tendency to choose such methods and algorithms which demand lower computing costs for the same accuracy [1, 2]. This applies in full to both discrete and fast Fourier transforms which are the most widely used algorithms for harmonic analysis on account of their simplicity and immunity from interference [3, 4].

Fourier coefficients are calculated using the formulas

$$a_k = \frac{2}{N}\sum_{i=0}^{N-1} U_i \cos(2\pi k i \Delta t f);$$

$$b_k = \frac{2}{N}\sum_{i=0}^{N-1} U_i \sin(2\pi k i \Delta t f),$$

where N is the number of segments of the digitized signal U_i in a realization interval T_r of the signal considered. In the ideal case, T_r is equal to the period T_s. The digitizing step Δt of the considered signal is often not a multiple of the period T_s, and this affects the accuracy of calculating the coefficients a_k and b_k and consequently the accuracy of the parameters being sought:

$$C_k = \sqrt{a_k^2 + b_k^2}; \quad \varphi_k = \arctan(b_k / a_k).$$

The accuracy of a discrete Fourier transform is also reduced in the presence of interference and strongly depends on the extent to which the signal period is known. Therefore, in practice, discrete Fourier transforms are used with an increased realization interval of the unknown signal $T_r \geq 10T_s$. However, this requires a far greater calculation time and corresponding apparatus costs, and this can be avoided by introducing into the discrete Fourier transform a procedure in which a prior determination is made of the signal period. In this case, it is possible to perform calculations on short samples which do not exceed two periods of the signal being analyzed. When choosing the method for determining the period of a signal represented by digital read outs [4–7] and taking account of the evident requirements of a low sensitivity to phase shifts, the presence of a dc component in the signal and, mainly, the presence of interference (noise), preference was given to a method which is a variety of the least-squares method.

Translated from Izmeritel'naya Tekhnika, No. 4, pp. 11–14, April, 2004. Original article submitted November 26, 2003.

TABLE 1. Results of Calculating Errors of Discrete Fourier Transforms with Prior Determination of T_s for Different Interference Levels

R, %	δT_s	δU_0	δU_{m1}	δU_{m2}	δU_{m3}	$\delta\varphi_1$	$\delta\varphi_2$	$\delta\varphi_3$	σ
0	0	0	0	0	0	0	0	0	0
5	0.078	0.207	0.26	0.865	5.158	0.907	0.7	1.497	0.827
10	0.292	0.424	0.529	2.194	13.948	2.457	1.63	3.038	1.989
15	0.464	0.649	0.75	3.239	20.832	3.674	2.438	4.885	2.96
20	0.591	0.828	1.088	4.123	25.622	4.808	3.099	7.187	3.832

The proposed method consists in seeking out the minimum from an array of root-mean-square deviations D_j which is formed by comparing an initial subarray taking the form of the first few readings of the considered signal and a subarray of the same size which is displaced relative to the initial time in the recorded interval:

$$D_j = \sqrt{\frac{1}{n+1}\sum_{m=0}^{n}[U(t_m) - U(t_{m+j})]^2} \,,$$

where n is the number of points of comparison (on the basis of experimental data this was taken to be $n = 100$); $U(t_m)$, $U(t_{m+j})$ are the mth readings of respectively the initial and compared subarrays of the considered signal; $j \in [n_{\min}; n_{\max}]$ is a parameter of the displacement of the compared subarray; $n_{\min}$ and $n_{\max}$ are the minimum and maximum boundaries of the anticipated range of values of the period.

When modeling noisy signals, we utilize centralized white noise (having zero mathematical expectation) U_n, normalized to the amplitude of the fundamental harmonic U_{fund}:

$$R_{\max} = \frac{U_{\text{fund}} - U_n}{U_{\text{fund}}} \cdot 100 \,.$$

This action is quite simply implemented in various high-level and low-level software media in accordance with a uniform distribution law:

$$p_\xi(x) = \begin{cases} \dfrac{1}{d-c}, & x \in [c, d]; \\ 0, & x \notin [c, d], \end{cases}$$

where $p_\xi(x)$ is the distribution density of the random quantity ξ in the segment $[c, d]$.

The influence of an inaccurate knowledge of the period and of the presence of noise will be estimated according to the well-known formulas:

$$\delta X = \frac{1}{M}\sum_{1}^{M}\left|\frac{X_{\text{known}} - X_{\text{calc}}}{X_{\text{known}}}\right| \cdot 100 \,;$$

$$\sigma = \frac{1}{M}\sum_{1}^{M}\sqrt{\sum_{i=1}^{N-1}(U_{ti} - U_{\text{calc}i})^2} \,,$$

where δX, X_{known}, X_{calc} are the relative averaged error and the known and calculated values of the sought-after quantities; $M = 1000$ is the number of experiments sufficient for eliminating the influence of the random nature of the noise on the results of the investigations; σ is the root-mean-square deviation; $U_{t\,i}$ and $U_{\text{calc}\,i}$ are the test and calculated signals.

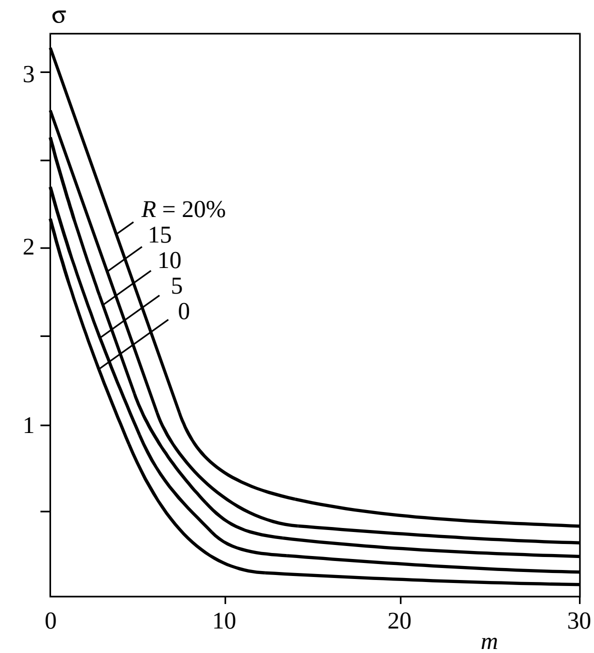

Fig. 1. Influence of noise on the root-mean-square deviation for $T_r = mT_s \pm \Delta t$.

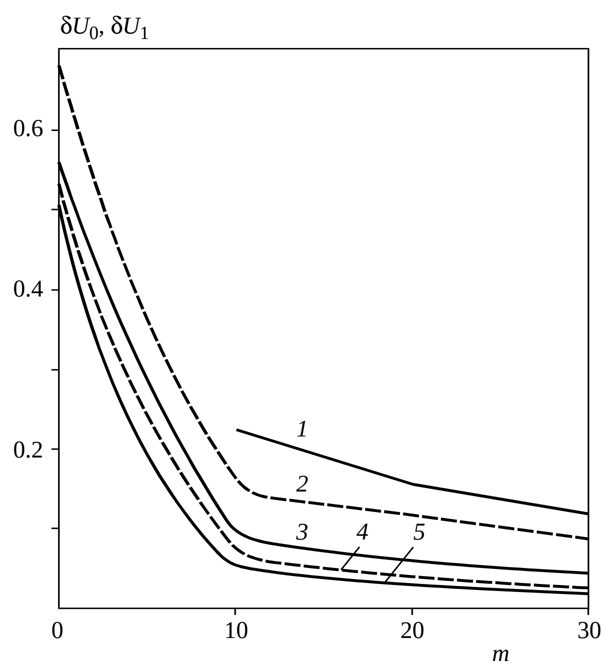

Fig. 2. Influence of noise on the errors δU_0, δU_1 for $T_r = mT_s \pm \Delta t$: 1) δU_1 for R = 15%; 2) δU_0 for R = 15%; 3) δU_1 for R = 5%; 4) δU_0 for R = 5%; 5) δU_0, δU_1 for R = 0.

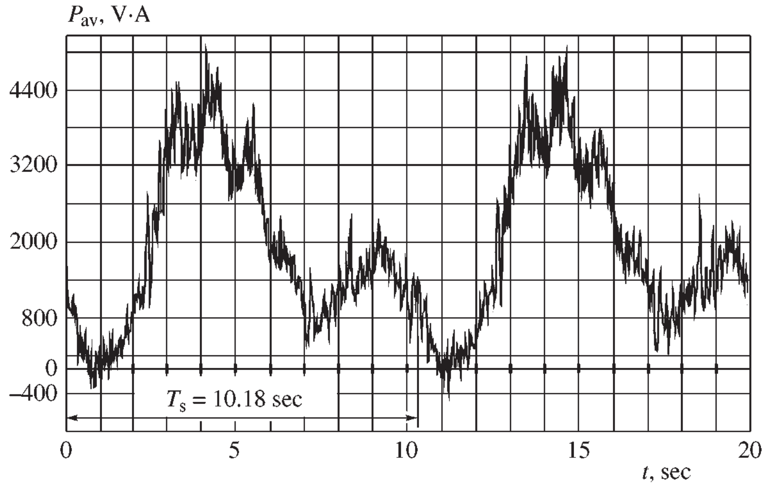

Fig. 3. Wattmeter graph of a suck-pump well-pumping device (well No. 850).

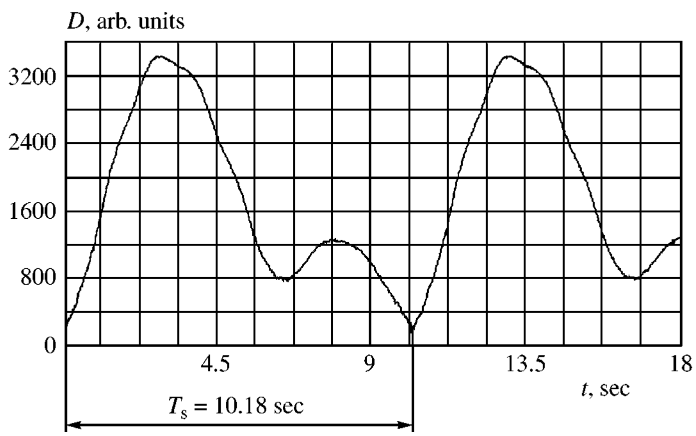

Fig. 4. Array of root-mean-square deviations.

As a rule, the problem of a lack of knowledge or of inaccurate specification of T_s when calculating the Fourier coefficients is eliminated by increasing the realization interval of the signal being analyzed.

A calculation experiment was performed in order to elucidate the relationship of the errors of a discrete Fourier transform to the realization interval of the analyzed array T_r and the noise level. In this experiment, a deviation $\Delta t = 10^{-4}$ sec from the nominal value of T_s equal to one digitization step was considered, i.e., $T_r = mT_s \pm \Delta t$, with $m \leq 30$. As an example, Table 1 and Figs. 1 and 2 give the results of a discrete Fourier transform of a three-frequency signal having a period $T_s = 0.02$ sec, in the presence of a dc component and noise, the level of the latter being 0–20%:

$$u(t) = U_0 + \sum_{i=1}^{3} U_{mi} \sin(2\pi i f + \varphi_i)$$

for $U_0 = 100$ V; $U_{m1} = 100$ V; $U_{m2} = 50$ V; $U_{m3} = 10$ V; $f = 50$ Hz; $\varphi_1 = 30°$; $\varphi_2 = 60°$; $\varphi_3 = 90°$.

As one would expect, in the absence of noise the discrete Fourier transform with prior determination of the period gives practically no error and only as the noise level is increased does the error increase. An increase in T_r does indeed promote a reduction in the error of the discrete Fourier transform, the order of which is inversely proportional to the order of T_r. Here it must be taken into account that the error in specifying T_r was a minimum whereas for a discrete Fourier transform with prior determination of T_s the maximum error in determining the period was $5\Delta t$ for $R = 20\%$. However, it must be remembered that an increase in T_r inevitably leads to an increase in the computer costs and the expenditure of time and consequently to the need to introduce additional apparatus (memory, coprocessor).

The procedure described above of a discrete Fourier transform with prior determination of the period was successfully applied for the functional monitoring and diagnostics of sucker-rod well-pumping devices using wattmeter graphs in the form of the dependence of the average supply of active power P_{av} over a period on the piston stroke (see Figs. 3 and 4).

Using the period found (in our specific case $T_s = 10.18$ sec), it was easy to perform the discrete Fourier transform, to find the amplitudes of the fundamental harmonics of the wattmeter graphs, to determine the necessary ratios of the amplitudes of the harmonics of the power, and to draw conclusions in terms of the diagnosing of the monitored pumping device [8–10].

The described procedure for determining T_s was also used in a set of programs for the functional monitoring and diagnostics of electrotechnical and electromechanical devices when calculating the acting values of multifrequency signals and is of interest in itself when identifying informative data streams [11, 12].

REFERENCES

1. S. A. Labutin and M. V. Pugin, *Izmerit. Tekh.*, No. 9, 34 (1998).
2. V. A. Dvinskikh, *Izmerit. Tekh.*, No. 2, 10 (1999).
3. M. Yu. Evsikov, *Élektrosvyaz'*, No. 11, 19 (1998).
4. E. I. Gol'dshtein and T. G. Danilenko, *Izmerit. Tekh.*, No. 6, 47 (2001).
5. E. I. Kulikov, *Methods of Measuring Random Processes* [in Russian], Radio i Svyaz', Moscow (1986).
6. A. P. Trifonov and Yu. S. Shinakov, *Joint Discrimination of Signals and Estimation of Their Parameters on an Interference Background* [in Russian], Radio i Svyaz', Moscow (1986).
7. D. V. Li, *Proc. Seventh Intern. Sci. Pract. Conf. for Students, Postgraduates, and Young Scientists, Vol. 2* [in Russian], State University, Tomsk (2001), p. 30.
8. E. I. Gol'dshtein, B. N. Daurikh, and L. K. Konradi, Authors' Patent Certificate No. 1707239 USSR, *Byull. Izobret.*, No. 3, (1992).
9. E. I. Gol'dshtein and E. N. Ermakova, Patent No. 2129666 Russian Federation, *Byull. Izobret.*, No. 12 (1999).
10. E. I. Gol'dshtein and E. N. Ermakova, Patent No. 2129667 Russian Federation, *Byull. Izobret.*, No. 12 (1999).
11. E. I. Gol'dshtein and D. V. Li, Patent No. 2189703 Russian Federation, *Byull. Izobret.*, No. 26 (2002).
12. E. I. Gol'dshtein and D. V. Li, Patent No. 2208293 Russian Federation, *Byull. Izobret.*, No. 19 (2003).