Integrating Urban Economics and Cellular Automata to model Periurbanisation

Spatial dynamics of residential choice in the presence of neighbourhood externalities

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Thèse présentée en vue
de l'obtention du grade de
Docteur en Sciences

Université catholique de Louvain

Louvain-la-Neuve, February 2005

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Le périurbain, sa ville et sa campagne en couleurs

Aquarelle, Laurence Godefroid, Marchovelette, October 2004.

Acknowledgements

First, I would like to thank Professors Isabelle Thomas, Mike Batty and Jacques Thisse who accepted to join the jury and have taken an interest into this thesis. Thank you to Professors Mark Rounsevell and Dominique Peeters, and to Jean Cavailhès, the members of my thesis committee, who continuously suggested improvements and new ideas for this research.

Especially, I would like to thank Jean Cavailhès for having offered me many opportunities to develop this research e.g. with a participation into the DATAR project or with longer and shorter stays in Dijon. Thank you very much, Jean, for your enthusiasm in this project. Thank you for keeping thinking and proposing answers to my hesitations regarding the right economic sense to give to these cellular dynamics. Thank you for showing me a way to think periurban processes, as well as a way to clarify ideas and conduct a doctoral research. I feel that I have learned a lot from the many discussions we have had with you and Dominique, each time giving new food for thought. I would like to thank Dominique Peeters for that as well, and because his door was open every time I was thinking to be stuck into a serious problem. I would like to thank Mark Rounsevell, my supervisor, for his suggestions, help and support in this research. Thank you too, Mark, for your confidence in me concerning both the research and the teaching activities during the years of my assistantship. Thank you to George Cojocaru for the development of the computing platform for the model.

Tank you to Isabelle Thomas for her comments on this thesis, but also for her daily encouragements. Thank you to Professor Hubert Beguin who introduced me to economic geography and showed me the interest of quantifying and clearly identifying processes in geography. Thank you to Professor Henry Zoller for introducing me to urban economic processes and for the interest he showed for my research through small discussions.

My thanks go also to the Dijon team. I have appreciated very much my stay in Dijon where I think I have learned a lot. Special thanks to Aurélie, Gabriel and Mohammed for welcoming me and for many interesting discussions. Thank you also to Professor Pierre Frankhauser and Cécile Tannier in Besançon for discussions and comments on my research.

Then, I would like to thank my colleagues in Mercator. I thank them very much for the general sphere in the unit. Of course, above all, I thank those who brought cakes, cookies, 'tartes' and chocolates. But, many thanks to all of them for the scientific and less scientific discussions. I have appreciated many moments and felt at home where I worked. Special thanks go to Sabine and Pierre with whom I started as an assistant and who shared parts, and thus shortened, my daily rail commuting trips. Special thanks to Annette and Pierre, and not only for the coffee or the computing help. I would also like to thank Isabelle R who, supportively, is always in a good mood. Thank you very much as well to you, Claire, for your help and support, especially at the end of this thesis. Many thanks to Elizabeth and Fred for your friendship. Thank you very much, Nadège, for your support, your friendship and confidence.

Many thanks to my parents and brothers for their continuous support. Thank you to Vincianne, Laurence (thanks for the aquarelle), Hugo and Mathéo (for the other drawings), and thanks to my old Saint-Aubain friends for entertainment moments.

Thank you very much, Marie, for your patience and for being with me during all moments.

Contents

Chapter 1

Introduction

1.1 General objective

This thesis addresses the issue of urban expansion by exploring the emergence and morphology of a periurban zone at the periphery of a city, where residents and agricultural activities mix.

In Western Europe, periurban areas have experienced rapid land use change and involve an increasing number of people. Areas at the interface between the city and the countryside, where a large part of the active population commutes to the centre, represent about 25 to 40 % of the total population (depending on national classification criteria). Commuters undertake increasingly longer daily trips from city centres, which results in the continued conversion of agricultural areas into urban use. Periurban areas, therefore, challenge important social and environmental spatial policy goals.

As a result of urban expansion into surrounding rural zones, the gradient of population density with respect to the distance to city centres flattens, and the morphological limit between the city and the countryside becomes increasingly blurred. This thesis analyses the morphology of the spatial patterns within this fuzzy interface. The general objective is to understand how residential choice gives rise to particular geographical forms through time. Thus, the thesis seeks to **understand the morphogenesis of periurban residential patterns and the underlying processes that lead to these patterns**.

Periurban morphologies in Western Europe are found to range from compact clustered settlements to highly scattered developments. **This thesis tests whether the diversity of periurban spatial patterns can result from**

changes in individual residential choices. What forms of urban expansion can emerge from various residential behaviours ? More precisely, what is the impact on the shape of the city of increasing residential preference for the quality of the local environment with respect to commuting distances ?

In order to answer these questions, a theoretical spatio-dynamic model is developed. The model considers the commuting periphery of a monocentric city and **integrates two different modelling paradigms of urban land use change: *Urban Economics* and *Cellular Automata***. In this way, emerging periurban structures are explicitly spatial and founded on residential micro-economic theory. Moreover, the framework is dynamic and combines the making of both short- and long-run spatial equilibria. Furthermore, the role of past history in shaping current settlement patterns is emphasized in the model because irreversibility of urban land conversion is assumed, as well as the sequential location of utility maximizing households. These key principles are further discussed in the next section. It is hypothesized that the combination of these principles within the model can lead to a satisfactory representation of the periurban morphogenesis process.

While the thematic objective of this research is to gain further understanding of *Periurbanisation* processes, the integration of *Urban Economics* and *Cellular Automata* is also an objective and a challenge, as little research has been undertaken that relates these scientific fields.

1.2 Key modelling assumptions and the need for integration

1.2.1 Understanding periurban forms

Periurbanisation refers to the process of residential growth towards the rural periphery of a city. This process leads to the emergence of a spatial zone characterized by a mix of agricultural activities and commuting households (Cavailhès et al., 2004b).

In this thesis, the mix of land uses is assumed to be taken into account as an environmental quality by residents. Residents are attracted by periurban areas, as these provide pleasant living conditions at a commuting distance from the city centre. Using similar hypotheses, Cavailhès et al. (2004b) have developed a model of a *Periurban City*. The authors determine the conditions for the emergence of a mixed agricultural-residential belt at equilibrium. Within this mixed zone, however, the local spatial arrangement of households and agricultural activities is unknown because the model is uni-dimensional. Therefore, this belt could still include a range of spatial structures that are observed today around European cities, including compact settlements, leapfrogging clusters, scattered dissemination of dwellings, ribbon developments along networks, etc.

This thesis aims to further examine the 2D spatial arrangement of such a mixed belt, the focus being on the emergence of various spatial forms. The formation of an equilibrium within a particular 2D spatial arrangement of residential and agricultural activities has been analysed by Cavailhès et al. (2004a). However, the authors use a pre-determined geometric arrangement while, in this thesis, the form of residential settlements is endogenous.

Although the 'compact city' ideal is disputed in the literature and the costs of urban dispersion are very much debated (see e.g. Anderson et al., 1996; Brueckner, 2000b; Breheny, 1992, 1995; Camagni et al., 2002; Gordon and Richardson, 1989; Newman and Kenworthy, 1989), it is believed that the spatial form of urban development remains an important issue to address when proposing land use planning policies that are socially and environmentally desirable. Moreover, it is assumed that in order to understand the sustainability of various urban forms, their emergence should be related to economic processes.

Therefore, the model that is presented in this thesis is founded on urban economics, and is able to generate spatial forms through time. The model aims to answer the question 'What is the spatial pattern of residents and agricultural activities in periurban areas ?', and is, therefore based on the following three general assumptions. Periurban spatial patterns emerge from

(i) the residential trade-off between accessibility and housing consumptions (i.e. the main assumption of urban economics)

(ii) the valuation of spatial externalities that are related to the mix of uses (as in periurban models) and based on neighbourhood interactions (as in Cellular Automata)

(iii) the irreversibility of urban land use and the sequential location of residents (as in dynamic urban economic models where the role of path-dependency is emphasized)

Consequently, the model can either be regarded as a discrete spatio-dynamic version of an urban economic model with periurban externalities, or as a Cellular Automata model where land conversion is driven by a land market that captures the effect of neighbourhood externalities.

1.2.2 Space and time in Urban Economics

Urban economics is a part of economic theory that originates from the need to solve urban problems (e.g. congestion, segregation, suburbanisation and the decline of city centres). It is the extension to the urban context (by Alonso, 1964; Muth, 1969; Mills, 1967) of the bid-rent approach that was introduced in the agricultural context by Von Thünen (1826). From the residential viewpoint, urban economic theory focuses on the trade-off in households location choice between accessibility (commuting costs) and space consumption.

Using this framework, urban economists have clearly identified the main drivers of urban expansion: increasing numbers of households, increasing income and decreasing transport costs (Brueckner, 2000b; Mieszkowski and Mills, 1993). Moreover, urban economists have enriched this analysis by considering a set of distance related externalities, for instance the aversion to crowding (e.g. Richardson, 1977; Fujita, 1989) that allows for mixed or spatially discontinuous equilibria to arise. This suggests that periurban areas, which by definition are mixed, can result from both commuting related processes and spatial externalities. Cavailhès et al. (2004b,a) introduced residential preferences for the proximity to agricultural activities in order to obtain a mix of agricultural and residential land uses within the framework of urban economic theory.

In the residential model, however, space is distance and all locations are points. This is an important advantage for deriving mathematical propositions and thus robust understanding of urban processes (see Fujita, 1989), but a more geographical perspective is needed when one is interested in the spatial form of urban development. In practice, it is much more difficult to identify differences between clustered or scattered developments within a single spatial dimension. Moreover, the importance of complementing an economic perspective with a more spatial perspective in modelling periurban land use change was also emphasized in empirical research (e.g. Bockstael, 1996; Irwin and Geoghegan, 2001; Irwin and Bockstael, 2002).

One solution to adding more geography to the economic study of periurban processes is to build a model that still represents the standard trade-off of the monocentric economic model, but within a discrete cellular framework. The disadvantage, however, of modelling within such a two dimensional setting, is that numerical simulations must be used to obtain spatial equilibria (or very strong assumptions must be made such as circular symmetry, see Ogawa and Fujita, 1989).

However, the choice is made in this thesis to undertake simulations within a lattice of locations, because this solution offers two additionally important perspectives. First, the local externalities (i.e. non-market goods that qualify any location) that enter the preferences of periurban residents are measured in an explicitly spatial manner (as a GIS neighbourhood function) independently of its distance to the centre. Second, the model is dynamic. Therefore, the 2D spatial forms that are generated by the model presented here emerge through time. They result from a fully endogenous process and, hence, remain in accordance with the main findings of the standard monocentric model of residential microeconomics.

As mentioned by Fujita (1989, p.3), *'many spatial phenomena can be treated in a satisfactory way only within a dynamic framework'*. The emergence of spatial forms should not be given instantaneously, but generated and examined through time. From the end of the 1970s, urban economists have developed a set of 1D models where *'the land use for a given parcel cannot be inferred simply by observing its distance from the CBD*[1] *'*(Brueckner, 2000a, p.287). Interestingly for the study of mixed periurban patterns, discontinuous urban patterns and reversed spatial expansion processes can be found within this set of models, that emphasize the irreversible nature of land conversion by urban use. Yacovissi and Kern (1995) conducted an empirical test of an urban economic model that takes into account the role of history in determining the overall urban pattern. The authors showed the greater explanatory power of these dynamic models.

The model that is developed here can be related to this dynamic theory of urban growth, as it also assumes that urban land conversion is irreversible. The assumption of irreversibility is a source of important *path-dependency*, which, in this context, represents the role of existing settlements in shaping the future evolution of the periurban landscape. The decision of a household at one moment in time to locate to a particular location within the commuting field of the city depends on the location decision of previous households and their spatial arrangement. Step by step, the built-up form of the city is created from this path-dependent trajectory.

Irreversibility plays an essential role in the formation of structures in physics or biology (Prigogine, 1996), but many other disciplines have also acknowledged the important role of Prigogine's *'arrow of time'*. This has given rise to the emergence of the *complexity theory* (Nicolis and Prigogine, 1989) within various research fields, including the study of cities. The principle of *self-organisation* was introduced and developed in urban and regional geography by Wilson (1981); Allen and Sanglier (1981); Weidlich and Haag (1983); Pumain et al. (1989); Allen (1997) and in spatial economics by (Arthur, 1994; Krugman, 1996a). These works emphasize the *path-dependent* and *emerging* nature of spatial organisation. More often, however, these studies are made at the scale of multiple city-regions. In this thesis, the scale of a single city or a functional region is considered.

[1]CBD: Central Business District

1.2.3 Neighbourhood externalities in Cellular Automata

Cellular Automata are simulation tools that are also part of the self-organisation paradigm. Geographers have mainly used Cellular Automata at the scale of a city-region, although more widespread applications also exist. Dating back to the 1950's and the work of Von Neumann (1966), Cellular Automata (CA) were established ten to twenty years ago as a potentially useful way of representing geographical worlds or cities by a grid of changing-state cells (Couclelis, 1985; Batty and Xie, 1994). The technique includes space and time in a discrete manner and is particularly well suited to the study of the endogenous formation of spatial morphologies, which is the case in this thesis. CA have been extended (Batty, 1998) and constrained (White et al., 1997) to better take account of geographic processes.

CA emphasize the role of local processes that lead to aggregate dynamic spatial patterns. A key notion in CA is the *neighbourhood*. As mentioned above, it is assumed in this thesis that households in a periurban area value the quality of their environment when choosing their residential location. It is not a major additional assumption to make that the environmental quality is perceived at some limited distance from a location, i.e. a neighbourhood. Or, written differently, that what is at the proximity of a localized person has more impact on this person than what is far away[2]. It is worth noting that Schelling (1971) showed how neighbourhood preferences of people transform a checker board structure into complete spatial segregation as time progresses. In Schelling's model, as in CA, the dynamics originate from the neighbourhood. The neighbourhood is assumed in this thesis to be a place of attraction and repulsion forces between residents and agricultural activities, as well as between household types.

While CA are generally recognised as an effective tool for replicating the evolution of urban and regional land use change, they often lack an explicit formulation of the theoretical underlying processes. Although most CA model pure neighbourhood effects, the meaning of these local interactions is rarely explained. More specifically few references are made to location preferences or decision making. Introducing economic theory within a CA model would, therefore, increase its explanatory power, which is a goal of this thesis.

Urban economists have also explored neighbourhood effects and more generally *spatial externalities*(see Kanemoto, 1980). Agglomeration economies and externalities are essential elements of spatial economics and especially when a dynamic perspective is chosen. For example, spatial concentration can reinforce through time because of increasing returns to scale (Arthur, 1994). A small lo-

[2]The First Law of Geography states that *'Everything is related to everything else but nearby things are more related than distant things'* (Tobler, 1979).

calized advantage can therefore develop into the strong heterogeneity of space. Path-dependency is then related to the existence of spatial externalities.

Agglomeration spillovers (spatial non-market interactions or externalities) are also recognised in residential choice (Durlauf, 2003; Fujita and Thisse, 2002; Ioannides, 2002): *'the propensity to interact with others is a fundamental human attribute, as is the tendency to derive pleasure in discussing ideas with others. Distance is an impediment to such interactions, and thus cities are the ideal institution for the development of social contacts.'* (Fujita and Thisse, 2002, p.9).

However, the existence of periurban areas support that there is also some desire to live outside of the city or not in a fully agglomerated setting. Is this because of the negative aspects of agglomeration (congestion) or the positive aspects of the countryside (greenness, open-space) ? Periurban households may well want to agglomerate, but not too much. What 'not too much' means for a household is assumed here to result from a local trade-off between clustering and dispersion. The land market, therefore, is assumed to reflect the effect of these opposing neighbourhood externalities and the effect of transport costs.

In summary, it is assumed that agglomeration-dispersion forces at the level of the neighbourhood (CA or Schelling-type forces) interplay with agglomeration-dispersion forces at the scale of the urban region (the fundamental trade-off of residential economics). Moreover, the research is based on the assumption that time and space not only matter, but work hand in hand in explaining the emergence and morphology of periurban areas. Integrating CA and urban economics, therefore, allows for dynamic patterns to be related to economic processes.

Finally, it is assumed that neighbourhood externalities is a key concept for integrating CA and urban economics. This idea is not new. Papageorgiou (1990), for example, presented a conceptual framework for modelling environmental quality in spatial choice. The following few sentences, although not referring to CA, reveal what this thesis is attempting to do when trying to integrate an economic, a spatial, and a dynamic perspective to model periurban residential choice:

> *'Geographical analysis has traditionally neglected to take into account how choice behaviour depends on preferences. One way of dealing with this problem is through the idea of a spatial externality: the mere concept of an externality implies preferences; and the word 'spatial' embodies the truth that the environment at a particular location is a set rather than a point-determined process. [...] Consider a distribution of individuals over some landscape. Every*

> *individual emits an externality which somehow diffuses its impact to other individuals. [...] A distribution of the spatial externality in [the landscape] prompts adjustments which alter the associated distribution of individuals. In this manner, one obtains two interacting surfaces unfolding over the landscape - a population surface and an externality surface. The nature of such interactions [...] depends on the structure of the spatial externality.'* (Papageorgiou, 1990, p.314).

Thus, the aim of this thesis is to model periurban preferences (*choice behaviour*) that include neighbourhood externalities (*the environment is a set rather than a point*) in a dynamic framework (*externality prompts adjustments in the population distribution*).

1.3 Outline of the thesis and the main results

This thesis is based on the development of a cellular-dynamic model of periurban morphogenesis that is grounded in residential choice theory. A model is proposed in chapter 3, which is then further developed in the two subsequent chapters (chapter 4 and 5). Theoretical findings in these three chapters are systematically summarized into different propositions named 'Results' as they do not derive from any analytical proof. These 'results' describe the most general processes that are observed from the simulation experiments. Periurban concepts and processes are reviewed in chapter 2, from which the need to develop a model is derived. Chapter 6 explores a calibration procedure of the model in order to assess its applicability. Chapter 7 concludes by indicating the advantages and drawbacks of the method.

Each chapter has specific methodological or thematic objectives. These are presented in more detail below. As each chapter has been written in order to be read on its own, the reader may find some repetition in the background or the assumptions. However, a slightly different perspective is taken in each chapter and the literature reviews focus on different aspects of periurbanisation processes or method.

Chapter 2 - Overview and the need to develop a model. This chapter aims to provide a conceptual framework for the analysis of periurbanisation. A definition of periurban areas is given and some spatial and demographical observations in Europe are analysed in light of this definition.

International comparisons of periurban processes are made difficult because spatial typologies vary from one country to another. However, it was found that residential choice behaviour shares common characteristics across Western Europe. The most attractive residential locations combine both a pleasant green neighbourhood with proximity to jobs and services. Furthermore, similar demographical and spatial trends indicate that periurbanisation is widespread in Europe. There are also, however, important variations in the spatial morphology of periurban patterns. Some of these differences can certainly be explained by local characteristics, including original settlement type, timing of periurbanisation, spatial planning policies, etc. However, little attempt has been undertaken in the literature to relate 2D spatial patterns (compact, clustered, scattered developments) to residential preferences. Thus, it is simply unknown whether the observed variety of periurban morphologies in Europe could already be explained by the variation in household preferences.

The model that is constructed in the next chapters is an attempt to fill this gap. The thesis tests therefore whether the observed diversity of periurban forms can arise, independently of local characteristics, from a stylized behaviour that is common throughout Europe: residents make their location choice by considering the distance to the job centre, and the level of both open-space and services provided by their local environment. It is concluded that a model of periurbanisation should (i) include space explicitly in order to investigate forms, (ii) be based on micro-economics that represent choice behaviour and (iii) bare a certain degree of path-dependency because the number of periurban households is increasing and periurbanisation occurs in different initial configurations.

Chapter 3 - Emergent periurban configurations. This chapter presents the construction of a dynamic open-city model where households are commuters and take into account two opposing externalities (greenness or open-space and social interactions or local public goods) as a function of local residential density. The objective is to explore the emergence of different spatial morphologies from varying neighbourhood preferences. The chapter presents the characteristics of the stationary (long-run) equilibrium and the short-run mechanism (utility adjustment process). This first model is run on a 2D theoretical grid and households, all of a single type, do not consider housing lot size in their location choice.

A continuum of patterns is obtained from changing preference for greenness externality with respect to local public goods. Local agglomeration forces expand the city and flatten its shape, while the preference for local dispersion leads to the emergence of a mixed periurban belt and a less populated city. Discontinuities in the urban built-up area arise in the shape of striped and clustered developments. Moreover, the level of fragmentation of the periurban belt increases with the preference for open-space and the spatial horizon of residents.

Interesting findings are also made on the evolution of the land rent profile through time within the periurban belt. Moreover, the mixed periurban belt arises as a long-run equilibrium configuration despite large local variations in land rent and without assuming the presence of any planning zones or land developer. This land rent pattern is explained by the irreversibility of urban conversion and the path-dependent nature of the spatial patterns. Because of path-dependency in the development of structures, the exact pixel state at equilibrium is unknown, but the emergent spatial patterns are robust from one simulation to another.

Finally, the chapter highlights the different effects of commuting costs and income change on the urban structure. These findings complement the standard suburbanisation effect of the monocentric approach by showing how increasing income enlarges the specialised core of the city, while decreasing transport cost enlarges the periurban belt. This finding is the opposite of that proposed by Cavailhès et al. (2004b).

Chapter 4 - 1D and dynamic perspective. This chapter focuses on the expansion dynamics of different kinds of periurban local density structures and therefore uses a 1D theoretical space. The chapter provides a visualization method analogous to a 1D CA representation. The aim is to further understand the effects on the dynamic patterning of urban spread of changing preferences, income, and transport cost. Moreover, the impact of implementing a green belt policy that seeks to reduce residential dispersion is also analysed.

By contrast to the model presented in chapter 3, residents can trade-off commuting costs and externalities with housing consumption, and the neighbourhood externalities are calculated on the basis of population density rather than land use density. The model is therefore more complex.

The results obtained from these simulations agree with the results of chapter 3 and complements its findings. For instance, the preference for local greenness or open-space is shown to create a discontinuous fringe from the start of urban development when no local agglomeration is valued by households. Local agglomeration delays the equilibrium and the creation of a mixed periphery. This suggests that cities should reach a certain size before being surrounded by a periurban belt.

It is also found that the periurban belt is wider, expands more rapidly and contains a greater diversity of local patterns when the value of local open-space is increased, which is more difficult to see in a 2D setting. Moreover, by considering population density rather than land use density, the size of residential clusters in the periurban belt is no longer directly dependent on distance. Finally, a green belt is shown to be able to delay urbanisation and reduce fragmentation in the most rural areas.

Chapter 5 - Segregation in periurban areas. This chapter aims to understand the spatial arrangement of two household types within the city-region. As most models describe segregation through a core-periphery approach (distance to CBD), it is unknown whether the mix of urban and agricultural uses can impact on the income sorting structure.

The model investigates the emergence of spatial segregation and dynamic processes of filtering (well-off people replacing poor people and vice-versa) within a periurban morphology which is endogenously scattered. Different types of neighbourhood preferences are included in the model to explore the effect of social discrimination. Simulation experiments are run to test the effect of different policy options that could reduce spatial segregation. The 2D theoretical space is the same as in chapter 3, while the residential choice includes housing lot size as in chapter 4. Filtering processes, however, make the model even more complex as the internal immobility of people, assumed in previous chapters, is partly relaxed.

Simulation results show that in the absence of greenness externalities, the equilibrium city is compact and concentric with the higher income group at the periphery. In contrast to the standard residential model of urban economics, however, short-run equilibria can show the emergence of a ring with less well-off people beyond the location of the rich group. Moreover, when segregation behaviour is assumed, the contact fringe between income groups becomes flatter.

With greenness externalities, the same scattered structures as in chapter 3 are found, with more well-off households who benefit from more open-space amenities as they are located at the periphery. Segregation behaviour can then disrupt the built-up morphology of the periurban belt. Moreover, because it is coupled with path-dependency, segregation can lead to non concentric patterns. Green buffer zones can for example arise and separate income groups in space.

Finally, income and transport subsidies are shown to be unable to de-segregate groups in space, but increase the suburbanisation of the poorer group, thus helping these households to compete with rich households at the periphery.

Chapter 6 - A calibration procedure. This chapter aims to construct a methodology for the calibration of the model. The experiment is not designed as a falsification test of the model, but is an attempt to assess whether the theoretical archetypes generated in chapter 3 can fit observed spatial patterns. More precisely, the objective is to assess the capacity of the model to generate global spatial patterns that are similar to real patterns when some heterogeneity is assumed in space, i.e. the transport network and planning regulations.

For the sake of simplicity, the model presented in chapter 3 is used, where housing consumption is constant. This is a reasonable assumption as the model is applied only to the periurban belt without considering the agglomerated part of the city. Moreover, the segregation model would need very precise information on the location of households by income in order to be calibrated.

The calibration procedure is applied to the Southern periphery of Brussels, at two spatial resolutions. The procedure consists in varying the relative weight of the greenness and social neighbourhood externalities for the residential choice in order to minimize the difference between observed and generated spatial patterns. The patterns are measured by a set of macro-structural indices.

It was found that the model performs correctly concerning the level of fragmentation and the fractality of periurban developments. This result suggests that it is useful to implement the two proposed neighbourhood externalities within a monocentric model. However, it was also found that the model could not account for the growth of larger subcenters within the periurban belt.

Chapter 7 - Critique and conclusion. This is the general conclusion of the thesis. The results of the research with respect to periurbanisation processes and the methodology are presented. It is believed that the integration of CA and urban economics is innovative and rich as shown by the different experiments undertaken. A general critique of the methodology is made and some perspectives are discussed.

Chapter 2

Periurbanisation in Europe: an overview

Outline

This chapter uses a European-wide perspective to shed light on the spatial processes that occur at the interface between urban and rural areas. There is a clear trend to the diffusion of population towards commuting fields and rural areas. A growing share of national territories and population are concerned with this process. Changes are driven by similar residential location behaviour throughout Europe, although slight differences can be observed. Nevertheless, despite strong similarities in the processes, differences remain in the morphology of urban development patterns. Local geographic and demographic characteristics, regional economic dynamism, historic settlements, as well as spatial policies differentiate the evolutions and morphologies of European periurban areas today.

A modelling effort is necessary to better relate periurban forms with underlying processes. That is why a model is proposed in the next chapters, which can generate various periurban morphologies independently of location characteristics. This first chapter aims to provide a conceptual framework and stylized facts to feed such a model of residential growth and morphogenesis.[1]

[1]This chapter is based on a research made for the DATAR (France) that was part of a prospective research on the evolution of rural areas in France and was sought to provide a European perspective. More specifically it was asked to assess the extent of periurban areas in Europe and identify the most recent demographic trends. The research surveyed works in demography, geography and economy. For comparative purpose, it was also necessary to

2.1 Introduction

The zone between urban and rural areas in Western countries has experienced large changes during the past fifty years. As a result, the distinction between the city and the countryside is becoming less clear. Urban and rural land uses intermingle. The urban way of life has spread out of the city so that the definition of the city itself has become a difficult task.

On the one hand, if a continuous built-up area is used to depict a city, this is only a morphological definition that no longer fits the reality of human interactions. Decreasing transport costs and increasing income have allowed for longer commuting beyond the morphological limit of the city (e.g. Brueckner, 2000b; Mieszkowski and Mills, 1993). On the other hand, agriculture is no longer able to completely define a rural area. The consumption of agricultural land for residential and amenity uses has increased, due to the preference for 'greener' areas and attractiveness of a pleasant landscape. This has also been accompanied by structural changes in the agricultural sector, including diversification (see e.g. Perrier-Cornet and Hervieu, 2002; Cavailhès et al., 1994).

These processes can be referred to as 'periurbanisation' (mainly as in the French-speaking literature), but the processes are also recognized by many authors across Europe and North America independently of the name they give to the process (deconcentration and decentralisation, extended suburbanisation, exurbanisation, ...). Periurbanisation raises important questions for the future structure of territories and urban regions, and for rural societies at the periphery of cities. Provision of services, commuting intensity, car-dependence and energy use, costs of infrastructure are major concerns (Breheny, 1995; Camagni et al., 2002; Ewing, 1994). The consumption and fragmentation of agricultural land and the proximity to residents also induce changes in the agricultural activities (Heimlich and Anderson, 2001). The expansion mode of urban developments impacts on natural and rural areas and questions the preservation of open-space and farmland (Alterman, 1997; Gordon and Richardson, 1998). Periurbanisation can also be argued to impact on city centres by changing demographic and social profiles and, thus, is not independent of central city decay and segregation (Glaeser and Kahn, 2003).

In the remainder of this chapter, a definition of 'periurbanisation' is intro-

know whether urban expansion processes were described in a similar manner in other European countries. A detailed analysis of spatial and demographical trends per country can be found in this report for France, the Netherlands, Belgium, Germany, Great Britain and Italy as well as a set of references for Sweden, Norway, Finland and Denmark (see Caruso, 2002b). A synthesis of this work has been published as a chapter in a collective book (Caruso, 2002a, in French). The elements presented in this thesis chapter are essentially issued from the book chapter. However, additional background references are provided and the perspective of the chapter is changed to show the need of building a model of periurbanisation that links forms and processes.

duced and discussed with other related concepts of urban change (section 2.2). This definition is used as a referential framework to describe the characteristics of periurbanisation in Europe in section 2.3. The level of periurbanisation is first assessed at the European scale using a land cover map and aggregate classifications that emphasize the morphology of urban developments (section 2.3.1). Then the importance of periurbanisation is shown for some particular countries using national functional typologies (section 2.3.2). In the conclusion of the chapter (section 2.4) it is suggested to build a model that relates the morphology of urban developments with the observed determinants of residential choice in Europe in order to better understand the variety of periurban patterns.

2.2 The periurbanisation concept

There are many concepts that are used to describe urban change in the literature, but some of them seem to dominate the debate in Europe: *suburbanisation* is the most widespread, *counter-urbanisation* is also very common, while *periurbanisation* is more and more common in the French-speaking literature (and in Southern Europe case studies). We have chosen to use the *periurban* concept throughout the thesis because its general understanding in the literature is closely related to the spatial characteristics that we want to analyse, and also because there is some sort of consensus about its sense (which is maybe due to the specific definition given by INSEE in France).

We choose to define periurban areas according to the following two characteristics:

(i) Periurban areas are under urban influence. The nature of the link between the periurban zone and the centre is functional and is characterized by commuting flows.

(ii) Periurban areas show rural character due to the presence of an agro-forestry sector which represents an important part of the total surface and implies low population densities.

In economic terms periurban areas can be defined as 'mixed' or 'integrated' areas where consumption and production activities compete for land. Residential consumers and agricultural producers co-exist (Cavailhès et al., 2004b).

It is much more difficult to find a general agreement on what is *suburbanisation* in the literature. Bourne (1996) discusses the complexity and diversity of this concept. He mentions ten different interpretations and points to differences in the USA and Europe, especially for the UK. Some of these interpretations can be linked to *periurbanisation* as they emphasise agent preferences for more space and natural amenity consumption in their residential choice, as well as the connection to an urban core (see Bourne, 1996, table 3).

The rural aspect of periurban zones is probably one of the main differences between the suburban and the periurban concepts. In our understanding, the former appears more agglomerated or dense. In fact, some authors differentiate *suburban* from *periurban* (often more distant from the city) in the same study, while others do not (Vandermotten, 1991). The differentiating criteria can be, for example, the commuting mode: individual car for *periurbanisation* and public transport for *suburbanisation* (Halleux, 2001).

The *periurban* concept is neither comparable with the *counter-urbanisation* concept (as defined in Champion, 1989; Fielding, 1989), which deals with change in the urban hierarchy due to migration flows toward medium and small-sized cities. This phenomenon, therefore, occurs at a different scale than periurbanisation by considering the whole urban network rather than a single functional region. It has however been doubted whether *counter-urbanisation* is a separate concept of *suburbanisation* (Nelson and Sanchez, 1999).

The problem of comparisons between *periurban*, *suburban* and *counter-urban* concepts is probably as ancient as the recognition of the processes themselves. Kurtz and Eicher (1958) already attempted to differentiate *fringe* and *suburb* (in Thomas, 1990). The *urban fringe* concept constitutes another important part of the literature mainly in the USA and later in Europe (Van Oort, 1996). Although the urban fringe definition is similar to the periurban concept because it deals with a mix of urban and rural functions within the same geographic zone (Lucas and Van Oort, 1993), its extent seems however too limited to account for the whole periurban process. *Urban fringe* is more focused on land conversion at the limits of the agglomeration (i.e. a continuously built-up area).

Finally, the concept of *urban sprawl* can be seen as a very close concept to *periurbanisation* as it generally refers to low density settlements, leapfrogging morphologies or scattered developments and, therefore, situations where urban and rural land uses intermingle. However, there is *'no common definition of sprawl and relatively few attempts to operationally define it in a manner that would lead to useful comparisons of areas to determine which had experienced greater or less degrees of sprawl'* (Galster et al., 2001, p.682). Moreover, the concept does not seem to be neutral in the debate on the causes and impacts of urban expansion. It is often regarded as undesirable and unsustainable (see a discussion in Batty et al., 2003).

Some recent works tried to define urban sprawl in a more precise manner by using the detailed location of land uses, and propose to measure sprawl with a set of spatial indices (Galster et al., 2001; Malpezzi and Guo, 2001; Torrens and Alberti, 2001; Burchfield et al., 2002). Previously the measure of *sprawl* or *suburbanisation* was mainly done by testing change in the density/distance gradient, with density being an aggregated value over spatial administrative units (see a survey in Mc Donald, 1989). This method provides no sense of the various spatial morphologies that can occur within equally dense areas. However, the method has the greater advantage to be related to known urban economic processes (Clark, 1958; Fujita, 1989). This is not the case for the most recent morphological metrics of sprawl, which are not related to economic processes (Burchfield et al. (2002) is a recent attempt in that sense, which relates land cover based measures of sprawl to population growth and household formation, and shows the increase of land use consumption per capita in the USA).

2.3 Overview of periurbanisation in Europe

This section first assesses the spatial extent and morphology of periurban areas by using a set of classifications applied homogeneously across Europe. Then, an overview of periurbanisation at a country level is given using specific national typologies. This second analysis focuses mainly on demographical mobility (migration flows by socio-economic group), the conversion of rural land into residential use, and the growth in the part of the territory that is polarised by an urban centre (e.g. evolution of commuting). A synthetic table is provided in order to draw some comparisons between different countries. However, distinctions should be addressed carefully because of a lack of comparability of national periurban concepts and measures.

2.3.1 Periurban morphology at the European scale

There exists no classification at the European scale that covers the concept of periurbanisation defined above in this thesis, and thus deals with both the commuting characteristic of periurban areas and the mix of their land uses. The level of periurbanisation is portrayed in this section by means of a land cover map and two classifications. They cover the whole Western Europe but emphasize only its morphological aspect. A third typology (by Vandermotten et al., 1999) is also discussed, which accounts for the functional (commuting) part of the definition, but covers only some European urban regions.[2]

[2]The *European Functional Urban Areas* (EUFA) is another European-wide classification, but is not considered in this discussion. It is based on the UN definition of built-up areas applied to NUTS5 units and on a threshold of 40% of people commuting to the core amongst the active population. It is also worth noting that similar classifications of the urban-rural continuum have been made in the USA (using population size/density, adjacency to metropolitan areas, degree of urbanisation and daily commuting), e.g. Butler and Beale (1994), Commuting zones (CZ) and Labour Market Area (LMA) (Tolbert and Sizer, 1996), rural-urban commuting areas (RUCA's) (Morill et al., 1999).

2.3.1.1 Urban land cover

The CORINE Land Cover European database (EEA, 1993) defines two urban classes: discontinuous urban land and continuous urban land (the definition of these classes includes photo-interpretation). These classes are presented in Fig.2.1 and summarized at the country level in Fig.2.2a as a percentage of national surface. In Europe, taken as a whole (EU 15 except Austria and Sweden), continuous urban land represents 0.37% of the territory and discontinuous urban land 2.56%.

The land cover map (Fig.2.1) provides a first overview of periurban patterns in Europe and shows the diversity of the spatial organisation around cities. The size of residential clusters, their shape, their density and dispersion within the non-built areas are potentially indicators of underlying location choice processes. For example, the difference is striking between the compact aspect of cities in the Netherlands and the strong dispersion of urban developments in Belgium. Both are very densely populated countries in Europe, but the settlement pattern is very different. The contrast and particularities of these countries are also clear from the graphic (Fig.2.2a) where discontinuous urban land is below 1% in the Netherlands, while it is about 6 times greater in Belgium than in Europe in average. Arguably this contrast reflects different spatial planning policies. Belgium can be seen as 'a very under-planned country' that gives rise to a widespread diffusion of dwellings and ribbon-like spatial development (Holden and Turner, 2001, p.323). This type of urban spread is avoided in the Netherlands as well as in Great Britain where compact city or green belt policies have been implemented [3].

If the intensity of periurbanisation is approximated by the share of discontinuous urban land in the total surface of a country, then Belgium, Luxembourg, Germany and the United Kingdom are the most periurbanised countries. This 'ranking' is however not the same if different typologies are used. Two other examples are shown below.

2.3.1.2 Urban typology

The statistical definition of cities varies throughout Europe (Pumain et al., 1992; Le Gléau et al., 1996) and it is difficult to find a definition that would be uncontroversial throughout Europe (EUROSTAT, 1999). However, a classification of the degree of urbanisation is proposed by EUROSTAT (1999) where three types of areas are defined, based on population and density thresholds.

[3] Different policy approaches in relation to urban spread are discussed by Fouchier (1999) for England, Norway and the Netherlands, as well as by Kreukels and Pollé (1997).

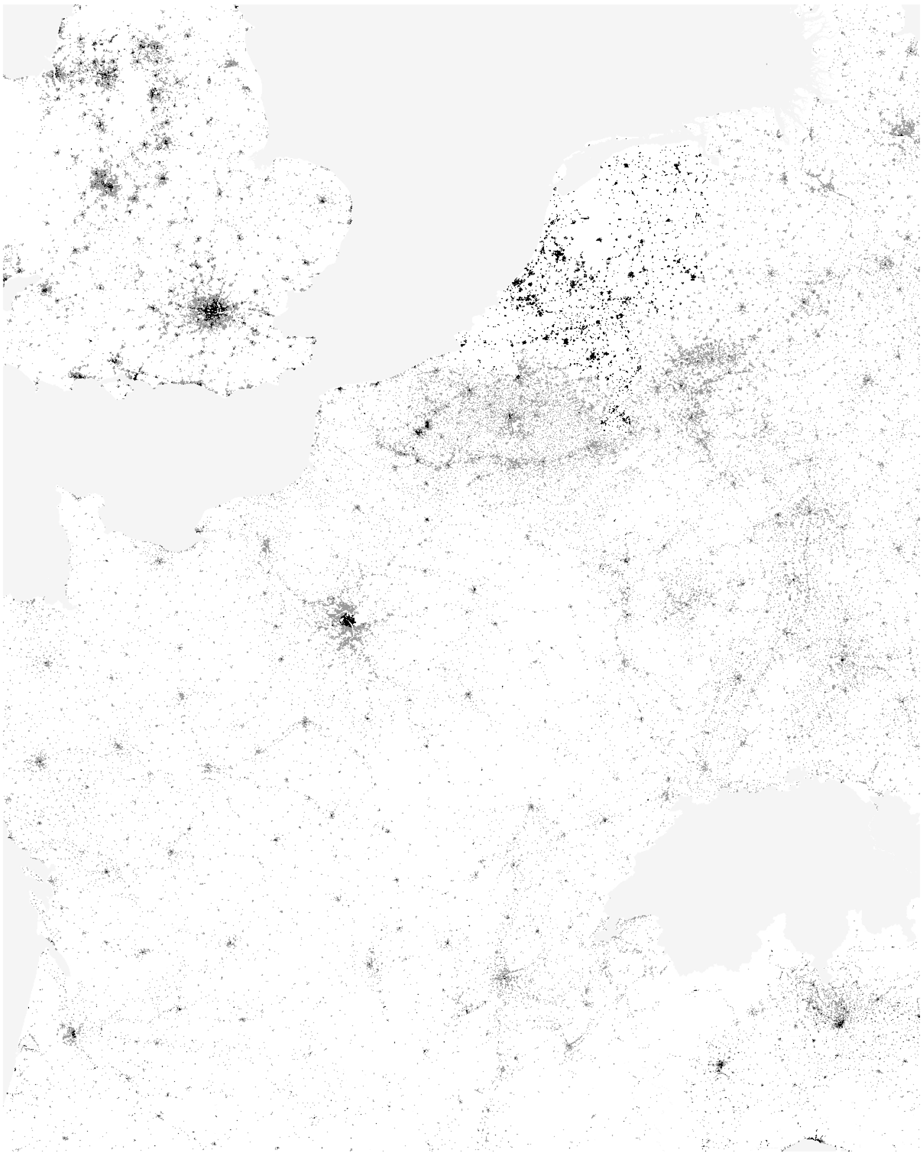
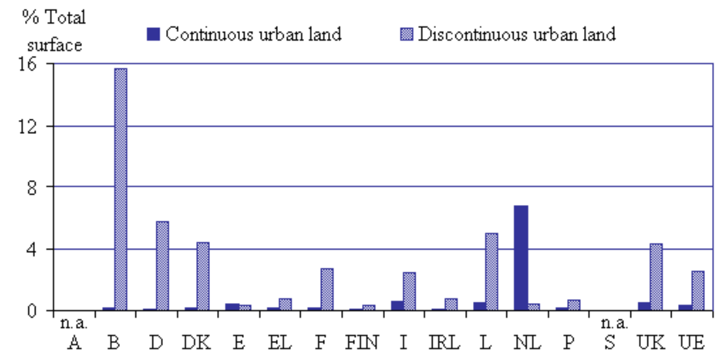
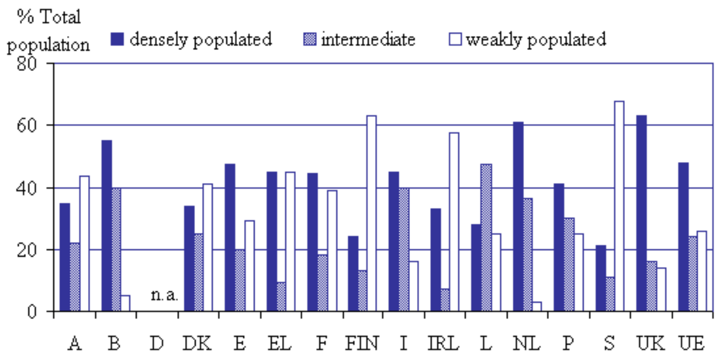


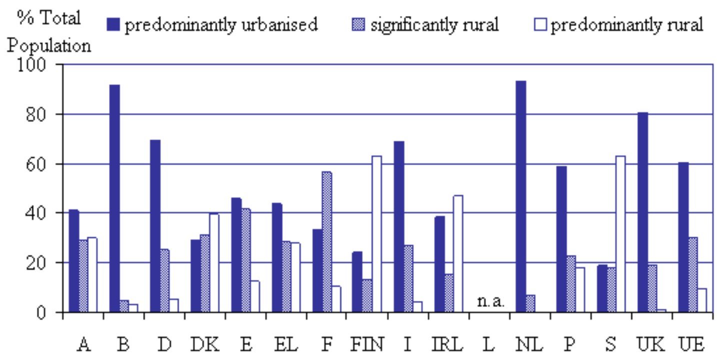
Figure 2.1: Urban classification CORINE Land Cover (EEA, 1993) (continuous urban land in black and discontinuous urban land in dark grey)



(a) Urban classification based on CORINE Land Cover (EEA, 1993) (Values summarized to national boundaries by the author)



(b) Eurostat classification of the urban-rural gradient, per country (data: EUROSTAT 1999)



(c) OECD based classification of the urban-rural gradient, per country (data: DGIV, 1997)

Figure 2.2: Three spatial classifications at the European scale

The *densely populated areas* are defined as groups of territorial units (NUTS 5 level) with more than 50,000 inhabitants and composed by units each with a population density of over 500 inhab./km^2. The *Intermediate areas* are groups with more than 50,000 inhabitants or contiguous to a *densely populated area* and within which units present a population density of over 100 inhab./km^2. The remaining areas are named *weakly populated areas*. The percentage of population for each class at the country level is presented below in Fig.2.2b.

According to these definitions, some important differences can again be observed between the different European countries. The share of rural population (through the *weakly urbanised areas*) is most significant in the Scandinavian countries (above 60% in Finland and Sweden). The opposite situation occurs for the United Kingdom and for the Netherlands which concentrate their population (more than 60%) in the *densely urbanised areas*. The *intermediate zone* covers *'rural character situations but overall an urban structure based on small and medium size agglomerations'* (see EUROSTAT, 1999, p.2.). Although there is nothing about a functional link with the city centre within their definition, these zones can therefore be assumed to contain most of the periurban situations. Thus, in the sense of the *intermediate zones*, the Benelux countries and Italy are the most periurbanised, while Greece and Ireland are the less periurbanised.

2.3.1.3 Rural typology

Another classification that is also based on population size and density thresholds is proposed by the OECD (see DGIV, 1997). This classification first identifies rural communities as spatial entities (at NUTS 5 level) where population density is below 150 inhab./km^2. Then (at the NUTS 3 level) three types of areas are defined : *predominantly rural* (at least 50% of the population live in rural units), *significantly rural* (15% to 50%) and *predominantly urban* (less than 15%).

This classification is reported in Fig.2.2c. Like the EUROSTAT example, this typology also suggests that the urban-rural dichotomy varies a lot from one country to another in Europe. The urbanised aspects of the Netherlands and Belgium but also of the United Kingdom, Germany and Italy are more strongly highlighted in this OECD classification. Also, a lower relative share of the most rural class may indicate that mixed areas are of great importance. France shows the highest value for the *significantly rural areas* whilst in this case Sweden, Finland and Denmark are more characterised by *predominantly rural areas*. This would suggest that periurbanisation is more likely to be important in France than in Nordic countries.

Analyses of population change along this OECD classification (see OECD, 1996; Champion, 1998) demonstrated that, during the 1980's, the *significantly rural* areas have been recording the highest growth. Belgium and Germany were the only countries in Europe where the *predominantly rural areas* experienced higher gains. This suggests that population deconcentration in both countries is a very extended process.

2.3.1.4 Morphological and functional urban areas

Vandermotten et al. (1999) give three overlaying definitions of urban areas: morphological agglomerations, functional urban areas and administrative urban areas. The former is defined according to a density threshold (650 inhab./km^2), and the second corresponds to the workforce catchment area where at least 10% of the active population works in the agglomeration. Vandermotten et al. (1999) compare the situation of seven North-West European cities in the early 1990's. A ratio between the population in the economic centre and the population of the functional region is reported in their analysis. This ratio is very high for the Rhine-Rhur region, demonstrating its polycentricity and a lower polarisation of the workforce (each centre functions with its own but limited commuting field). The dominance of London over its functional region is shown to be less important than in Paris where the functional region is nearly two times greater than London. Brussels is also very extended with respect to the population of its centre and the number of employments.

The ranking of the seven cities differs when one consider either the surface or the population of the functional and morphological areas. The variation in density is therefore important to consider. According to the authors, different densities are the expression of differentiated residential behaviour, specific conditions for access to housing, urban policies, and land and planning regulations (Vandermotten et al., 1999, p.91).

2.3.2 Overview of periurbanisation by country

As mentioned by Champion (1992), the evolution of the settlement pattern in Europe is led by changes in demographical characteristics (ageing, new family types, reducing household size,...). In many European countries the rather uniformity of natural growth increases the importance of internal migration in explaining the spatial distribution of the population (Rees and Kupiszewski, 1999; Champion, 1998). Furthermore, strong similarities in migration patterns are observed throughout Europe. The attractiveness of city centres, periurban areas and rural areas varies with the age of migrants, the size of the households, and their socio-economic status in the same way across the different countries.

However, there are also some differences. For example, the rate and phasing of deconcentration and concentration periods vary throughout European regions (Champion, 1992, 1998; Cheshire, 1995). Moreover, migration patterns can be influenced by the regional economic growth, but not necessarily in the same way in different regions. For instance, migration patterns are highly correlated with unemployment level in the United Kingdom or Germany, while it is not the case in the Netherlands where residential migration is substituted by daily commuting (which, in turn, can be explained by the scarcity of housing and the density of road networks) (Kupiszewski, 1999). Different expansion processes can also occur because of differences in planning policies and housing markets. For example, while the role of the public sector has been of great importance to explain suburbanisation in the United Kingdom, France, the Netherlands and Denmark, this role has been played by private firms in Belgium, Germany or Spain (Cooke, 1990).

Therefore, although similar residential behaviour is observed in Europe, the pattern of migrations can be influenced by national characteristics. In the remainder of this section, an overview of periurbanisation trends is given from population size and change within national typologies. Each European country defines one or several typologies to describe the urban-rural gradient, and some classes might represent the periurban concept given above. Examples are given for France, The Netherlands, Belgium and Great-Britain. These typologies are based on different spatial units and combinations of morphological and functional variables. Thus, it is difficult to base comparisons on these 'periurban' classes. However, a synthetic table reports key figures of periurban population and extent (see table 2.1) that are discussed below.

2.3.2.1 France

According to the spatial nomenclature adopted by INSEE (Institut National de la Statistique et des Etudes Economiques)(see e.g. Le Jeannic, 1997; Schmitt et al., 1998), a French commune is either a *rural commune* or belongs to an *urban unit*. An *urban unit* is a group of communes in which there is a built-up area with at least 2000 inhabitants and where the maximum distance between two dwellings is 200m. Then, INSEE defines *urban areas* as a group of contiguous communes, rural or urban, that surround an *urban pole*. In these *urban areas* at least 40 % of the active population commute to the *urban pole* [4]. By definition an *urban pole* is a group of contiguous *urban units* that contains at least 5000 jobs). The *periurban* ring is made of the urban area except for the urban pole. Moreover, *rural areas under weak urban influence* have been defined and

[4] and to other poles in the case of *multi-polarised communes*.

charaterised by a threshold of 20% of commuters within the working population of the commune.

Both the *periurban ring* and the *rural areas under urban influence* can include urban units and rural communes and fit the concept of periurbanisation proposed above. Together, they represent about a quarter of the population (24.6%) and half (52.8%) of the French communes. Residential housing increased by 70% in periurban rings within the inter census period 1962-1999. The peak of periurbanisation was reached in the late 1980's and a bit earlier in the urban area of Paris (Le Jeannic, 1997; Cavailhes and Schmitt, 2002). The 1990's have seen continuing but slowing down periurbanisation (Bessy-Pietri, 2000). This weaker residential mobility is explained by ageing, economic crisis, the level of homeownership and change in housing policy that reduces subsidies for homeownership access (Baccaini, 2001).

In France, periurban areas are younger than cities and specialise in high level workers and intermediate employees (Schmitt et al., 1998). Young periurbanising families search for a natural environment with large house and garden, and a link to the city where they have their job (Le Jeannic, 1997).

2.3.2.2 The Netherlands

From the EUROSTAT and OECD European classifications, the Netherlands are characterised by a very low level of predominantly rural areas and weakly populated areas. As shown on the land cover map, the Netherlands are also characterised by a high level of urban compactness. Spatial planning has strongly orientated urban development, towards deconcentration in the 1950's and 1960's (although avoiding extensive dispersion forms), and towards compact city forms from the 1980's in order to protect green areas (see e.g. Jansen et al., 1997). The Randstad cities, for instance, encircle a 'Green Heart' where the land use is mainly agricultural and households mainly commuters, like in periurban areas. Planners also strictly design growth areas at the periphery of cities in order to accommodate the housing demand. The necessity to protect green areas is a lively debate in the Netherlands (Lorzing, 1996) because it contrasts with the preference of people for more spacious housing, and induces strong tensions on the housing market (which is also a source of increased social segregation (Van Kempen and Van Weesep, 1998)).

The Dutch Bureau of Statistics (CBS) defines (see Rees et al., 1998) three categories of municipalities along the urban-rural gradient: *urban*, *rural* and *urbanised rural.* The definition of *urban* municipalities is based on the population that resides in an agglomerated nucleus (continuous built-up). Subcategories of these *urban* municipalities are defined and based on the total population of the agglomeration. In *Rural* municipalities, at least 20% of the labour force is in the agricultural sector. Between *urban* and *rural* municipalities, an *urbanised rural class* is defined, where the agricultural workforce is less important and the largest residential nucleus comprises a maximum of 30,000 inhabitants. A specific commuter subclass is also defined within the *urbanised rural* group. The *urbanised rural* class can approximate the periurban concept given above. Periurbanisation would therefore account for 38.2% of population and cover 56.7% of the territory.

The socio-economic characteristics of Dutch periurban areas contrast with centres because the households that went into peripheries before the compact-city policy are now getting older, while the migration of young families in these areas is now more limited. However, deconcentration towards less dense areas and smaller centres has been a continuing process during the last 20 years, although it has occurred within compact morphologies.

2.3.2.3 Belgium

Van der Haegen et al. (1996) define 17 *urban regions* in Belgium, based on a variety of functional, morphological and demographical criteria. A *morphological agglomeration* is defined by the continuity (250m) of urban fabric around a central city, which defined by a concentration of shops, services, population as well as other characteristics like the size of housing and the age of population. A *'banlieue'* is then drawn around the morphological agglomeration and defined by another large set of variables (including residential migration, land use conversion, work and school commuting, population growth, income,...). This *'banlieue area is functionally urban but, morphologically, can appear to be rural'* (translated from Merenne et al., 1997, p.14). It corresponds, therefore, to our periurban definition. Altogether, the *morphological agglomeration* and the *'banlieue'* form an *urban region* if there is at least 80,000 inhabitants. Thus, small rural centres are not included in this typology.

Moreover, a larger commuting ring is defined by Van der Haegen et al. (1996) where at least 15% of the active population commutes to the centre. Like the *rural under urban influence* class in France (although with a smaller commuting threshold) this commuter zone might also correspond to the periurban concept. The periurban areas, in the sense of *'banlieues'* and *commuter zones*, account for 33.6% of the population and 39.4% of the total surface. In population terms, it is slightly lower but comparable to the Dutch case and more than in France. In surface terms, it is surprisingly below the level of these two countries. However, the whole Belgian territory is not covered by the *urban region* typology.

The dispersed form of the Belgian urban settlement has been emphasized previously by the land cover map and graph (Figure 2.1 and 2.2a). The analysis of dwelling nodes, defined only by a criteria of built-up contiguity (see Halleux et al., 1998) complements this observation and shows the importance of small urban nodes (75% of the nodes have less than 1000 inhabitants). The population who live in dispersed dwellings, out of any of these nodes, is also very important (mainly in the Northern part of the country). Moreover, this diffusion of the population has been increasing in the 1980's, while in the 1970's there was a stronger encroachment of new houses on dwelling nodes.

Although weaker than in the period 1950-1980, the conversion rate of agricultural land into residential use was strong in the period 1980-1995 (40 % increase) (Jehin, 1998). In the same period, population growth was the highest in *'banlieues'* and accelerated in *commuter zones*, showing the continued attractiveness of periurban areas. In the 1990's, about 50% of new migrants in periurban areas came from an urban agglomeration and 40% from another periurban zone (Eggerickx, 1999). As in the two previous examples, Belgian periurban areas are younger than cities but are more rapidly ageing. In Belgium, the income level is also higher (Eggerickx, 1999). However, there is a contrast between periurban areas because the access to homeownership for the less well-off households is made difficult in the more economically dynamic areas and in the areas of early periurbanisation.

2.3.2.4 Great Britain.

Within the *functional regions* typology proposed by Coombes et al. (1982), *urban centres* are defined by a minimum concentration of shops and employments. These *urban centres* are then extended to the limit of the continuously built-up zone to form *urban cores.* A commuting *ring* (15% of active population) surrounds the *urban core* in order to form a *daily urban system.* Remaining zones, the *outer areas*, are linked to the core where the highest number of residents commute. If the *daily urban system* contains more than 50,000 inhabitants, it is an *urban region.* If there is less than 50,000 inhabitants, it is a *rural area* and it is attached to the *urban area* where there is more commuting. An urban area and the attached *outer* and *rural areas* form a *functional region.*

Similarly to the French and Belgian cases, the *rings* are here assumed to represent periurban areas. Therefore, periurbanisation represents 26.6%[5] of population, which is similar to France, but accounts only for 36.3% of the territory. This is the lowest percentage in the four examples presented. One can see from table 2.1 that the urban core part is much more expanded in Great Britain than in the other countries. This might be related to spatial policies, which minimize the conversion or rural space. For more than fifty years, spatial planning

[5] 1981 Census

has sought to contain urban developments by implementing *Green Belts*, and beyond, by concentrating new dwellings in small towns and villages. Recent planning policy guidances (see DETR, 1998) encourage the reuse of derelict urban land for residential use (this represented 47% of new dwellings in 1998) and prevent the creation of new urban nodes that could not reach 10,000 dwellings in 20 years.

The analysis of demographical change shows that population of *urban cores* decreased while *outer*, *rings* and *rural areas* increased during the 1980's. The deconcentration process towards less densely populated areas was thus continuing, but more slowly than during the previous decade. New peripheral towns also recorded important demographical gains. They also seem, therefore, to provide wanted residential environments and proximity to the countryside.

2.4 Conclusion

There are clear deconcentration trends towards rural areas and commuting fields in Europe. The periurban zone, defined as a mixed area (agricultural and residential) with a functional link to an employment centre through commuting trips, accounts for an important part of Europe's national territories and a growing part of its population. As shown on table 2.1 for four countries, periurban areas can represent about a quarter to more than a third of population depending on countries. In terms of surface, it represents at least a third of the surface but can account for more than half of national territory.

Similar migration patterns are found across European countries. Spatial preferences vary along the different stages of the life cycle. Young adults migrate towards urban locations and upward the settlement hierarchy because of education and employment needs. Young families with children seek for more spacious housing and migrate into periurban areas, which is the main internal migration flow. European periurban zones are thus specific in terms of demographic characteristics (age profile, type of households) and, in this respect, contrast strongly with cities. In terms of social status, the difference between cities and periurban areas is less clear, and differences between periurban areas can also be found.

Actual periurban processes are similar across Europe, with the main residential migration flows heading towards green and pleasant periurban locations at commuting distance from employment and proximity to services. However, these processes superimpose on diverse histories and geographic spaces. Different demographic potentials and urban networks, late or early periurbanisation, regional economic growth, or planning and housing market characteristics,

Classes	Subclasses (+ criteria[6])	Spatial extent (%)[7]	Population (%)
	France - Urban areas (INSEE)		
Urban areas	Urban pole (min 5,000 jobs)	7.7 (90)	60.7 (90)
		8.5 (99)	61.0 (99)
	Periurban and multi-polarised communes (contiguous + 40% active pop commuting)	**28.5** (90)	**15.7** (90)
		40.8 (99)	**21.0** (99)
Rural areas	**Rural under urban influence** (20% active pop commuting)	**24.3** (90)	**8.9** (90)
	Other rural	39.5 (90)	14.6 (90)
	The Netherlands - Urbanization categories (CBS)		
Urban	Pop > 50,000	7.9	33.7 (94)
	Pop < 50,000 (min 2000 inh.)	28.1	17.3 (94)
Urbanised rural	**Commuter** (commuting + max 20% male active in agric.)	**16.2**	**14.9** (94)
	Urban Rural (max 20% male active in agric.)	**40.5**	**23.3** (94)
Rural		7.3	10.8 (94)
	Belgium - Urban regions (Van der Haegen et al., 1997)		
Urban Region	Agglomeration (continuous built-up, services, density,...)	10.8 (91)	43.0 (91)
	Banlieue (50% total commuting to agglo + 25% active pop)	**15.5** (91)	**13.7** (91)
Commuter zone (15% active pop commuting)		**23.9** (91)	**19.9** (91)
Other		49.8 (91)	23.4 (91)
	Great Britain - Functional regions (Coombes et al., 1992)		
Urban Region (Cores + Rings $> 50,000$ inh.)	Cores (continuous built-up)	40.1	61.7 (81)
			60.1 (91)
	Rings (15% active pop commuting)	**36.3**	**26.6** (81)
	Outer areas (max % active pop commuting to cores)	12.2	6.5 (81)
Rural areas		11.4	5.4 (91)

Table 2.1: Four national spatial typologies: spatial extent and population of periurban areas. (Classes in bold are likely to cover the periurban concept chosen in this thesis.

seem to lead to slightly different periurbanisation intensity and varying spatial forms. The morphology of periurban areas appear to be heterogeneous in Europe. Residential settlements range from dispersed and leapfrogging developments to compact and clustered settings. It is not known, however, whether these forms result from different local constraints (urban network, planning,...) or can result from changes in the determinants of residential choice from place to place. In order to know that, a link must be drawn between the overall form of city expansion and residential choice processes.

The assessment of the form of the city and its expansion has long been made by analysing the density decreasing function (Mc Donald, 1989). It presents the advantage to be deduced directly from residential micro-economics. Moreover, the role of spatial externalities in changing the density gradient has also been shown (Alperovich, 1980). However, these measures are 1D by essence and generally use spatially aggregated measures that cannot render various dispersion patterns. Nowadays, urban form and expansion are also assessed with more details from 2D landscape indices (e.g. Torrens and Alberti, 2001; Galster et al., 2001) or fractals (Batty and Longley, 1994; Frankhauser, 1994). However, conversely to density gradient, these indices cannot be deduced from residential choice. Nevertheless, recent attempts by De Keersmaecker et al. (2003); Thomas et al. (2004) relate fractal metrics with socio-economic variables.

Actually, only few research have been undertaken to relate the global spatial morphology of urban developments with individual residential choice. More particularly, while the effect of changing income and transportation costs on the expansion and density of cities are well understood (Brueckner, 2000b; Mieszkowski and Mills, 1993), less is known about the role of periurban externalities and open-space in shaping mixed patterns at the periphery of cities. Different empirical works have shown the role of rural-type environments with respect to distance in residential choice (see e.g. Geoghegan, 2002; Geoghegan et al., 1997; Sullivan, 1994). Nevertheless, it is needed to further assess the implications of these preferences on the overall form of the city. Cavailhès et al. (2004b) show the emergence of a mixed area when rural amenities are valued, however in a 1D setting that cannot generate different forms of land use mix. In Wu and Plantinga (2003), open-spaces shape a 2D urban form, but spatial heterogeneity is given exogenously (and no account is made of the impact of an individual household's location on the other).

In order to clearly understand the origin of periurban forms, a model is needed that can create the different morphologies observed in this chapter from the determinants of residential choice, without any exogenous perturbation.

The model proposed in the next chapter is explicitly spatial in order for the results to be comparable with an urban land cover map. A cellular framework is thus used, from which urban form can be measured. Moreover, as periurbanisation is a dynamic process that converts agricultural land use through time along with residential migration flows, the model is dynamic. Also, as the model is path-dependent, it can potentially be used to show the impact of different initial configurations (e.g. different traditional settlement patterns), and thus, to test whether this can also explain European morphological diversity. Finally, it has been shown that residents in Europe make their location choice by considering the distance to the job centre and the quality of their local environment in terms of greenness and proximal services. Therefore, the monocentric model can be used to represent this residential choice as soon as local externalities are added in the preferences of households.

Chapter 3

Understanding emergent spatial configurations: a 2D model with single household type and fixed housing lot

Outline

In this chapter a theoretical model of residential spread is presented that aims to explore the emergence of a periurban zone where agricultural and residential activities coexist between a city and surrounding rural area. The behaviour of residents in the model corresponds to the general features of the decision making process that were highlighted in the previous chapter. Households are assumed to be commuters who take into account neighbourhood externalities as a function of local residential density. The model is based on integrating the monocentric urban model with 2D cellular automata (CA). The model is therefore explicitly spatial and dynamic and, moreover, is founded on micro-economics. Consecutive short-run equilibria are generated from incremental population growth and a utility adjustment process, which lead to specialized or mixed long-run equilibria.

After a short review of CA and urban economics, the effect of the behavioural assumptions on the long-run equilibrium market is described.

This allows for some analytical results to be given in this chapter. The dynamic functioning of the model is then further explained. The long- and short-run mechanisms still hold in the next chapters. For the sake of simplicity, the assumption of a fixed housing consumption by households is made, which will be relaxed later in chapter 4 and 5.

Several simulations are run on a theoretical 2D lattice. First, a sensitivity analysis is undertaken on the preferences of the household and on the size of the neighbourhood that they consider when choosing a location. Different morphologies emerge from this experiment, that go from compact to scattered cities. Second, the effect of changing the level of income, commuting cost and rural rent is analysed.[1]

[1]This chapter is based on Caruso et al. (2004b)

3.1 Introduction

The diffusion of residents into rural hinterlands is a major spatial change affecting Western metropolitan areas. For several decades, residential density gradients have not only decreased with time, but residential spread has also impacted on local spatial patterns (Anas et al., 1998). Urban expansion locates residential land use within rural areas. A mixed spatial arrangement results, which is the characteristic of a 'periurban' area (Cavailhès et al., 2004b)[2]. However, various spatial morphologies of urban development can be found in periurban areas ranging from compact patterns to clusters, and very scattered residential settlements (Caruso, 2002a; Dieleman and Faludi, 1998). Because the social and environmental costs of these possible urban forms are questioned (Brueckner, 2000b; Camagni et al., 2002), it is necessary to further understand the emergence of various urban spatial morphologies at the periphery of cities, as a result of economic processes. This is the objective of the work presented here.

In some places, urban planning has also played a role in shaping periurban morphologies (Kreukels and Pollé, 1997; Holden and Turner, 2001). Spatial policies can restrict building, organize new developments, and so affect the observed urban forms. Local particularities, however, make planning behaviour difficult to generalize and model. Urban expansion is also a dynamic process, and therefore, the urban geography of today is - at least partly - 'locked-in' by history (Arthur, 1994). In the same sense, the development of models that account for the durability of housing (Brueckner, 2000a), demonstrates the spatio-temporal dependency of spreading urban structures.

Within the monocentric model, urban economists have clearly identified the three major drivers of urban expansion: (i) rising incomes, (ii) transportation improvements and (iii) increasing households number (Brueckner, 2000b; Mieszkowski and Mills, 1993). In addition, it is known that residential decision making also considers the presence of different externalities in space, including local public goods and open space (or inversely, crowding)(Fujita, 1989). In Europe, attractive residential locations often combine spacious and 'green' environments with good accessibility to jobs and services (Caruso, 2002a). Furthermore, within areas where commuters are located close to farming activities, spatially mixed equilibria can emerge from farmer-resident non-market interactions (Cavailhès et al., 2004b).

A model that aims to explore the mechanisms of residential spread and the morphogenesis of a periurban area needs to be founded on residential economics,

[2] As explained in the previous chapter, the word 'periurban' differs from 'suburban'. 'Suburbanisation' is considered to be a contiguous built-up extension of the city without emphasizing any mixes of land uses. Suburban areas have, in contrast to periurban areas, a more 'agglomerated' aspect.

and include location externalities. However the residential monocentric model is rather limited in its capacity to represent diverse spatial arrangements. A solution to this problem is to take advantage of simulation through a discrete bi-dimensional approach referred to as *Cellular Automata* (CA). CA are explicitly spatial models that can generate complex spatial forms. Moreover, CA are dynamic models and display self-organizing (path-dependent) structures. The challenge addressed by this chapter is to articulate, within a CA structure, a monocentric residential model with externalities.

A review of the literature (section 3.2) has shown that there have been few, or no, efforts to integrate CA and urban economic theory. The model is described in section 3.3 by outlining the assumptions related to the discrete framework, the long-run equilibrium land market, and the short-run dynamics. Then simulations and sensitivity analysis are undertaken on a theoretical 2D lattice (section 3.4). Because the spatial forms relate to residential preferences and economic parameters, the model can contribute to understanding the relationship between economic processes and geographical patterns.

3.2 CA and urban dynamic models

Cellular automata (CA) are decentralized spatially extended systems consisting of large numbers of simple identical components with local connectivity [, . . . that] *have the potential to model the behaviour of complex systems* (Mitchell, 1998, p.95). A CA model comprises a lattice of cells each with an identical pattern of local connections to other cells and each with a state taken from a finite set. The set of cells to which a cell is connected is the *neighbourhood* of a cell. A *transition function* (transition rule) returns the updated state of a cell, depending on the neighbourhood. Therefore, the dynamics of cellular states shows recursiveness in space and time.

The early theoretical developments of CA came during the infancy of computer science and were attributed mostly to Von Neumann (1966) based on work on self-reproducing machines. Nowadays, CA become increasingly applied to modelling natural and human systems. They are attractive because they emphasize the role of local, neighbourhood-type interactions in shaping aggregate properties. Hence, intricate patterns and complex behaviour can emerge from a very simple set of rules.

Early cellular dynamic models of socio-economic systems and human interactions in space were not necessary referred to as CA. Sakoda (1971) and Schelling (1971, 1978) developed segregation models where individuals of two groups are located on a grid. A version of this model was developed in the urban context by Miyao (1978) who, disregarded the cellular framework, but included an explicit utility function and considered transport costs. Miyao demonstrated the instability of a mixed long-run equilibrium when there is no transport cost. Furthermore, he found that, within the monocentric framework, mixed patterns can be locally stable if the neighbourhood externalities are weak relative to the elasticity of utility with respect to land consumption. More recently, Axelrod (1984) and Nowak and May (1992) have aimed to understand social dynamics and relate CA to *Game Theory*.

CA passed from abstract games to geographical models after the work of Tobler (1979) and Couclelis (1985)(see Batty et al., 1997). Within the last ten years CA applications for urban and regional land use change have been prolific[3], ranging from metaphorical works to empirical examples used to forecast land use change in particular regions. However, despite the many examples of CA in geography, very few attempts have been made to explicitly link CA to economic theory relative to location decisions. Although CA have succeeded in simulating the development of cities and changes in land use, they suffer from the lack of a theoretical foundation. Like other dynamic geographical models, CA have been confined to an *ad hoc* formulation of spatial externalities, or failed to account for the role of preferences on location choice. In the present chapter, it is argued that spatial externalities are the bridge between CA and urban economics, because the interdependencies between the spatial distribution of externalities and the population distribution can force agents to adjust their location choices (Papageorgiou, 1990).

Cellular models can be found that follow a somewhat similar perspective. Page (1999) developed a model where agents have either negative or positive preferences for agglomeration and for the average distance to other people. Whilst some outcomes meet Beckman's dispersed city (Beckmann, 1976), the model considers economic variables only indirectly. Page (1999) does not formulate agent behaviour within a micro-economic context, and does not consider the land market. Webster and Wu (1999a,b, 2001) propose a micro-economic model of externality bargaining between polluting firms and residents within a CA framework, that may lead to spatial segregation of firms and residents. Although intra-group attraction is modelled and the negative externality of firms affects the utility of residents in inverse proportion to their distance from the firms, no transport cost is explicitly modelled. Furthermore, utility is used as a probability of state change, and land use conversion is driven by a Monte Carlo process. It would seem interesting, therefore, to formulate residential behaviour within a monocentric framework, with an explicit commuting cost, and

[3]The reader can have an overview of this field from the special issues edited by Batty et al. (1997) and Benenson and Torrens (2004). The *generalized and constrained CA* approach of White and Engelen (1993, 1997) is one of the better known examples of a CA application that is combined with a regional model.

use utility levels as incentives to land use change in a deterministic manner. Parker (1999); Parker and Meretsky (2004) model conversion from agricultural to urban use within a profit maximizing framework with positive or negative neighbourhood externalities. The model is monocentric but operates as a search algorithm for a static equilibrium land use allocation which does not emphasize the role of path-dependency and the succession of intermediate short-run equilibria of the land market. One can find a comparison of Parker's model with the framework presented in this chapter in (Parker and Caruso, 2003).

Within the urban monocentric framework, urban growth models with irreversible housing (surveyed by Brueckner (2000a)) are the most similar to dynamic cellular models, as the spatial structure is not adjusted instantaneously (i.e. it is path-dependent). Although they do not consider externalities[4], these models provide interesting results related to the pattern of urban sprawl, and properties that differ from the standard Alonso-Muth-Mills model. Anas (1976, 1978) has analysed housing durability under myopic landownership in a model where population growth is accommodated at the periphery of the city. Others have developed models with irreversibility and perfect foresight (e.g. Mills, 1981; Fujita, 1982; Wheaton, 1982; Turnbull, 1988), that show how positive density and rent gradients can occur, how land development can take place from the outside inward, and how leapfrogged, mixed or scattered development patterns can arise as equilibrium configurations.

Capozza and Helsley (1989) have developed an urban growth model with fixed lot size. They showed that the sequencing and structure of development is no different in this case than with malleable housing. This also suggests that it may be important to model other sources of discontinuities, such as, for example, neighbourhood externalities. Mixed urban configurations can also be found in non dynamic models that include externalities. Richardson (1977) showed how the preference for low densities (the aversion to crowding) can produce increasing bid rents and spatially discontinuous equilibria.

The *periurban city* model (Cavailhès et al., 2004b) formalizes the existence of a mixed spatial configuration when households are sensitive to rural amenities. A similar model was also developed within a specific bidimensional structure (a multi-fractal geometry) allowing residents to choose between a variety of rural externalities and a hierarchy of urban goods (Cavailhès et al., 2004a). The model presented here belongs to the same family of models of periurban interactions, but has a number of key differences. First, the model is dynamic. Secondly, a different formulation is used for household interactions and the farmer behaviour is exogenous. Thirdly, the model can explore 2D spatial configurations that are not *a priori* defined (contrary to Cavailhès et al. (2004a)). Finally it

[4]To our knowledge, there is no residential model with urban growth dynamics (perfect foresight or not) **and** neighbourhood externalities.

was developped to further understanding of the path-dependent behaviour of a growing city with externalities. The model connects therefore different fields of urban geography and urban economics concerned with spatial explicit interactions. The relevance of these spillover interactions for the understanding of residential dynamics has been recently re-emphasized (e.g. Ioannides, 2002; Durlauf, 2003).

3.3 The model

The model considers a monocentric region where farmers and residents compete within a land market. Households are utility maximisers who consume a non spatial composite good, housing, and externalities. Farmers are producers of agricultural goods. Absentee landowners allocate land to the highest bidder. Proximity interactions between households and farmers are external economies that are taken into account in the location decision of residential agents. The intensity of these interactions is related to the density of residents within their neighbourhood.

The model is therefore very simple: only two land uses are considered and, moreover, the role of land developers and urban planners is not considered. However, the model takes into account the main general processes of periurban areas. On the one hand, as shown in the previous chapter, the conversion rate of agricultural land into residential use has been very important in the past decades (e.g 40 % increase in 15 years in Belgium). On the other hand, agricultural land is by far the major land use component of periurban areas. The presence and impact of other urban land uses (e.g. services, infrastructures) is aproximated by residential density.

For what concern urban planners, it is a choice not to consider zoning systems in order to analyse the impact of individual preferences without any other constraint that would induce some spatial heterogeneity *a priori*. Finally, in this model, it is equivalent to consider rent maximising land owners (price discriminant monopoly) or utility maximising households (for this alternative understanding of the space allocation process, see Caruso, 2003b).

Because the neighbourhood is central to CA models, and because commuting costs are central to urban economics, the integration of both techniques is well suited to the analysis of urban expansion processes. The spatial structure and the evolution of the periurban area is assumed to derive from the interplay of an agglomeration force at the scale of the metropolitan region as a whole, and local agglomeration/dispersion forces exhibited at the scale of the neighbourhoods: residents trade-off commuting costs, local service amenities and green area amenities. These amenities being simply modelled as a function of local density, the framework is quite general and other interpretation of local externalities can be given (congestion, open-space, social contacts, public goods,...).

3.3.1 The cellular space

Assumption C1. The region is represented by a discrete space where there is a finite number of locations, each with a single use. Consider a lattice with I columns and J lines. There are $G = I \times J$ cells ij, all of which are the same size and shape. Each location (or cell) is represented as a square, which is the most common form in spatial simulation and can easily be adapted to raster data. The housing lot size is a constant chosen without loss of generality as the surface unit. This theoretical model is thus independent to scale as long as no calibration to real situation is undertaken.

The area is featureless (except for the city centre) and isotropic, and there are no exogenous amenities. The resulting spatial patterns will therefore show symmetry and repetition. Each location is characterized by the presence of one, and only one, type of agent: *Household* (H) or *Farmer* (A). The state of a cell ij at time t is either residential or agricultural: $C^t_{ij} \in \{H, A\}$. The location of each decision-maker coincides perfectly with the land use. The total number of agents in the area is therefore given by $G = H^t_G + A^t_G$, with H^t_G, the total number of households, and A^t_G, the total number of farmers within the grid at time t. This assumption is characteristic of CA models compared to multi-agent systems. The implications of this assumption in a real world application will depend on spatial resolution.

Assumption C2. A dimensionless CBD is located at the origin $(0, 0)$ **of the grid.** The CBD provides all non agricultural jobs and consumption goods. Each cell ij is also characterized by an Euclidean distance to the CBD, d_{ij}, that is the commuting distance. By definition, periurbanisation occurs within the spatial extent of a functional urban region, i.e. the maximum commuting distance. The use of a monocentric assumption in the study of overall urban expansion form is suggested from empirical observations that the distance to the main centre is still a major variable even in a polycentric city (see Péguy et al., 2000; Nelson and Sanchez, 1999, and chapter 2)

Assumption C3. The level of externalities at a given location is a function of the land use in the neighbouring cells, as well as of the distance between the location under consideration and its neighbours.

Therefore, both the number and the configuration of residents within a neighbourhood is important. The neighbourhood of a cell ij is a set of cells kl, and is denoted $\mathcal{N}_{ij}$. Each cell kl belonging to $\mathcal{N}_{ij}$ is characterized by an Euclidean distance x_{kl} separating ij and kl, called the focal distance. The extent of the neighbourhood is denoted by $\hat{x}$, i.e the maximum focal distance within a neighbourhood ($x_{kl} \leq \hat{x}$). A neighbourhood, $\mathcal{N}_{ij}$ contains n cells, and $n = n_{Hij}+n_{Aij}$ with n_{Hij} being the number of households and n_{Aij} the number of farmers in $\mathcal{N}_{ij}$. $\hat{x}$ and therefore n are exeogenous and homogeneous in space and time.

3.3.2 Long-run equilibrium

The allocation, C^t_{ij}, of a cell into a farmer or residential state, and the land rent, R^t_{ij}, are determined through the confrontation of bid rents. The addition to the model of location-specific externalities, that are considered by residents, leads to a multiple bid rent curve. Long-run equilibrium bid rent profiles are described in this section after the description of agent behaviour. Short-run dynamics are described in section 3.3.3. For the sake of clarity, the superscript t has been dropped in this section.

3.3.2.1 Farmers

The model focuses on residential behaviour. Therefore, the agricultural bid rent, Φ_{ij}, is given exogenously according to a linear Thünen type equation:

$$\Phi_{ij} = \Phi_{00} - bd_{ij} \tag{3.1}$$

In the first set of simulations, the agricultural bid rent is considered to be a constant ($\Phi_{ij} = \Phi_{00}$). Later, the slope b is changed exogenously.

There are arguments and evidence to suggest that the agricultural rent decreases with distance in periurban areas. The slope b could be interpreted as a unitary transport cost for a (Thünen type) farmer who sells his production directly at the CBD. As in a Thünen type framework, land located close to the central market generate positive rents due to savings in transportation costs. It could also reflect the expectation of a landowner in the context of urban growth. *'Uncertainty and growth help explain why agricultural land near the boundary of an urban area sells for a large premium over the value of agricultural land rents'. 'This option value decreases as distance from the boundary of the urban area increases'* (Capozza and Helsley, 1990, p.201, p.199).

Shi et al. (1997) show that in areas accessible to the urban centre, farmland values are affected upward by the size of the urban pole and the squared distance.

Plantinga et al. (2002) conducted an empirical test of the model of Capozza and Helsley, and showed that option values associated with delaying irreversible land development (as a function of population change variance) are significant and capitalized into farmland values. Cavailhes and Wavresky (2003) also shows that land prices decrease with distance in periurban areas. They further suggest that rural amenities can represent an important part of farmland price in the periurban belt.

There are also evidences to suggest that when agricultural land values are high, cities tend to be more compact (Brueckner and Fansler, 1983; Brueckner, 2000b). Whether changes in agricultural land value can affect periurban spatial structure is therefore an important issue. It is important therefore to keep parametric control on the agricultural rent and undertake sensitivity analysis.

3.3.2.2 Households

The residential bit rent, Ψ_{ij}, of households (all identical in tastes and income[5]) is given by the solution of the maximization of a utility function under a budget constraint. Residential utility, U, is expressed as a function of a non spatial composite good consumption, Z, a land consumption, and externalities. The local neighbourhood at a given location is a source of positive and negative externalities. The externalities are divided into two groups: *Environmental Externalities*, E_{ij}, representing preferences for a scenic landscape, proximity to crops and pastures, greenness, low density... and *Social Externalities*, S_{ij}, representing preferences for social services and contacts, schools, public transport, network services,... Both types of amenities are integrated in a Cobb-Douglas function:

$$MaxU(Z_{ij}, E_{ij}, S_{ij}) = Z_{ij} E_{ij}^{\beta} S_{ij}^{\gamma} \tag{3.2}$$

Land consumption is normalized to 1. β and γ represent the elasticity of U with respect to E_{ij} and S_{ij}. β and γ are positive, although limit cases are also presented in the discussion. In most simulations, γ was set to 1, so that β changes can be interpreted in relative terms.

Let L_{ij} be defined as the total amount of *Local Externalities* gleaned in the neighbourhood of ij:

$$L_{ij} = E_{ij}^{\beta} S_{ij}^{\gamma} \tag{3.3}$$

[5]This willbe partly relaxed in chapter 5.

Each household commutes to the CBD for work and shopping. A household earns a fixed income, Y, a part of which is allocated to commuting costs, $T(d_{ij}) = ad_{ij}$, with a the cost per unit of distance and d_{ij} the distance to the CBD. With their budget net of transport cost, households purchase a composite good (chosen as the numéraire) and a unit of land. Z_{ij} is the level of consumption and R_{ij} the rent per unit of land at ij. The budget constraint is therefore

$$Y = ad_{ij} + Z_{ij} + R_{ij} \tag{3.4}$$

and the indirect utility function (from Eq. 3.2,3.3, and 3.4)

$$V_{ij} = (Y - ad_{ij} - R_{ij})L_{ij} \tag{3.5}$$

At long-run equilibrium, all residents enjoy the same utility level, $\bar{u}$, corresponding to the utility of the rest of the world. This *Open City* assumption is further discussed in section 3.3.3. The bid rent function is the maximum rent that a resident is ready to pay for a location in order to reach the equilibrium utility level, $\bar{u}$. It is therefore

$$\Psi_{ij} = Y - ad_{ij} - \bar{u}L_{ij}^{-1} \tag{3.6}$$

3.3.2.3 Neighbourhood externalities

The neighbourhood externalities, E_{ij} and S_{ij}, are assumed to be a function of the density, ρ_{ij}, of households in the neighbourhood, $\mathcal{N}_{ij}$. More precisely, ρ_{ij} is a local interaction potential because it accounts for a distance decay effect: a weight w_{kl} is given to each cell in $\mathcal{N}_{ij}$, depending on the focal distance x_{kl} and the extent of the neighbourhood, $\hat{x}$. The following decreasing function is chosen so that $1 \geq w_{kl} > 0$:

$$w_{kl} = 1 - \left(\frac{x_{kl} - 1}{\hat{x}}\right)^{\sigma} \tag{3.7}$$

σ is positive, and the distance decay is convex with $\sigma \in]0, 1[$, linear with $\sigma = 1$, and there is no distance decay effect with $\sigma = \infty$ ($w_{kl} = 1\ \forall kl$, the potential is then properly said to be a density).

The local potential of interactions, or the local weighted residential density is

$$\rho_{ij} = \frac{\sum_{kl \in \mathcal{N}_{ij}} w_{kl} H_{kl}}{\sum_{kl \in \mathcal{N}_{ij}} w_{kl}} \tag{3.8}$$

where $H_{kl} = 1$ if $C_{kl} = H$, and 0 otherwise. And $\rho_{ij} \in [0, 1]$.

E_{ij} and S_{ij} can now be defined as functions of ρ_{ij}. $E(\rho_{ij})$, the neighbourhood environmental externality, is assumed to decrease with increasing residential density. This process represents losses of greenness and closure of the landscape caused by the presence of additional residents in the neighbourhood. E is therefore a local dispersion force. Conversely, $S(\rho_{ij})$, the neighbourhood social externalities, increases with increasing residential density. S is therefore a local agglomeration force. Additional residents in the neighbourhood are assumed to provide more personal contacts as well as additional commodities and services.

Both externalities are given an exponential form:

$$E_{ij} = e^{-(\rho_{ij})^{\theta}} \qquad S_{ij} = e^{(\rho_{ij})^{\phi}} \tag{3.9}$$

where both θ and ϕ are positive values. Then, Eq. 3.3 becomes

$$L_{ij} = E_{ij}^{\beta} S_{ij}^{\gamma} = e^{\left(\gamma \rho_{ij}^{\phi} - \beta \rho_{ij}^{\theta}\right)} \tag{3.10}$$

It is further assumed that each marginal gain in local density has less impact on the volume of externalities perceived by a household. $E(\rho)$ should therefore be a strictly convex function of the neighbourhood density, while $S(\rho)$ should be concave. Given that ρ_{ij} ranges from 0 to 1, this leads to the following restrictions on the parameters: $\theta \in]0, 1], \phi \in]0, 0.5]$ (details in appendix). These conditions can be considered without loss of generality, whatever the value of β and γ, as utility is ordinal, and any monotonic transformation of a given utility function represents the same preferences.

Given these functional forms, different configurations of the total neighbourhood amenity, L_{ρ}, are found when varying the liking of households for a 'green neighbourhood' (β) or for neighbourhood social amenities (γ). The examples presented in Fig.3.1[6] were used in the simulations. In the case of the upper curve ($\beta = 0$, $\gamma = 1$), households are not sensitive to green externalities. The denser the neighbourhood, the greater the amenity. For the bottom curve, ($\beta = 1$, $\gamma = 0$), households prefer null density neighbourhoods. In the three intermediate curves, households trade-off the two externalities. When density is low, the total externality increases with density because of social needs, but this

[6]See the enclosed CD for an animated version of the figure.

becomes less because of the concave shape of $S(\rho)$ and because of the decrease in greenness ($E(\rho)$). A maximum externality is achieved at a certain level of density. Further increasing the density leads to more social interactions, but the decrease in greenness becomes increasingly important and decreases the total benefit.

In the appendix we show that the existence of such a maximum for $L(\rho)$ (Eq. 3.10) is conditioned to $\theta > \phi$. Denote ρ^* the optimal local density:

$$\rho^* = \left(\frac{\phi\gamma}{\theta\beta}\right)^{1/(\theta-\phi)} \tag{3.11}$$

$\theta\beta \geq \phi\gamma$ is needed as an additional condition to have $\rho^* \in [0, 1]$ (see appendix).

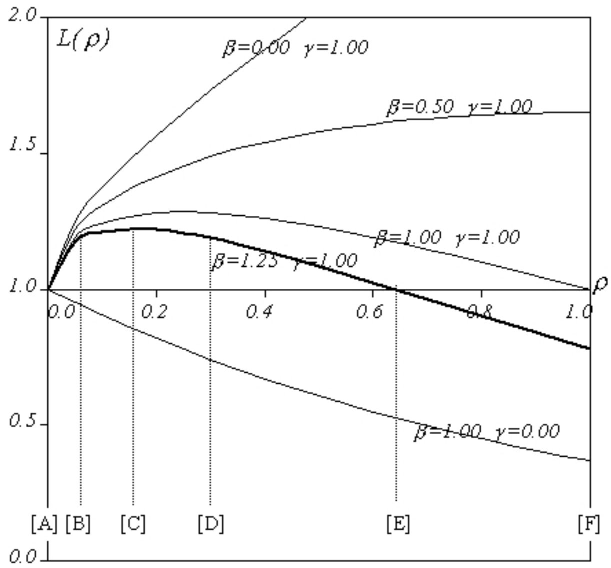
With a low preference for greenness (low β), the optimal neighbourhood density is high. In this case, even within a completely urbanised environment ($\rho = 1$) households have neighbourhood benefits. With more preference for open environments (higher β), the optimal neighbourhood density is achieved more rapidly and the decrease in total externality comes earlier. However, negative externalities ($L(\rho) < 1$) can only be generated when $\beta > \gamma$, i.e. when the taste for greenness is high, and the neighbourhood densely occupied. When $\beta = \gamma$, households are indifferent between a fully urbanised neighbourhood ($\rho = 1$) and a completely empty neighbourhood ($\rho = 0$), but prefer intermediate densities. Finally, decreasing θ accentuates the convexity of the open space externality and gives more importance to very low densities. Similarly, decreasing ϕ accentuates the concavity of the social amenity and also gives more importance to very low densities. In both cases, the optimal local density would be lower[7].

3.3.2.4 Emergence of a periurban belt

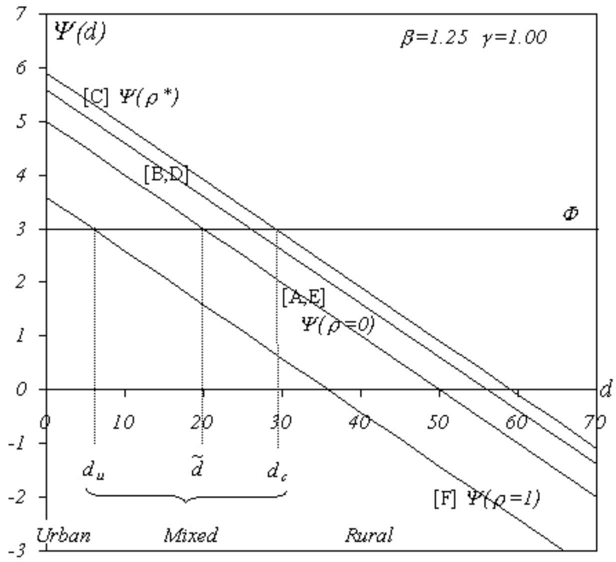
Multiple residential bid rents. The residential bid rent can now be completely formulated as a function of the weighted neighbourhood density, ρ_{ij}: (from Eq. 3.6 and 3.9)

$$\Psi_{ij} = Y - ad_{ij} - \bar{u}e^{\left(\beta\rho_{ij}^{\theta} - \gamma\rho_{ij}^{\phi}\right)} \tag{3.12}$$

[7]No sensitivity to θ and ϕ is undertaken within the simulations presented here, the parameters are considered as given parameters of households perception. The limit values $\theta = 1$ and $\phi = 0.5$ have been used to keep ρ^* sufficiently far from 0 and so better differentiate the cases as the possible ρ values are discrete. A larger magnitude of L would be obtained with lower ϕ, but this would imply lower ρ^*. Also, the magnitude of L is already affected by β and γ. In the appendix, the reader will find a table with sensitivity of the optimal density and externalities to changes in θ and ϕ. Moreover, additional graphical examples showing the effect of changing β and γ on ρ^* and on the externalities are provided.



(a) $L(\rho)$ with $\theta = 1.0$ and $\phi = 0.5$ for five different sets of preferences.



(b) $\Psi(d, \rho)$ with $\theta = 1.0$ and $\phi = 0.5$, and $\beta = 1.25$ and $\gamma = 1.00$.

Figure 3.1: Total externality and neighbourhood density for different preferences. Multiple residential rent curve and emergence of a mixed periurban zone at long-run equilibrium.

This is a location specific bid that varies with distance and locally according to the characteristics of the neighbourhood. The model produces, therefore, multiple residential bid rent curves, each corresponding to a possible neighbourhood density. In Fig.3.1b, six of these curves have been plotted for the case where the preference for open space amenities is slightly larger than the taste for social amenities ($\beta = 1.25 > \gamma = 1.00$). It was shown before that in this case disamenities are possible.

From point A to F (Fig.3.1a), the neighbourhood density increases. For an individual located at a location ij, and provided that $\rho_{ij} < \rho^*$ (i.e, from point A to C), a marginal increase in neighbourhood density benefits the agent and increases his residential bid. The highest residential bid rent curve corresponds with a location ij having optimal neighbourhood conditions (C in Fig.3.1a). From point C to E, a marginal increase in neighbourhood density corresponds to a loss of amenities. Bid rents are again lower than for the optimal condition. At E, the bid rent superimposes the bid rent with null density (which is also the bid rent for the standard model without externality). Beyond E, given that households have important preferences for rural amenities, the neighbourhood is too urbanised and this leads to disamenities ($L(\rho) < 1$ in Fig.3.1a from point E to F). The bid rent curve is at its lowest when the neighbourhood is fully urbanised (at point F, with $\rho_{ij} = 1$). A household would only accept such a location if it were closer to the city in order to reduce commuting costs.

Moreover, when the preference for open spaces is low with respect to social amenities ($\beta < \gamma$), the lowest bid rent curve corresponds to a situation where no social interactions can be found, i.e. an empty neighbourhood ($\rho = 0$). If $\beta = \gamma$, the lowest bid rent corresponds to an empty neighbourhood or to a fully occupied neighbourhood ($\rho = 0$, $\rho = 1$). Also, the more a household values neighbourhood amenities in general (increasing both β and γ), the larger the difference between the level of utility for optimal neighbourhood conditions and the worst neighbourhood conditions. The bundle of residential bid rent curves is also wider.

Market equilibrium. The equilibrium land rent R_{ij} is defined by the market by soluting the maximum value for Eq. 3.1 and 3.12 at each location:

$$R_{ij} = Max(\Psi_{ij}, \Phi_{ij}) \tag{3.13}$$

The capacity of a resident to outbid a farmer depends on the commuting cost and on the quality of the neighbourhood. The equilibrium configuration can no longer be thought of as a specialized residential city contiguous to a specialized agricultural area. The classic fringe between urban and rural areas when there is no externality is denoted $\tilde{d}$, which is the solution to $\Psi(L = 1) = \Psi(\rho = 0) = \Phi$.

When households benefit from positive externalities ($L > 1$), their bid rent increases and the specialized agricultural area moves further away from the CBD. The distance to which the specialized agricultural area is pushed away is the maximum distance where a resident can outbid a farmer. The crossing point of the agricultural bid rent and the residential bid rent for an optimal density is denoted by d_c, which is the longest distance commuted by a household at long-run equilibrium, and is the solution to $\Psi(\rho^*) = \Phi$. From Eq. 3.1, 3.11 and 3.12:

$$d_c = \tilde{d} + \frac{\bar{u}(L(\rho^*) - 1)}{(a-b)L(\rho^*)} > \tilde{d} \tag{3.14}$$

with $a > b$, and the previous parametric conditions for a maximum in $L(\rho^*)$.

The higher the utility at optimal neighbourhood conditions ($L(\rho^*)$), the longer the maximum commuting distance. With increasing preference for local social interactions (increasing γ), ρ^* increases and residents prefer more dense neighbourhoods. In the meantime $L(\rho^*)$ also increases and the maximum commuting distance (d_c) is longer.

Likewise, when households have a strong preference for green neighbourhoods ($\beta > \gamma$) and when the local density is high, they can encounter negative externalities ($L < 1$). Their bid rent can therefore fall under the classic bid rent (curve E=A in Fig.3.1b), and a farmer can outbid the resident at a distance $d_{ij} < \tilde{d}$. The limit of the specialized urban area is closer to the CBD than in the standard model. Denote this limit d_u, which is the solution to $\Psi(\rho = 1) = \Phi$

$$d_u = \tilde{d} + \frac{\bar{u}(L(\rho = 1) - 1)}{(a-b)L(\rho = 1)} \tag{3.15}$$

When the preference for $\beta > \gamma$, then the specialized residential area is smaller than in the standard model ($d_u < \tilde{d}$). When $\beta = \gamma$, the specialized residential area is the same size ($d_u = \tilde{d}$), and, when the preference for social interactions increases further, $\beta < \gamma$, the specialized residential area is larger ($d_u > \tilde{d}$).

An important, but rather counter-intuitive, property of the model results therefore from this discussion:

Result 3.1. *The preference for local social interactions is a local centripetal force (ρ^* increases), but also a regional centrifugal force that pushes the specialized residential and agricultural areas away from the CBD (d_u and d_c increases). The preference for local open space amenities is a local centrifugal force (ρ^* decreases), but also a regional centripetal force, that brings the specialized residential and agricultural areas closer to the CBD (d_u and d_c decreases).*

In the area that ranges from d_u to d_c, it is not known whether residents outbid farmers because this depends on the location-specific neighbourhood density. Farmers and households can therefore coexist in this area. Let $p = d_c - d_u > 0$ denote the width of this theoretical periurban belt that varies with β and γ, as does its location from the CBD. The periurban belt includes the classic fringe $\tilde{d}$ when the preference for open space is relatively high ($\beta \geq \gamma$). The belt starts from beyond $\tilde{d}$ when $\beta < \gamma$. In this case, a household benefits from any increase in local density up to $\rho = 1$, although the benefit decreases from ρ^*. This process leads to residential specialization in the area contained within $\tilde{d}$ and d_u.

While $p > 0$ is a necessary condition to obtain a mixed belt, it is not a sufficient condition when space is discrete. There are several ways to accommodate a density ranging from ρ^* to $\rho = 1$ within the cells of a neighbourhood. Residents can be grouped within a neighbourhood, leaving all of the rural cells on the same side of the neighbourhood. Moreover, in a discrete setting, the local density changes by given intervals (which are smaller with larger neighbourhoods). Thus, when the magnitude of the change in L between ρ^* and $\rho = 1$ is small and the neighbourhood is small, it is more difficult to obtain a mixed belt. Parameter values were chosen consequently in the simulation and, because of the discrete characteristic of space, a post-simulation definition of the mixed belt was also necessary. Nevertheless, the following general property of the model can be stated from this analytical description in continuous space, with continuous functions:

Result 3.2. *A mixed periurban belt, where residential and agricultural activities coexist emerges at the periphery of a city at long-run equilibrium, as a result of a trade-off between open space and social neighbourhood externalities and commuting costs.*

3.3.3 Short-run dynamics

The model is intended to simulate rural land conversion from the confrontation of agricultural and residential bid rents, when households arrive from elsewhere (i.e. the rest of the world). A long-run equilibrium is achieved from successive short-run equilibria and from the progressive adjustment of the regional utility to the world utility, $\bar{u}$. The model is made dynamic by incremental population growth (at discrete time steps) as long as the regional utility is higher than the world utility. Newcomers can settle in any available cell. Therefore, in order to obtain a short-run equilibrium, the utility of all households equals the utility of the last migrant wherever his location (i.e. not necessarily at the fringe). The assumptions related to these short-run dynamics follow.

Assumption D1. The model considers an *Open City* framework where utility adjusts progressively to external utility. At any given time, the level of utility, u^t, within the city is the same for any resident. New households are attracted to the city as long as they can achieve a level of utility that is higher than in the rest of the world. Let this utility surplus be denoted by $\Delta u^t = u^t - \bar{u} > 0$. The framework implies that moving has no cost for new migrants, who are also perfectly informed of the wages and prices in the city. The economy is considered to be in a long-run stationary state at t^* as soon as the level of utility within the city equals the level of utility external to the city $\bar{u}$ ($\Delta u^{t^*} = 0$). Two types of equilibria are therefore defined: a short-run equilibrium that is characterized by no residential migration within the region, and a long-run equilibrium where there is no residential migration between the region and the rest of the world.

Assumption D2. The model starts from an agricultural plain ($H_G^{t=0} = 0$), and the growth of the city is given exogenously. The idea of an area which receives a certain number of migrants as time goes by, is well suited to the reality of most functional regions in Europe, where the number of households is increasing. g is the exogenous growth rate, or the number of agricultural cells that are converted into residential use at each time step. The growth rate is positive and constant through time until the long-run equilibrium is achieved at t^*. From this moment, neither the number of residential cells nor the land values change.

Assumption D3. New migrants make their location choice only from the observation of the previous land use. Migrants do not anticipate future population growth and changes in their neighbourhood. At time t, they settle in an unoccupied cell where they will achieve the highest utility level, given the land use structure at $t-1$. This leads to path-dependency in the dynamics of the city, the structure of the model being *time-space recursive* (Anselin, 2001). The time lagged (first order) variable is the local density, ρ_{ij} (d_{ij} and Φ_{ij} do not change through time). The level of externalities that the new migrant expects at time t, is $L(\rho_{ij}^{t-1})$. The residential bid rent (Eq. 3.6 or 3.12) can be re-formulated in these spatio-temporal terms:

$$\Psi_{ij}^t = Y - ad_{ij} - \bar{u}(L_{ij}^t)^{-1} \tag{3.16}$$

with $L_{ij}^t = L(\rho_{ij}^{t-1})$, and according to (3.8)

$$\rho_{ij}^{t-1} = \frac{\sum_{kl \in \mathcal{N}_{ij}} w_{kl} H_{kl}^{t-1}}{\sum_{kl \in \mathcal{N}_{ij}} w_{kl}} \tag{3.17}$$

where $H_{kl}^{t-1} = 1$ if $C_{kl}^{t-1} = H$, and 0 otherwise.

Assumption D4. Asynchrony of location decisions. The residential dynamics is completely asynchronous: only one new resident seeks to settle in the region at each step. Without loss of generality, the time unit is therefore chosen so that $g = 1$. Making time discrete in this manner does not necessitate the assumption of coordination between agents. Consequently, as well, the time to reach the long-run equilibrium equals the residential population at the long-run equilibrium ($t^* = H_G^{t*}$). The issue of non-coordination has already been developed in inter-regional migration models by Weidlich and Haag (1983, p.12). The issue of simultaneous development of several locations has been shown to have an important impact on the patterns of sprawl in irreversible housing models with perfect foresight (Brueckner, 2000a).

Assumption D5. Residential utility incentive and the competition between myopic maximizing landowners. As long as $\Delta u^t > 0$, a household migrates to the city at time t and locates at cell ij in order to maximize its utility, denoted u^t. The (long-run) bid-rent of the household at that moment is $\Psi_{ij}^t(\bar{u})$, which is the maximum land rent the household is willing to pay for a cell ij in order to obtain the utility of the rest of the world. $\Psi_{ij}^t(\bar{u})$ is given by Eq. 3.16. However, the new migrant will actually pay a lower rent than $\Psi_{ij}^t(\bar{u})$ and, therefore, will pocket a utility surplus, $\Delta u^t = u^t - \bar{u}$, which is maximized and represents the incentive of migrating to the city.

More precisely, the land rent paid by the new migrant always equals the agricultural rent Φ_{ij}. Indeed, in a lattice grid, two agricultural cells often provide the same maximum utility level u^t to a household. If not, the difference between the best and the second best cells can at least be overlooked. The arrival of a new resident leads therefore several landowners to be in perfect competition, within an economy where there is an excess supply for a single new demand. The cells that are not chosen by the new resident will only provide Φ to the landowners. Thus these landowners will propose to the new migrant that they rent their own cell ij as long as

$$\Psi_{ij}^t(\bar{u}) > \Psi_{ij}(u^t) = R_{ij}^t = \Phi_{ij} + \epsilon > \Phi_{ij} \tag{3.18}$$

with ϵ strictly positive and converging to 0. At the time of making a location decision, therefore, the new resident will pay $R_{ij}^t = \Phi_{ij}$, which is the agricultural rent. The utility is then derived from the indirect utility function (3.5):

$$u^t = (Y - ad_{ij} - \Phi_{ij})\, L_{ij}^t \tag{3.19}$$

This process further assumes that the landowners are myopic (relaxing this assumption will be considered later) and do not consider the impact of the new locations on neighbouring rent values. They maximize the actual gain in each cell taken separately.

Assumption D6. The production of housing in the city follows *putty-clay* technology[8] **and landowners adjust the residential rents immediately.** Urban growth occurs only at the expense of agricultural areas. Housing is a durable good, i.e. the conversion of agricultural parcels into residential use is irreversible and every building is occupied. As a result of further population growth, changes may occur in the neighbourhood of a resident after his own location decision has been made. These changes impact on the local density level and, thus, on the level of externalities perceived by the household. The difference in local externalities needs therefore to be compensated with a change in Z consumption and in the land rent, in order for the household to achieve the same utility as all of the other residents at the same moment, u^t (assumption D1, *Open City*). Landowners are assumed therefore to adapt immediately (by increasing or decreasing) the rent of the residents who have settled previously with respect to the utility level of the last migrant but also in relation to the neighbourhood changes. When the neighbourhood is degraded, it is even possible for a household to rent a cell at a lower level than the agricultural bid rent ($\Psi_{ij}^t < \Phi_{ij}$) without inducing a return to agriculture. If the rent was not adapted, the resident would move and the unoccupied building would result in a zero rent for the landowner. Because of the rent adjustment, therefore, the propensity to change location is null[9]. There is no internal migration[10].

The bid-rent of the residents who have settled before time t in cells $i'j' \neq ij$ is similar to Eq. 3.16:

$$\Psi_{i'j'}(u^t) = Y - ad_{i'j'} - u^t(L_{i'j'})^{-1} \tag{3.20}$$

Finally, the land rent at time t for the set of cells in agricultural use at $t-1$ is

[8]The irreversibility of housing choice is analogous to the choice of a certain production factor by a firm through time. In *putty-clay* models, substitution of inputs is possible *ex-ante* but not *ex-post*. Contrarily, in *putty-putty* models, factors are always substitutable, while they are never in *clay-clay* models.

[9]All residential cells provide the same level of utility. Therefore, there is no reason to change location as soon as an infinitesimal but positive internal moving cost is assumed. Then, a fixed interregional cost of migration can also be assumed, which would not change the reasoning, but the long-run equilibrium utility would be different between regions.

[10]Except if the neighbourhood becomes so degraded that the bid-rent is negative. The housing cell would then be unoccupied. Parameters are chosen therefore so that $\Psi_{ij}^t > 0 \ \forall ij$.

$$R_{ij}^t = \Phi_{ij} = \Psi_{ij}^t(u^t) \;\; with \; C_{ij}^{t-1} = A \tag{3.21}$$

and the land rent at time t for the set of cells in residential use at $t-1$ is

$$R_{i'j'}^t = \Psi_{i'j'}(u^t) \;\; with \; C_{i'j'}^{t-1} = H \tag{3.22}$$

In practice, by assuming $g = 1$, land conversion proceeds by the following logical expression that summarizes the three necessary conditions assumed for a cell ij to be developed at time t: (i) conversion occurs only at the expense of a rural cell, (ii) the utility of a household at ij is higher than in the rest of the world (which is an incentive to migrate into the region), and (iii) from all the available cells, ij provides the highest level of utility.

if $C_{ij}^{t-1} = A$
and $u_{ij}^t = (Y - ad_{ij} - \Phi_{ij})\, L_{ij}^t > \bar{u}$
and $u_{ij}^t > u_{kl}^t \; \forall kl \neq ij, \; C_{kl}^{t-1} = A$
then $C_{ij}^t = H$
else $C_{ij}^t = C_{ij}^{t-1}$

One can note that in a quadrant city, two locations at the same distance from the CBD and with the same neighbourhood density, can provide the highest utility level. Because $g = 1$ is chosen (assumption D4), only one can be converted into urban use. To resolve this problem, a tie-breaker is applied arbitrarily using a random number. Whilst this random element makes small changes to individual locations, it does not affect the spatial patterns. The resulting structures remain the same from one simulation run to another, but a change in their orientation or a radial shift may occur. Therefore, despite path-dependency, it is not necessary to conduct multiple run (Monte-Carlo type) analysis in this case. Overall patterns are deterministic but cell states are not because of the random tie-breaker.

In addition, one can note the effect of choosing an asynchronous setting ($g = 1$, assumption D4). If $g > 1$ is chosen, the g best cells would be chosen, and tie-breaks within this set of g cells would not need to be solved randomly. However, as no coordination is assumed between our myopic households, new migrants could have unexpected neighbourhood characteristics when they make their location choice. Therefore, they might not have the utility surplus in t which they expected from their migration choice. The level of utility at t would be fixed by the worst location chosen by one of the g households. Indeed, if $g = \infty$ is chosen, there is no constraint on the number of households that can enter the city-region. Thus in $t = 1$, the spatial pattern would be exactly the pattern obtained from the standard urban economic model, with the level of utility fixed at the urban-rural fringe. As there is no difference between cells in the initial configuration $t = 0$, this pattern would be obtain whatever the preference for externalities. However, the long-run equilibrium would not necessarily be reached. Depending on their preference for externalities, migrants in $t = 2$ and subsequent steps could find interesting locations at the fringe of the standard city pattern, where they can bid over the agricultural rent.

In brief, if $g > 1$ is chosen, the greater is g, the closer the pattern would be from the standard model in the first step, as these g first migrants minimize transport cost and thus organise themselves in a compact manner. Equilibrium residential developments would be more compact, with less leapfrogged cells (if these are going to appear because of the greenness preference). The greater is g, the more rapid is also the decrease in utility. However, because the absence of coordination between migrants is probably a more reasonable assumption than coordination, it is preferred to use $g = 1$ throughout this work. This assumption implies that if discontinuous patterns are going to occur, these discontinuities will correspond directly to the wishes of migrants. Furthermore, this assumption implies a greater dependence on the random tie-breaker, but we know that it does not affect the overall urban structure.

3.4 Simulations

At $t = 0$, there are 2500 agricultural cells. At $t = 1$, the first household locates where its utility is maximum, i.e. the cell in contact with the CBD (no commuting and null local density). At period $t = 2$ and in subsequent steps, the change in local densities, due to previous location decisions, results in a cellular space that is differentiated in terms of externalities. Therefore, the new residential locations may not follow the sequence of minimizing transport costs as new residents prefer some of the observed local densities to others. Varying the respective weight of the two local preferences leads to different spatial configurations both in short-run and long-run equilibrium.

The five pairs of β, γ parameters in Fig.3.1 are used in the following discussion, and compared to the standard model with no externalities. Three different neighbourhood sizes are considered: $\hat{x} = 1.42; 3.00; 5.00$, i.e. respectively neighbourhood of 8, 28 and 80 cells[11].

[11] These neighbourhood are represented in appendix. Also, for this set of simulations, no distance decay effect has been implemented within the neighbourhoods: $\sigma = \infty$. The aim is to provide a typology of patterns. This typology does not change qualitatively with σ. However, the impact of changing σ certainly merits a deeper analysis because the spatial arrangement of residents within the neighbourhood is then taken into account in the formation the externalities. The CA is thus no longer of the 'totalistic' type. An example of changing σ is provided in appendix, where it is shown how the parameter changes the relationship between the externalities and the density of residents within the neighbourhood.

In this section, first, a typology of the configurations at stationary state is presented. Spatial characteristics of these configurations are discussed along with the limits of specialized areas and indices of fragmentation and density. Secondly, a simulation is used to show the evolution of land use patterns and rent profiles through time. The effect of relaxing the landowner myopia assumption is also discussed. The third part consists in describing city aggregates and a comparative statics analysis.

3.4.1 Archetyp cities

Long-run spatial configurations can be categorized according to their morphological characteristics. The limits of the specialized and mixed area were measured at t^*. The upper limit of the mixed periurban area, denoted by D_c, is defined as the longest distance that a resident travels in t^*. Beyond D_c, all cells are agricultural. The shortest distance at which an agricultural cell is found at t^* is denoted D_u. From the CBD to D_u, the area is specialized with residents. $P = D_c - D_u$ is the extent of the mixed periurban belt. These limits are reported in Table 3.1 and were used to classify the resulting configurations. When the relative importance given to local open space amenities increases compared to local social amenities (β relative to γ), a 'Compact City' occurs (tiles a and b in Fig.3.2), then a 'Periurban Belt City' (tiles c and d), and finally a 'Dispersed City' (tile e).

A Compact City shows a clear break between urban and rural cells, as in the standard residential economic model. A Compact City always occurs if no greenness externalities are included in the model (tiles a are cases where $\beta = 0$) and when the preference for social amenities is so high that the optimal density ρ^* is beyond 1 (the condition $\theta\beta > \phi\gamma$ is not respected). In the standard city, however, the maximum commuting distance (D_c) coincides with the minimum agricultural distance (D_u), and the size of the mixed zone is null ($P = 0$). This is not exactly true for a Compact City where households have a strong interest in neighbourhood social externalities, as P is small, but positive (tiles a and b, and Table 3.1). However, although $D_u < D_c$ there is no mix and the perimeter of the city is flattened. Compared to a circular case, this 'octagonal' configuration allows households at the fringe to benefit from a denser neighbourhood (closer to 0.5 like in a 1D continuous setting) and more social interactions. With a concave limit, there are more contacts with agricultural activities, which households disregard when β is small. The flattening of the pattern is less important when households have a larger spatial horizon. Within a neighbourhood that includes more cells, it is less critical for a single resident to settle along a line rather than a curve.

The flattening of the city is also weaker when households, in addition to social contacts, value open spaces (β increases). The second row in Fig.3.2 is a Compact City, that is less flattened as, along the perimeter, households benefits from greenness. With respect to Result 3.1., the role of social amenities is noticeable in tiles a and b in comparison with the standard model (tile f). These Compact Cities are more extended and populated. The increase in commuting cost is compensated for by the consumption of the contact externality.

The second row in Fig.3.2 is also a theoretical limit case where the analytical periurban extent is null ($p = 0$, as $\theta\beta = \phi\gamma$ and $\rho^* = 1$). However, this is not a simulation bifurcation point between two qualitatively different morphologies. In fact, from this point onwards, a marginal increase in the preference for greenness does not transform the 'Compact City' into a 'Periurban Belt City', although it is expected from the long-run equilibrium analysis.

As mentioned earlier, this is due to the arrangement of cells within the neighbourhood, which can be completely segregated without reaching $\rho = 1$ and because space and density intervals are discrete. A residential location along the perimeter of a circular city has less than half of its neighbourhood occupied with residents. *A fortiori* $\rho^* = 1$ cannot be a bifurcation point. When trying to approximate the bifurcation point, two agricultural cells were observed to indent the urban perimeter and two others were completely leapfrogged with $\beta = 0.785$ (i.e. $\rho^* = 0.406$), but none with $\beta = 0.780$. This approximation only holds for the same set of parameters including the unitary transport cost, with a 9 cell neighbourhood and no distance decay in the neighbourhood.

With a stronger preference for rural amenities (in this case $\beta \geq 0.785$), the Compact City transforms into a Periurban Belt City. The configurations are characterized by the insertion of a mixed belt within the urban and rural specialized areas (Fig.3.2c and d). In the neutral preference case ($\beta = \gamma$), the extent of the compact urban core coincides with the standard urban rural fringe (Table 3.1). When the preference for green amenities increases further, the size of the periurban belt increases because of (i) a reduction in the specialized urban core ($D_u < \tilde{d}$ as $L(\rho = 1) < L(\rho = 0)$), and (ii) a smaller decrease in the commuting limit D_c (the upper bid-rent, $\Psi(L(\rho^*))$ decreases with increasing β). Fig.3.2d exemplifies this case and is the numerical expression of the long-run equilibrium bid-rent profile drawn previously (Fig.3.1b).

When continuing to increase the preference for local greenness (β), households do not accept a fully occupied neighbourhood, even at close distances to the CBD ($\Psi(L(\rho_{ij} = 1) < \Phi_{ij} \ \forall ij)$), and the specialized residential area disappears ($D_u = 0$). This is the 'Dispersed City'. The spatial extent of this completely mixed city is determined by the level of externalities under optimal neighbourhood conditions. Therefore, when households do not show preference for local social contacts ($\gamma = 0$), any increase in local density is a disamenity and the extent of the mixed area coincides with the urban-rural fringe of the standard model ($\Psi(L(\rho = 0)$ is the upper bid rent). A Dispersed City occurs when the optimal neighbourhood density is null. Increasing the density automatically leads to negative externalities so that the farmer can bid over the resident at very short distances (lower than $\hat{x}$) from the CBD. This is always the case if residential preferences are only dedicated to environmental externalities ($\gamma = 0$ and $\beta > 0$). In this case, D_c equalizes the classic fringe $\tilde{d}$. A Dispersed City is not possible provided $\gamma > \beta$.

There are different morphologies of mixed uses in the Periurban Belt and Dispersed cities, including spotted leapfrogs of different width, or clustered and striped settlements. There is a sort of a continuum of periurban forms with changing preferences. Increasing the preference for greenness generates 'agricultural holes in the suburbs', then larger holes, then these larger holes open up towards the rural area and create large residential bands, then thinner stripes, then small clusters, and then spots. As mentioned earlier, however, qualitative bifurcation points are difficult to establish theoretically. Moreover one would need a precise definition of what is a stripe (a zebra type development), what is a cluster, etc... in order to measure these qualitatively different morphologies.

One can also notice (e.g. from tiles d in Fig.3.2) that the morphology of the periurban belt exhibits a transition from compactness to smaller spots, or thinner stripes with increasing distance to the CBD. However, this is made in a discrete setting and, therefore, the distance effect is not smoothed. Moreover, this distance effect is partly broken by the fact that externalities are taken within a finite neighbourhood. There is no reason for a household to leap beyond the size of its neighbourhood in order to find optimal density. Therefore, bands and spots tend to be separated by a constant distance although the CBD distance increases.

Finally, it is possible to compare these forms with observed land cover data (e.g. Fig.2.1). At that scale, the Netherlands resemble probably tiles a and b (Fig.3.2), while Belgium resemble more the spotted developments of tile e. However, at an other scale, Belgium is also characterised by ribbon like developments, which therefore, would be closer to tile d. Further comparing the theoretical results with observed urban patterns requires a serious pattern matching procedure and the use of well founded spatial pattern indices. This will be explored in chapter 6. For the moment, we retain that the model is able to generate a diversity of urban structures, ranging from scattered to compact settlements, and that a similar diversity is observed in Europe.

(a) Exclusive pref. for S: ($\beta = 0.00, \gamma = 1.00$). From left to right: $\hat{x} = 1.42$, $t^* = 1577$; $\hat{x} = 3.00$, $t^* = 1606$; $\hat{x} = 5.00$, $t^* = 1636$

(b) High pref. for S: ($\beta = 0.50, \gamma = 1.00$). From left to right: $\hat{x} = 1.42$, $t^* = 1169$; $\hat{x} = 3.00$, $t^* = 1181$; $\hat{x} = 5.00$, $t^* = 1189$

(c) Neutral case: ($\beta = \gamma = 1.00$). From left to right: $\hat{x} = 1.42$, $t^* = 649$; $\hat{x} = 3.00$, $t^* = 671$; $\hat{x} = 5.00$, $t^* = 684$

(d) High pref. for E: ($\beta = 1.25, \gamma = 1.00$). From left to right: $\hat{x} = 1.42$, $t^* = 418$; $\hat{x} = 3.00$, $t^* = 444$; $\hat{x} = 5.00$, $t^* = 453$

(e) Exclusive pref. for E: ($\beta = 1.00, \gamma = 0.00$). From left to right: $\hat{x} = 1.42$, $t^* = 93$; $\hat{x} = 3.00$, $t^* = 61$; $\hat{x} = 5.00$, $t^* = 61$

(f) Standard case, ($\beta = \gamma = 0.00$), $t^* = 339$

□ residential cell ■ agricultural cell

Figure 3.2: Long-run configurations for three neighbourhood sizes (increasing from left to right) and five sets of preferences (increasing the relative taste for local open space from top to bottom).

	β	γ	t^*	D_u	D_c	P	$\mathrm{P_{DENS}}$	$\mathrm{H_{FRAG}}$	TDR^{t^*} (/inh.)	TDR_{co}	TTC^{t*} (/inh.)	TTC_{co}	$\frac{\Delta TTC}{TTC_{co}}$ %	$\frac{\Delta TDR}{\Delta TTC}$
Standard case	0.00	0.00	339	20.10	20.10	0.00	0.00	0.0315	230 (0.68)	230	448 (1.32)	448	0.00	n.a.
Neighbourhood size $\hat{x} = 1.42$														
Excl. pref. for S	0.00	1.00	1577	43.14	45.31	2.17	0.46	0.0141	3495 (2.22)	3496	4614 (2.93)	4611	0.06	n.a.
High pref. for S	0.50	1.00	1169	37.48	38.33	0.84	0.51	0.0164	1701 (1.45)	1701	2933 (2.51)	2933	0.02	n.a.
Neutral case	1.00	1.00	649	20.25	31.11	10.86	0.68	0.1800	293 (0.45)	131	1244 (1.92)	1203	3.45	3.91
High pref. for E	1.25	1.00	418	6.71	29.00	22.29	0.58	0.4107	220 (0.53)	−319	737 (1.76)	616	19.51	4.48
Excl. pref. for E	1.00	0.00	93	1.00	20.02	19.02	0.28	0.9427	53 (0.57)	−599	115 (1.24)	62	87.20	12.14
Neighbourhood size $\hat{x} = 3.00$														
Excl. pref. for S	0.00	1.00	1606	44.00	45.22	1.22	0.49	0.0138	3494 (2.18)	3480	4741 (2.95)	4740	0.02	n.a.
High pref. for S	0.50	1.00	1181	38.00	38.33	0.33	0.52	0.0162	1700 (1.44)	1700	2978 (2.52)	2978	0.01	n.a.
Neutral case	1.00	1.00	671	20.22	31.11	10.89	0.73	0.1208	260 (0.39)	141	1299 (1.94)	1265	2.68	3.50
High pref. for E	1.25	1.00	444	6.71	29.07	22.36	0.62	0.2755	84 (0.19)	−317	771 (1.74)	676	14.09	4.22
Excl. pref. for E	1.00	0.00	61	1.00	19.65	18.65	0.19	0.7787	18 (0.30)	−323	68 (1.12)	32	113.14	9.43
Neighbourhood size $\hat{x} = 5.00$														
Excl. pref. for S	0.00	1.00	1636	45.00	45.12	0.12	0.25	0.0139	3480 (2.13)	3514	4874 (2.98)	4874	0.00	n.a.
High pref. for S	0.50	1.00	1189	38.18	38.33	0.14	0.43	0.0165	1697 (1.43)	1697	3009 (2.53)	3009	0.00	n.a.
Neutral case	1.00	1.00	684	21.40	30.89	9.49	0.76	0.0824	250 (0.37)	171	1331 (1.95)	1309	2.22	2.73
High pref. for E	1.25	1.00	453	9.43	28.84	19.41	0.62	0.1560	63 (0.14)	−267	787 (1.74)	697	12.94	3.66
Excl. pref. for E	1.00	0.00	61	2.24	19.10	16.87	0.18	0.4891	9 (0.15)	−255	65 (1.06)	32	102.01	8.08

Table 3.1: Long-run equilibrium for three neighbourhood sizes and five sets of preferences: Population (t^*), limits and extent of the mixed area (D_u, D_c, P), periurban density ($\mathrm{P_{DENS}}$), fragmentation ($\mathrm{H_{FRAG}}$), total differential rent (TDR^{t^*}), total differential rent of an equivalent circular city (TDR_{co}), total commuting cost (TTC^{t^*}), total commuting cost of an equivalent circular city (TTC_{co}), 'extra commuting', ($\frac{\Delta TTC}{TTC_{co}}$), ratio 'extra rent'-'extra commuting' ($\frac{\Delta TDR}{\Delta TTC}$)

Two indices were, however, computed to further measure the spatial patterns of the mixed structures and assess the impact of changing preferences. The periurban residential density, P_{DENS}, represents the ratio of the number of households to the number of cells within the mixed zone delimited by D_u and D_c. The level of urban fragmentation, H_{FRAG}, is the share of the perimeters of residential cells that is in contact with a farmer (adapted from edge density in Mc Garigal (2002)). A value of 1 indicates that every resident is completely isolated and the landscape is completely fragmented. A null value indicates that there is no adjacency between residents and farmers across the metropolitan area (i.e. 2500 residential cells).

Periurban density (P_{DENS}) decreases with an increasing preference for greenness, while fragmentation (H_{FRAG}) is increasing. However, for a given set of preferences, the morphology is determined by the level of spatial myopia of households. Varying neighbourhood size ($\hat{x}$) does not impact on the periurban density, but it does impact on the size of leapfrogs and the width of urban stripes. Fragmentation is lower for larger neighbourhood extents, the size of residential clusters and stripes being wider. When households are not sensitive to local agglomeration, but only to greenness and therefore separation from others, it is interesting to note that they still cluster when a large neighbourhood is considered (contrarily to Page (1999) the agglomeration here is due to the commuting cost).

One can conjecture that enlarging the neighbourhood to the whole region and implementing a distance decay function (see σ) would turn into an analysis that is closer to the one proposed by Page (1999), where cities emerge in different places within a lattice. This leads to polycentricity. However, the patterns here are also constrained by the commuting distance. If this commuting cost disappear, the first resident locates arbitrarily in space, then assuming that greenness is the only externality (as in tile e of Fig.3.2), the second resident locates as far as possible from the first resident, i.e. in one of the corners. The third resident then maximises distance to the other two, etc... . With a limited neighbourhood extent, the second household will locate beyond the neighbourhood of the first resident, but not necessarily in a corner of the lattice. Now, with commuting distance and local agglomeration force, the trade-off is more complex. Other simulations would be necessary to provide insights in the emergence of polycentric developments within this monocentric framework.

In summary of the descriptive analysis of long-run equilibria, Result 3.2. can be further developed:

Result 3.3. *With strong preferences for neighbourhood interactions between households, the classic circular shape of the city is flattened. When households* *increase the relative value attached to neighbourhood open space amenities, a mixed belt arises between the specialized residential core and the specialized agricultural area. When preferences are neutral ($\beta = \gamma$), the size of the specialized city is the same as for the standard urban model, and a mixed belt arises beyond this limit. When the preference for rural amenities is stronger, the periurban belt is closer to the CBD. When there is no preference for local social interactions, there is no compact core and the city is dispersed up to the standard fringe. Landscape fragmentation is reduced when households have a large spatial horizon.*

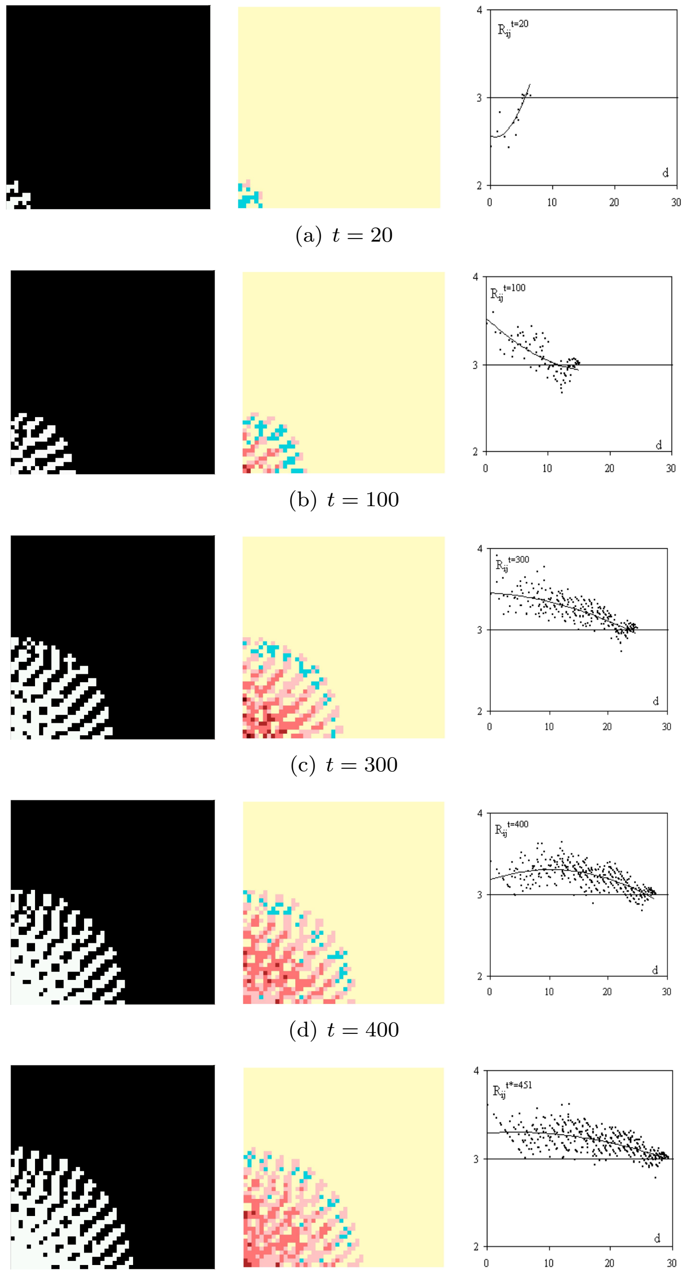
3.4.2 Trajectory of spatial equilibria and rents

Changing spatial configurations trough time are shown in Fig.3.3[12] for the case where negative externalities are possible (high preference for E: $\beta = 1.25, \gamma = 1.00$) with a 28 cell neighbourhood ($\hat{x} = 3.00$). The robustness of the model to the assumption of myopic landownership is addressed later in this section. Fig.3.3 reports changing maps of residential rent and changing distance profiles of rent. Table 3.2 reports parameters for the fit of rent against distance and square distance, that is shown on Fig.3.3.

At each period of time, one household seeks to locate in the area and converts an agricultural cell. As mentioned before, the path-dependent nature of the model implies that the area is not necessarily filled in by starting from the most accessible and ending with the least accessible place. Of course, because, there is no externalities in the initial configuration, the first household always locate close to the CBD where he can afford the highest utility surplus. It is different, however, for the next migrants. They look at the location of previous households in order to assess the neighbourhood quality of available agricultural cells. Trading-off this neighbourhood quality with commuting distance, these households make a choice in order to maximise their utility.

In this example, the second and third households choose a location that is contiguous to the first household, as if they were minimizing the CBD distance. The next ones, however, choose to leapfrog an agricultural cell in order to avoid high density neighbourhood and thus have a neighbourhood environment closer to ρ^*. As time progresses, the rural area is filled in with residents who choose to locate at the fringe, or to leapfrog an additional cell or even to locate closer to the CBD where, although they benefit from less externalities, compensate with commuting costs. At each time step, the agricultural cell that provides the highest utility level is urbanised. In this case, the process stops in $t^* = 451$. The city is therefore more populated and reaches equilibrium later than in the

[12] This simulation is also available as a video file in the enclosed CD.



(a) $t = 20$

(b) $t = 100$

(c) $t = 300$

(d) $t = 400$

(e) $t^* = 451$

Left: land use □ residential, ■ agricultural.
Middle: land rent ■ $R_{ij} < \Phi$, ■ $= \Phi = 3.00$, ■ < 3.25, ■ < 3.50, ■ < 3.75, ■ ≥ 3.75.
Right: land rent and distance to the CBD (see table 3.2 for parameters of fitted curve)

Figure 3.3: Short-run equilibria. Land use, land rents and rent profiles with $\beta = 1.25$, $\gamma = 1.00$, and $\hat{x} = 3.00$.

standard case because of the externalities. At $t^* + 1$, no household can outbid a farmer in an undeveloped cell, the pattern is fixed as long as no exogenous change occur.

From the morphological point of view, the fragmentation level is continuously decreasing as time goes on ($H_{FRAG} = 0.45$ ($t = 20$), $= 0.26$ ($t^* = 451$)), whereas the periurban density is continuously increasing ($P_{DENS} = 0.48$ ($t = 20$), $= 0.63$ ($t^* = 451$)). These values reflect the step by step generation of the compact part of the settlement, and the fact that in the meantime the mixed periurban zone is pushed outward. The cells that were leapfrogged in the first stages, are later urbanised, gradually leading to more compactness close to the CBD. This dynamic is also reflected by the evolution of the rent profile.

The very first resident locates in the bottom left corner and pays the agricultural rent. He does not benefit from any neighbourhood externality, but the transport cost is null. Therefore, his utility is $Y - \Phi_{00} = 7.00 > \bar{u}$[13]. The situation is different for the next migrants who can trade off transport cost with a better quality neighbourhood, according to the presence of previous residents. For instance, when the 20th resident settles down, his utility is $7.75 > Y - \Phi_{00} > \bar{u}$. The effect of this increase in utility and change in neighbourhood conditions, at the very start of the simulation, are shown by an important decrease in land rent values closer to the CBD (Fig.3.3a). The local environment of the first inhabitants has been degraded by the reduction of greenness due to the arrival of newcomers. Land rent then begins to rise gradually, following the increase in population and the downward adjustment in utility. For example, at $t = 100$, utility is $Y - \Phi_{00} > 6.00 > \bar{u}$, and the land rent profile decreases with distance in a more classical way.

As time goes on, the rent profile tends to become concave (compare $t = 300$ and $t = 400$ in Fig.3.3 and estimates for d^2 in Table 3.2). This is a consequence of filling in the locations that were leapfrogged in the first steps. The neighbouring cells incur a rise in local density and a fall in externalities. This process explains the second drop in land rent for cells located close to the CBD. More precisely, the sequence of rent values for a single cell can be described by four stages: (i) agricultural rent until conversion; (ii) fall in the rent because the cell is close to the fringe and encounters neighbourhood changes (this step can be preceded by an increase if the local density is not ρ^* at the moment of the location; (iii) continuous rent rise as the total population increases and utility decreases (because the last migrant has a high commuting cost), whereas there is an absence of change in the neighbourhood as newcomers settle further away; (iv) neighbouring leapfrogged cells (if they exist) are developed and rent decreases consecutively. The trajectory of a single cell can be stopped at any of these stages, when the long-run equilibrium is reached and population growth stopped.

[13]Parameter value for the simulations are: $Y = 10$, $\Phi_{00} = 3$, $a = 0.15$

	$t = 20$	$t = 100$	$t = 300$	$t = 400$	$t^* = 451$
Intercept	2.5648 °°°	3.5249 °°°	3.4546 °°°	3.1799 °°°	3.2882 °°°
	(0, 0885)	(0, 0586)	(0.0306)	(0.0261)	(0.0248)
Dist. d	−0.0297	−0.0657 °°°	−0.0044	0.0246 °°°	0.0047
	(0, 0618)	(0.0148)	(0.0047)	(0.0036)	(0.0033)
Sq.dist. d^2	0.0195 °	0.0017 °°	−0.0006 °°°	−0.0012 °°°	−0.0005 °°°
	(0, 0093)	(0.0008)	(0.0002)	(0.0001)	(0.0001)

Table 3.2: Parameter estimates of residential rent through time as a function of distance and square distance. (Standard errors are in parenthesis. ° denotes significance at the 10% level, °° at the 5% level and °°° at the 1% level or better). The fitted curve is presented on Fig.3.3.

Result 3.4. *Land values change through time as a result of both population growth and increasing neighbourhood density. Land rent rises with population and the consequent increase in commuting costs at the fringe. Land rent decreases or rises subsequently with urbanisation of the neighbourhoods. This affects the general decreasing shape of rent profiles beyond the compact urban core for all residential cells where* $\rho \neq 1$.

Moreover, as a result of the dynamic process, long-run equilibrium land rents within the mixed periurban area vary strongly from place to place. A given cell can have a high residential rent, while one of the contiguous cell (e.g. at similar distance from the CBD) is rented at the agricultural rent. This pattern is surprising for a long-run equilibrium. A landowner who rent to a farmer at t^* can have neighbours who rent at much higher level until $t = \infty$. However this landowner cannot change this situation as it has been constructed sequentially, and at t^* no households would agree to rent this cell at a higher level than Φ. The irreversibility of residential conversion and the myopia of landowners explain , therefore, the persistence of agricultural areas within the periurban belt. Furthermore, it shows that green open-spaces can be preserved at the periphery of the city without any *a priori* planning decision. Residents organize themselves according to their preferences. If they value green neighbourhoods, these will appear as long as the equilibrium is constructed sequentially.

Result 3.5. *When households value the greenness of their neighbourhood and landowners are myopic, the irreversibility of residential land conversion and the sequential immigration of residents can lead to mixed periurban belt at long-run equilibrium, even in the absence of a planned preservation of agricultural areas. As a result, land value in the mixed area may vary sharply with location.*

The importance of assumptions D5 and D6, stating the immobility of households and the myopia of landowners is also emphasized through another characteristic of the rent profiles. Up to the end state, there are groups of cells that remain below the level of the agricultural rent. These cells are outlined in the rent maps (Fig.3.3 central column). According to the stages mentioned above, these are cells at stage (ii), mainly located near the commuting fringe, but not at the commuting fringe. Some cells can also have a more central location. These cells are all located within the neighbourhood of the last newcomers. Although they were providing good neighbourhood conditions at the moment of their residential conversion, after the arrival of new residents, the level of externalities decreases as does the level of the rent. Only the fall in utility, accompanying population growth and increasing commuting, is able to counter-balance this decrease. However, the rent of the cells that are developed only a few steps before reaching the long-run equilibrium is not counter-balanced. This is how the rent can be lower than the agricultural bid rent at some locations from t^* to $t = \infty$. Clearly, no landowner would accept to rent the cells where $R_{ij} < \Phi_{ij}$ at t^*, except if they were myopic, i.e. the value they give to the present rent is infinitely higher than for any future steps.

In subsequent chapters, the problem is solved and residential land rent will not fall below the agricultural rent any more. In fact, it is assumed in the present chapter that housing consumption is a constant. In the next two chapters, this assumption is relaxed and thus, an household who settled in a given location, although its choice is irreversible, will be able to trade-off a loss in externality by an increase in housing consumption (as the cells are of fixed size, this would mean increase in housing quality or decreasing intra-cell density).

For the moment, however, in the fixed housing consumption framework, it is interesting to relax the myopic assumption and instead assume rational expectations from landowners. This means that cells cannot be developed where the differential rent $(\Psi_{ij} - \Phi)$ is negative at long-run equilibrium t^*. The model is then run until a new virtual long-run equilibrium is reached. This hypothetical city is less populated, but the problem is not solved as, despite a utility surplus, there is no further growth and some other cells have a negative differential rent. The reasoning can then be continued in two ways until no cells have negative differential rent at t^*: either these other cells are constrained and the constraint is maintained from previous time steps, or the constraint is set only on these new cells, while the previous landowners decide to remove their constraint. In the first case, i.e. when building constraints are accumulated, the city collapses step by step: a single resident remains at the bottom left corner and no other landowner wants to rent to residential use. In the second case, the iterated stationary states show a reorganization of residents at the periphery for the first five iterations. Then, as the reorganization uses more and more central cells, the process enters into a cyclical stability of period 2[14].

[14]Some cells apart, the process loops from a configuration close to t^*, with 420 residents,

Concepts concerning coordination from *Game Theory* are useful in improving the cellular model presented here[15]. For t^* configurations to be effective long-run equilibria under a non myopic assumption, some cooperation must be assumed between the landowners. For instance, the negative part of the differential rent must be shared between all landowners. Without cooperation, the city is doomed to extinction (except for a single household), or no static equilibrium can be identified as there is an endless game between landowners (no Nash equilibrium). The non-robustness of the model to the anticipation hypothesis is an important limitation. Only when it is certain that the location of a new resident in the neighbourhood of ij will not cause a decrease in the rent at ij, is the hypothesis robust, i.e. when $\rho^* \geq 1$ $(\theta\beta \leq \phi\gamma)$

Result 3.6. *Land rents can be lower than the agricultural rent in some residential locations at long-run equilibrium because of sequential conversion decisions and the locked-in spatial distribution caused by the immobility of residents. Landowners are therefore myopic or coordinate in order to distribute amongst themselves the negative part of the total differential rent. Alternatively, households are able to improve their housing consumption.*

3.4.3 City aggregates and static comparative analysis

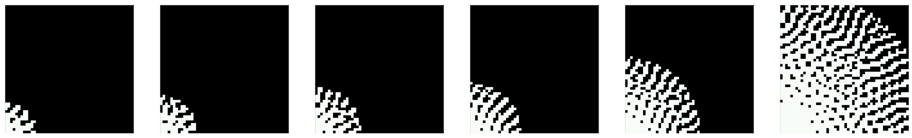
The global wealth and costs of the different configurations obtained in Fig.3.2 are analysed in this section. Results are shown in Table 3.1. A static comparative analysis is also undertaken for the case detailed before in the temporal analysis, where the liking for greenness amenities is high ($\beta = 1.25, \gamma = 1.00$, and $\hat{x} = 3.00$). Parameter values were gradually changed for the unit commuting cost (a), income level (Y) and the slope of the agricultural rent (b). Long-run equilibria resulting from the sensitivity analysis are shown in Fig.3.4, and morphological measures and economic aggregates are reported in Table 3.3.

3.4.3.1 Total differential rent and commuting cost.

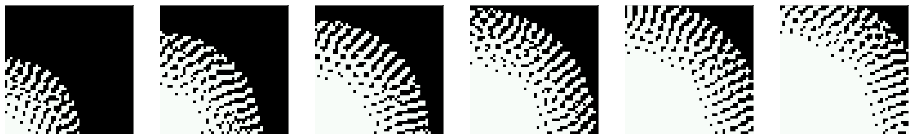
At long-run equilibrium, the Compact Cities are clearly characterized by a higher level of total differential rent (TDR^{t^*} in Table 3.1) than the Periurban Belt and Dispersed Cities, as Compact Cities are more populated. Also, because of the reduced spatial extent of the Periurban Belt and Dispersed Cities, the residential rent close to the CBD is lower than for Compact Cities. Therefore,

and a configuration with 250 residents. Between the two configurations, the population of the periphery shifts into empty stripes and the residential core is emptied, except for the leapfrogged cells.

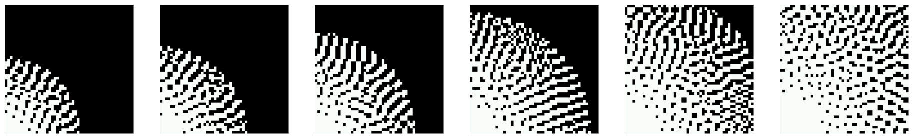
[15]Note that both *Game Theory* and *Cellular Automata* originate from Von Neumann.



(a) Decreasing unitary commuting cost. From left to right: $a = 0.30, 0.25, 0.20, 0.15, 0.10$



(b) Increasing residential income. From left to right: $Y = 10.0, 11.0, 11.5, 12.0, 12.50, 13.0$



(c) Decreasing slope of rural rent. From left to right: $b = 0.0000, 0.0125, 0.0250, 0.0375, 0.0500, 0.0625$

□ residential cell ■ agricultural cell

Figure 3.4: Comparative statics of long-run equilibrium configurations, with $\beta = 1.25$, $\gamma = 1.00$, and $\hat{x} = 3.00$. Base case: $a = 0.10$; $Y = 10$; $b = 0.0000$.

the differential rent per inhabitant is lower when residents state preferences for greenness.[16] Result 3.5. also suggested that when the preference for greenness is relatively high, the land rent of some residential cells can be lower than that of the agricultural cells.

The mean commuting cost ($TTC^{t*}/H_G^{t^*}$) in a standard city without externalities and fixed lot size, increases less than linearly with the population. Resulting values (see Table 3.1) for the mean commuting cost are therefore counter-intuitive: a higher level of fragmentation corresponds to a lower mean commuting cost. As the five cases does not have the same population, they cannot be compared in these terms. Aggregate values (TDR^{t*} and TTC^{t*}) have therefore been computed for a circular (standard city) of an equivalent population (TTC_{co} and TDR_{co}), in order to undertake comparisons (therefore with different preferences). The excess of commuting costs due to dispersion is measured by the difference between the total commuting cost in the city with externalities and in the circular case (see $\frac{\Delta TTC}{TTC_{co}}$) in Table 3.1) [17]. As a result, there is no (or little when octagonal) 'extra-commuting' in the Compact City. By contrast, in the cases where there is a mix of land uses, there is an

[16]One can note however that the differential rent per inhabitant is higher for the extreme case where there is no preference for social amenities, $\gamma = 0$, compared to the case where $\gamma = 1$.

[17]$\Delta TDR = TDR^{t^*} - TDR_{co}$ and $\Delta TTC = TTC^{t^*} - TTC_{co}$

	$H_G^{t^*}$	D_c	P	P_{DENS}	H_{FRAG}	$\frac{\Delta TTC}{TTC_{co}}(\%)$	$\frac{\Delta TDR}{\Delta TTC}$
Base case	444	29.07	22.36	0.62	0.2755	14.09	4.22
Commuting cost change							
$a = 0.05$	1550	58.14	46.14	0.64	0.2445	12.76	5.14
$a = 0.15$	204	19.24	15.11	0.63	0.2721	14.01	3.83
$a = 0.20$	168	17.69	14.69	0.63	0.2847	15.36	2.69
$a = 0.25$	114	13.89	10.89	0.68	0.2675	15.80	2.21
$a = 0.30$	81	11.70	8.54	0.64	0.2387	10.14	3.10
Income change							
$Y = 11.0$	874	39.12	23.12	0.64	0.2088	8.13	4.03
$Y = 11.5$	1153	44.10	22.72	0.65	0.1710	6.25	4.02
$Y = 12.0$	1461	49.04	23.04	0.65	0.1583	4.82	4.09
$Y = 12.5$	1746	53.91	22.89	0.68	0.1304	3.65	4.20
$Y = 13.0$	1980	59.03	22.98	0.70	0.1057	2.62	4.39
Rural rent change							
$b = 0.0125$	576	33.24	26.24	0.62	0.2772	14.13	3.76
$b = 0.0250$	779	38.64	30.58	0.62	0.2637	14.09	3.34
$b = 0.0375$	1111	46.53	36.33	0.62	0.2839	14.03	2.88
$b = 0.0500$	1554	58.14	45.77	0.64	0.2571	12.67	2.58
$b = 0.0625$	1903	68.59	52.78	0.74	0.1874	9.63	2.46

Table 3.3: Comparative statics of morphological indices and economic aggregates at long-run equilibrium, with $\beta = 1.25$, $\gamma = 1.00$, and $\hat{x} = 3.00$. Base case: $a = 0.10$; $Y = 10$; $b = 0.0000$.

excess of commuting. With a small neighbourhood size ($\hat{x} = 1.42$), this excess amounts to 3% in the first Periurban Belt case ($\beta = 1.00$) and 20% in the second ($\beta = 1.25$). For larger neighbourhoods, the level of extra-commuting is lower, because settlements are more clustered. In the extreme case of environmental preference, when households do not value social interactions, total commuting cost are double the level for the standard circular city.

The extra-commuting cost can be compared to the extra-differential rent provided by fragmentation. The value of $\frac{\Delta TDR}{\Delta TTC}$ being > 1 for the three cases where there is a mixed area indicates that rent surplus, due to mixing, always compensates for the extra-commuting cost at equilibrium. This is certainly an element to consider when assessing the social and economic costs of urban sprawl morphologies.

3.4.3.2 Change in commuting cost.

The impact of changing a on total population and on the size of the mixed periurban area (P) is non linear as the simulations were conducted in 2D. As shown in Fig.3.4 and Table 3.3, decreasing a leads to increasing periurban extent, because the commuting fringe (D_c) is attracted outward more rapidly than the fringe D_u. Hence, the commuting cost has no impact on the periurban mixed morphology itself: density $\mathrm{P}_{\mathrm{DENS}}$ and fragmentation $\mathrm{H}_{\mathrm{FRAG}}$ indices do not express any important variation or any trend. This result contrasts with the analytical Periurban City model of Cavailhès et al. (2004b) where decreasing commuting leads to a denser mixed periurban area.

Following population, total and mean commuting costs decrease. However, the change in total transport cost relative to the transport cost of an equally populated circular city is more complex. The extra-commuting cost ($\frac{\Delta TTC}{TTC_{co}}$) increases with increasing unit cost, up to a certain population threshold. It decreases for small cities. The ratio of extra-commuting to extra-differential rent ($\frac{\Delta TDR}{\Delta TTC}$) is even more difficult to understand. At first glance, it is inversely related to the extra-commuting, but for even smaller cities, its value varies without any trend (size effect variability). However, whatever the unit commuting cost, the ratio is always > 1 and therefore the benefits of fragmentation exceed its costs.

3.4.3.3 Change in income.

Rising individual income does not impact on the extent of the mixed periurban zone (constant Pin Table 3.3) because there is an upward translation of the whole system made of the bid rent parallels. The specialized residential area and the commuting fringe move outward together. The resulting configurations show a larger compact city core, which is how the overall fragmentation is reduced, but no increases in the periurban density (border effect excepted). Yet, rising income leads to a total population increase, as well as to a periurban population increase. The income effect on the morphologies is therefore very different from the effect of commuting costs.

The total and mean commuting costs grow with the city population. However, the extra-commuting due to fragmentation clearly decreases when income rises. This is explained by the growth of the compact part of the city. The ratio of extra-commuting to extra-differential rent is constant (see Table 3.3).

3.4.3.4 Change in rural rent.

The homogeneous agricultural rent can be changed into a Thünen type rent with $b > 0$ (Eq. 3.1). Increasing b, means that the agricultural rent falls in remote rural areas whereas it remains unchanged for periurban farmers closer to the CBD. Such a change impacts on the equilibrium configuration by increasing the population and the spatial extent of the city.

In morphological terms (see Fig.3.4), this is equivalent to decreasing the unit commuting cost, i.e there is no important variation in periurban density and fragmentation level. The commuting fringe moves away from the CBD more rapidly than the morphological fringe, and the mixed area extends. Although the total and mean transport costs increase, the extra-commuting ($\frac{\Delta TTC}{TTC_{co}}$) decreases as does the ratio $\frac{\Delta TDR}{\Delta TTC}$.

Result 3.7. *Increasing income causes further agglomeration of people within the compact part of the city and, therefore, reduces overall fragmentation. By contrast, decreasing commuting costs (or decreasing rural rent in remote areas) benefits the mixed part of the city and further expands the sprawling patterns with a similar degree of fragmentation.*

3.5 Conclusion

This chapter has presented a dynamic theoretical model of residential development in cellular space. The model integrates Urban Economics and Cellular Automata in order to simulate the emergence of different residential morphologies (Compact City, mixed Periurban Belt, Dispersed City). Residential location choice is based on the maximisation of a utility function that takes into account neighbourhood externalities. These externalities are determined by the density of households in a bi-dimensional neighbourhood. The dynamics of the system are generated by the sequential decision making of new migrants who locate themselves according to what they observe at the previous time step. The incentive to migrate arises from the existence in the region of a surplus to their reservation utility in a short-run equilibrium. Different spatial configurations have been observed and quantified. The compactness/fragmentation of space changes according to the relative importance that agents give to neighbourhood social externalities with respect to neighbourhood environmental externalities.

Simulations were run on a 2D square grid. It was found that the presence of local agglomeration forces may compensate for commuting costs and, therefore, expand the size and population of the city. Local agglomeration forces also flatten the shape of classic circular concentric patterns. When increasing the preference for low local density, a mixed area with farmers and residents emerges at the periphery of a compact urban core. Fragmentation arises as an equilibrium configuration in the form of clustered and striped settlements, which are wider when agents have a large spatial horizon. The succession of micro-morphologies with distance to the CBD was emphasized and shows how people self-organize when they trade-off two opposite neighbourhood externalities with commuting costs. Fragmented cities are also less populated and the surplus of commuting cost is more than compensated for by the aggregate differential rent.

In dynamic terms, the preference for low local densities generates a leapfrogging residential growth. When rural gaps are later filled in, the land rent at the margins of the compact core decreases. The general rent profile tends therefore to curve in the mixed periurban area. Furthermore, in a mixed periurban area, land values are shown to vary from place to place. These mixed areas are long-run equilibrium structures in which open-space is preserved without any planning system. This situation issues from the sequential location of households and the irreversibility of residential land use conversion.

Finally, it was found that decreasing commuting costs or increasing the slope of the rural rent enlarges the zone of mixed land uses. Conversely, increasing income leads to an enlarged compact city while the mixed periurban zone is pushed away from the centre. This result is in contradiction with Cavailhès et al. (2004b). A test of both models against a given periurban reality could give insights into the plausibility of these hypotheses.

The modelling framework has shown that it is possible to overcome the differences between spatially-explicit simulations of land use change and theoretical urban economics models. This approach (i) highlights the importance of space and spatial interactions in economic processes, including the appreciation of non market goods, (ii) emphasizes the relationship between dynamic processes of land conversion and the spatial morphology emerging from individual interactions ('morphogenesis'), (iii) brings a deductive approach to a CA model, when these are usually empirical, (iv) shows the importance of path-dependent processes in shaping long-run economic equilibria.

The model has provided insight into the relationship between resident preferences and spatial fragmentation. Future research could therefore address sustainability issues, not only on the basis of aggregate economic indicators, but also on an evaluation of the city forms (see chapter 6).The framework is also able to tackle other types of neighbourhood effects that are important for agents when making decisions in geographical space. For instance, in chapter 5, neighbourhood segregation processes will be explored.

Chapter 4

Analysing dynamic patterns and the impact of a green belt policy: a 1D model with single household type and varying housing lot

Outline

This chapter aims to further reveal the dynamic mechanisms of residential growth that can generate discontinuous urban spatial patterns. We draw on the spatio-dynamic model presented in chapter 3 but reduce it to a single spatial dimension in order to better visualize the timing of the process. The model is therefore closer to the standard monocentric urban model (with externalities). Moreover, while a single household type is still considered in this chapter (this will be relaxed in the next chapter), residents are now allowed to trade-off housing consumption with commuting and externalities, which brings the model closer to real situations. Conversely to the previous and the next chapters, the 1D setting also allows for externalities to be computed as a function of people density rather than land use density.

The evolution of spatial structures through time is examined by means of space-time diagrams as used for elementary cellular automata. An analysis is undertaken of the sensitivity of the dynamic patterns to changing households preferences and neighbourhood size, as well as income and commuting cost. The analysis finally considers the effect of implementing a green belt policy to limit residential dispersion in rural space.[1]

[1]This chapter is based on a working paper Caruso et al. (2004c). A previous version has been presented to the XL Colloquium of the *Association de Science Régionale de Langue Française*, Brussels, September 2004. The paper (in French) is available online (see Caruso, 2004).

4.1 Introduction

This chapter tackles the issue of urban spread and its different spatial forms. The objective is to shed new light on the emergence of non-compact urban morphologies, i.e. discontinuous patterns that consume more space at the periphery of cities. A monocentric periurban context is considered where residents commute to a single CBD and residential development mixes with agricultural land use (see Cavailhès et al., 2004b, for the concept of a periurban city). The dynamics of the phenomenon is related to the concept of urban sprawl, which is a major concern in the United States(Brueckner, 2000b; Galster et al., 2001) but also in Europe, where dispersion trends affect an always greater share of the population and territory (Caruso, 2002b).

Increasing income and decreasing commuting costs are the main drivers of urban expansion. However there is also general agreement within the European context that internal migration of active households with children is occurring toward the peripheries. These families are attracted by larger housing plots, lower density environments and landscape amenities, as well as accessible locations and well serviced neighbourhoods. These decisional characteristics are included into the model presented in this chapter.

Despite strong similarities, periurbanisation processes can have different morphological impacts within different regional contexts (e.g. in terms of population potential, economic growth, traditional settlement patterns, planning and housing policies). The degree of periurbanisation can partly be assessed through spatial typologies at the European scale based on administrative boundaries (e.g. EUROSTAT, 1999). However morphological differences are better measured at a finer scale, using land cover data. For example, Vandermotten et al. (1999) show how the respective share of continuously built-up and discontinuously built-up areas within commuting fields vary from one European city to another.

The objective of this chapter is to explore a residential growth process within an initially rural area. We focus on the morphogenesis of urban development rather than on the measurement of spatial forms. Also, although the socio-economic and environmental impacts of residential dispersion represent the ultimate goal of this research, such issues are not addressed in this analysis. More specifically, a discrete spatio-dynamic model is used to simulate land use conversion at the periphery of a city under a given urban growth rate. The use of space-time diagrams to visualize and describe the movement of mixed spatial configurations toward the periphery is assessed. The spatial configurations include different urban density levels, i.e. different proportions of residential and agricultural uses. Moreover, because land conversion in this model is driven by the land-market, it is also possible to analyse the evolution through time of land rents and housing consumption.

First, the different modelling methods that are incorporated in the model are presented. Second, the micro-economic characteristics of the model and its dynamic functioning are presented. Third, a sensitivity analysis is undertaken of the spatio-dynamic output to changes in individual preferences and budget-related parameters. Four, the impact of a green belt policy is analysed on the distribution of residents and land values.

Green belts have been implemented to contain the outward growth of a city and prevent the urbanisation of agricultural and recreational land. The green belt policy is closely related to the assumption that a compact city is a sustainable city. However supports for this assumption are still debated (see Breheny (1995) and the opposition between Newman and Kenworthy (1989) and Gordon and Richardson (1989)) but also side-effects of compaction have been underestimated (e.g. increased journey to work in new developments and supply shortage effect on land values, see Evans (1991); Hall (1997)). From a morphological viewpoint, the impact of a green belt on the geometry of urban settlements has been assessed by Longley et al. (1992) for South East England. Modelling experiments have also been conducted. For example Brown et al. (2003) constructed an agent based model to evaluate the effectiveness of green belts depending on width and location, while Wu and Plantinga (2003), with a 2D open city model, analysed how far restricted open-space can limit urban development. Both paper consider that households have preference for a type of landscape externality, which, conversely to what is done in the present paper, is localized exogenously. Finally, Lee and Fujita (1997) have analysed the economic efficiency of urban development beyond a green belt depending on its location, the distance-decaying nature of amenities provided, and the level of preference for these amenities.

4.2 Modelling background

The approach presented here is based on the integration of an urban economic model within a discrete spatio-dynamic framework inspired by elementary cellular automata (CA)(as recently popularized by Wolfram, 2002). The methodology is original because it is based on the use of one-dimensional CA (1D CA). These have not been used in urban studies (except by Lai (2003), see below). Conversely, two-dimensional CA have been used widely to simulate urban growth or to model the city as a complex system (see e.g. Batty et al., 1997; White and Engelen, 1997; Webster and Wu, 2001; Wu, 2003). Bi-dimensional CA also have certain similarities to preference models (the early development of which are due to Schelling (1971)) and more recently to spatial models of game theory and chaos (see e.g. Nowak and May, 1992; Nowak and Sigmund, 2000).

Two-dimensional CA (2D CA) more closely represent geographical forms. This probably explains their widespread use in spatial modelling since Hagerstrand's diffusion model (Hägerstrand, 1967), and the beginnings of raster GIS and cellular geography (Tobler, 1979). However, 1D CA have two advantages over 2D CA. First, they offer an easier way of visualizing the dynamics[2]. Second, their structure is closer to the standard urban economic model for which space is considered only by distance along a line segment.

Cellular automata are systems composed of a large number of simple and identical components interacting locally. Each element takes a single value in a finite set of values. The value of each element changes synchronously with discrete time steps, and is a function of the value of neighbouring elements at the previous time step. CA have been used to model complex natural systems and as an alternative to dynamic modelling with differential equations. Most of the models developed in geography to simulate land use change are extended CA. Additional constraints are often added in these models so that the state of a cell depends not only on the state of its neighbours but also on other local characteristics (e.g. transport accessibility, soil type, slope, age of development,...) (and thus the 'locality' condition of CA is not strictly respected). Furthermore, the definition of neighbourhoods varies a lot between applications and sometimes include distance decay effects. Conversely, 'parallelism '(or synchrony) conditions and 'homogeneity' of state conversion rules are often respected.

1D elementary (2 states) CA comprise a line of cells, each coloured black or white (which defines the cell state). At each time step a rule defines the colour of a cell on the basis of its original colour and the colour of its immediate left- and right-hand neighbours at the previous step. By superimposing the states of each cell on a diagram through multiple time steps, trajectories of spatial configurations can be observed. Fig.4.1 shows three examples of such CA diagrams. For each example, the decision rule is shown at the top (representing the state of the three neighbourhood cells and the results for the central cell). Then the five first sequences are shown. At the bottom, large triangles show time running downwards for longer period simulations. The left hand example (called rule 254) is one of the simplest examples as cell states become homogeneously black. A more interesting pattern emerges from the second example (rule 90), with nested triangles. This rule replicates a well known fractal pattern called the Sierpinski Sieve[3]. The third example (Rule 30) presents an irregular pattern with aperiodic structures.

[2]As mentioned by Wolfram (2002, p.24) when presenting his 1D experiments '*An important feature of cellular automata is that their behaviour can readily be presented in a visual way*'

[3]The Sierpinski Sieve or Sierpinski Gasket can be produced from a filled equilateral triangle

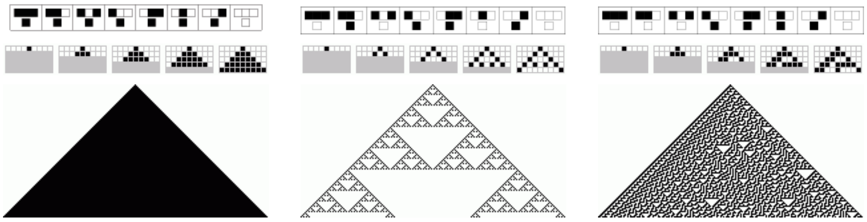


Figure 4.1: Space-time diagrams of elementary cellular automata (rule 254, 90 and 30). After Wolfram, 2002.

Wolfram (1984, 2002) described systematically the dynamic behaviour of all 256 possible sets of these simple CA. He proposed a qualitative typology of four classes roughly with reference to continuous dynamic system theory[4]. The first three classes are based on the type of attractor (limit point, limit cycle, strange attractors or random-looking behaviour), and the fourth class is defined by persistent structures that emerge locally.

Lai (2003) used the same set of elementary CA to gain insight into the types of conditions that can drive urban change in a linear city. His analysis was based on the assumption that (i) cities are structured with spatial overlaps and, therefore, are better represented in topological terms by semi-lattices rather than trees, and (ii) that cities are characterized by complex structures with localized emerging patterns and, therefore, behave like Wolfram's class 4 CA. He suggested that five sets of local rules fulfil these assumptions and thus can be used to determine urban change. However, as Lai acknowledges, the substantive meanings of these rules are not addressed. It is difficult, therefore, to understand what the rules imply regarding the behaviour of individual urban agents.

Within a 2D setting, Caruso et al. (2004b) developed a model of residential growth that aimed to overcome the lack of theoretical foundation of most 2D urban cellular automata applications. These applications often do not consider land use change as the result of individual economic actions, nor do they consider the meaning of transition rules that correctly reproduce observed land use change. Caruso et al. (2004b) coupled a CA with a dynamic monocentric model with neighbourhood externalities. By comparison with standard urban economics, the main advantage of the model is that it can generate spatial forms that can be compared with observed and geographically detailed land use patterns. Moreover, the resulting spatial configurations do not depend only on the commuting distance, but also on 2D spatial neighbourhoods. While static or dy-

decomposed into four triangles traced from the midpoints of the initial triangle's edges. Then the central triangle of these four is removed. The Sierpinski Gasket is formed by repeatedly removing the central triangle from every filled triangle. Its fractal dimension is 1.585.

[4]More general classification schemes and quantification have also been proposed (e.g. Langton, 1992).

namic urban economics can generate mixed zones surrounding cities, this model can further characterize the spatial arrangement of land uses within these zones. Different spatial measures (residential cluster size, fractal dimension, fragmentation,...) can then be used to evaluate the form of these structures and to calibrate the model, and eventually to test hypotheses about location decisions (Caruso et al., forthcoming).

Because the model is dynamic, it is useful for understanding the emergence of discontinuities in the expansion of cities (leapfrog developments). The model is therefore related to dynamic monocentric models with irreversible urban conversion (see reviews in Brueckner, 2000a; Miyao, 1987). For these models, in contrast to the static Alonso-Muth-Mills framework, the city structure is not adjusted instantaneously, but progressively and depends therefore on past development (path-dependency). Mixed structures, leapfrogs and reverse urban development (from the periphery to the centre) emerge throughout the sequence of equilibria. Yacovissi and Kern (1995) showed that these dynamic models better fit the observed urban density profiles than static models.

The dynamic functioning and the behavioural assumptions for the model presented below are similar to Caruso et al. (2004b), although (i) it is reduced to a single dimension and (ii) households also consider the size of housing plots in their location preferences.

4.3 The model

Consider a monocentric open-city where housing is a durable good (irreversibility of urbanisation). A utility differential in favour of the city leads to immigration. The city is characterised by population growth and the utility level within the city is adjusted through time to the external utility level. The urbanisation rate is defined exogenously as is, therefore, the utility adjustment speed. The utility of residents within the city is a function of two neighbourhood externalities. As in CA models, these two externalities create the dynamics in the model because they are lagged in both space and time.

The neighbourhood externalities are called 'periurban externalities' because the focus of the model is on residential-agricultural interactions that can generate mixed land uses in the commuting field of a city. It is hypothesized that migrants choose a location by considering both the cost of commuting and the quality of the neighbourhood at each location. Residential choice is a compromise, therefore, between four elements: transport cost, housing lot size, the number of local social interactions (or neighbourhood public goods), and the greenness of the local landscape (or local open-space, or low local density)[5].

The two externalities are a function of the residential density (or conversely of the density of rural parcels) within the neighbourhood. This local residential density is an endogenous variable which is calculated at the moment in time t when a decision is made on the basis of the spatial configuration at $t-1$. The double effect of agglomeration-dispersion relative to the urban centre (CBD) and agglomeration-dispersion at the neighbourhood scale generates urban spatial configurations that are still concentric, but also discontinuous.

4.3.1 Residential behaviour

Representation of residential behaviour is based on the standard residential model with the addition of two neighbourhood externalities: greenness and local public goods varying with local density. The model therefore integrates the 'neighbourhood goods' model (Fujita, 1989, p.200) and the 'crowding externalities' model (Fujita, 1989, p.227). The formulation is similar to Fujita except that the local density (ρ) for a given distance d is not only a characteristic of a point location, but of the neighbourhood around the point location. The limits of the neighbourhood are defined by $d-\hat{x}, d+\hat{x}$.

In contrast to Caruso et al. (2004b)[6], the model does not consider a fixed housing lot size. Therefore, the local density ρ_d is a population density rather than an urban use density (as in Fujita, 1989). Population and housing consumption can vary from one place to another. The weight for the preference for housing consumption (α) is an additional parameter. Furthermore, any change in housing lot size is considered to be a vertical change because horizontal changes affect distances and these changes are difficult to model in a given discrete spatial setting.

All individuals are identical in terms of income and preferences. They obtain a wage (Y) from their work in the CBD and pay a commuting cost which increases linearly with distance (d). The unitary transport cost is given by a. The economic program of a household is

$$\textit{max} \quad U(Z,H,E,S) = kZ^{1-\alpha}H^{\alpha}E^{\beta}S^{\gamma} \tag{4.1}$$

$$\textit{subject to} \quad Z + R(d)H = Y - ad \tag{4.2}$$

with $\alpha \in [0,1], \beta \geq 0, \gamma \geq 0$, and $k = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$ (which simplifies the

[5] According to the terminology in use for local public goods, the model considers 'super-neighbourhood' externalities. *If the benefits of a public good are confined within a city but vary among neighbourhoods in the city, we call it a super-neighbourhood good* (Fujita, 1989, p.177).

[6] Chapter 3

writing of the bid rent). $E \equiv E(\rho_d)$ and $S \equiv S(\rho_d)$ are respectively the greenness externalities, decreasing with density ($\delta E/\delta\rho < 0$), and the social or public good externalities, decreasing with density ($\delta S/\delta\rho > 0$). Z is the consumption of a composite good. H is the housing consumption. The price of Z is unitary and $R(d)$ is the land rent. The density of households in the neighbourhood of a location d is given by (no distance decay effect within the neighbourhood)

$$\rho_d = \frac{\sum_{d-\hat{x}}^{d+\hat{x}} H_d^{-1}}{2\hat{x}+1} \tag{4.3}$$

The externality functions are chosen so that green externalities are convex and public good externalities are concave, therefore reflecting a decreasing marginal effect of increasing density:

$$E_d = e^{-\rho_d} \tag{4.4}$$

$$S_d = e^{\rho_d^{1/2}} \tag{4.5}$$

At the optimum, demands for the two goods Z and H are

$$\hat{Z}_d = (1-\alpha)(Y-ad) \tag{4.6}$$

$$\hat{H}_d = \alpha(Y-ad)R_d^{-1} \tag{4.7}$$

and the indirect utility is given by

$$V_d = (Y-ad)R_d^{-\alpha}E_d^{\beta}S_d^{\gamma} \tag{4.8}$$

Households that migrate into the city are ready to pay Ψ_d to locate at distance d and obtain a level u of utility. Ψ_d, the bid rent of the household, is determined by

$$\Psi_d = (Y-ad)^{1/\alpha}u^{-1/\alpha}E_d^{\beta/\alpha}S_d^{\gamma/\alpha} \tag{4.9}$$

4.3.2 Residential conversion dynamics

The region under consideration is an array of cells with a central CBD. The city is growing and the rate of conversion of rural space into urban land is one cell on each side of the CBD at each time step. Households entering the city-region choose a location within the available rural cells in order to maximize utility. This utility is compared to the external utility. A utility surplus leads to further immigration. For a choice at time t, a household observes the city at $t-1$. This sequential approach is path-dependent. The bid rent equation (Eq.4.9) shows that the household makes its decision according to the distance to the CBD as well as a to the quality of the neighbourhood.

At each time step, it is assumed that at least two agricultural cells appear to be equivalent to the new resident (see Caruso et al., 2004b, or chapter 3). Therefore all agricultural landowners are in competition for the incoming household who, therefore, can obtain an agricultural cell by paying the agricultural rent. The newcomer reaps a utility surplus which is maximized when he locates where the utility is at its highest. One can also assume the presence of a land developer who acts as a perfectly discriminating monopoly and maximizes the additional land value at each time step. The cell developed at time each time t is the cell where the reservation bid rent of the households is the highest.

Because the city is open, the last migrant determines the utility level u^t for all households in the city. Landowners (who like the residents are myopic) adapt the land rent and housing lots immediately in every urbanised cell as a function of the new utility, but also according to possible changes in the quality of the neighbourhood. This hypothesis is necessary to avoid internal migration that would leave abandoned houses or a return to agricultural use.

Finally, the land rent at any time t is given by the following expression, where the externalities are lagged in time and, therefore, will be the source of spatial path-dependency.

$$R_d^t = \begin{cases} \Psi_d^t & = (Y - ad)^{1/\alpha} u^{t-1/\alpha} E\left(\rho_d^{t-1}\right)^{\beta/\alpha} S\left(\rho_d^{t-1}\right)^{\gamma/\alpha} \\ & \text{for any cell already urbanised at } t-1 \\ \Phi & \text{for the newly urbanised cell and agricultural cells} \end{cases} \tag{4.10}$$

4.3.3 Experiments

The following section presents results from several sensitivity analyses. The patterns that are produced are not as complex as rule 30 or 90 in Fig.4.1 because the locality condition is not strictly respected due to the commuting distance effect. Moreover a long-run equilibrium is made possible in the city and therefore plays the role of a point attractor. First, we look at variations of the open-space preference (changing β with $\gamma = 0$) and change in neighbourhood extent($\hat{x}$). Second, we analyse the land rent profile and housing consumption against distance and time for three pairs of neighbourhood preferences (with both β and $\gamma > 0$ or $= 0$). Third, we analyse the effect of changing the income (Y) and the unitary transport cost (a) as well as the preference for housing size (α). Finally, we show the impact of implementing a green belt policy on the development of the city.

4.4 Resulting space-time patterns and sensitivity

4.4.1 The effect of neighbourhood size and open-space preferences

When no neighbourhood externalities are considered in the model, each cell can only be differentiated from the others by distance. Therefore, when maximizing utility, households choose the location that minimizes the commuting cost,which is the closest agricultural cell from the CBD. At time $t = 1$, new migrants locate just next to the CBD; at time $t = 2$ they locate in the immediately adjacent cell; and so on. If the process is assumed to be identical on each side of a CBD located in the middle of a line segment composed of 2x100 cells, the evolution of the city can then be represented by an isosceles triangle with a 200 base cells and a height of 100 cells. Such a triangle represents a concentric and non-discontinuous growth. At time 100, the whole space is occupied homogeneously and the 200 cells of the base are black (urban) (see Fig.4.2).

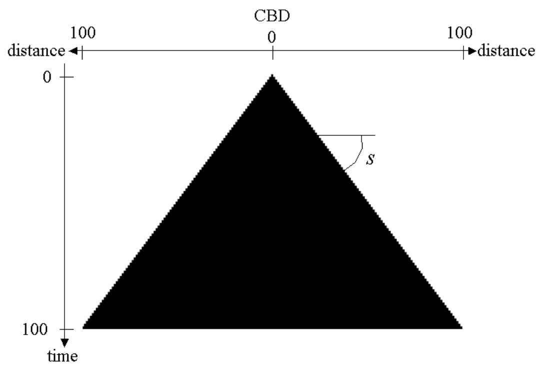


Figure 4.2: Space-time diagram of a classic continuous urban growth with no externalities

If a preference for green neighbourhoods is introduced into the model ($\beta > 0$), the triangle is transformed. Infill of the city progresses in a discontinuous manner in the first time steps and, more generally, at the fringes during the growth period. Leapfrogs emerge at the urban fringe. Fig.4.4.1 shows the impact of increasing β (while γ is null), i.e. the attraction of greenness, which increases from left to right. Fig.4.4.1 also shows (from top to bottom) the effect of increasing the size of the neighbourhood where local externalities can be found, i.e. the spatial horizon of households ($\hat{x}$).

In these examples, the agricultural rent is fixed at a level that is very much below the level of income (or equivalently, the external utility is very low compared to the utility in the city). Therefore, nothing can prevent full urbanisation of the cellular array. After 100 time steps, with an urban growth rate of 2 cells per time steps, the base of the diagram is always made up of 200 black cells as in homogeneously-ending elementary CA (i.e. Wolfram's class 1)[7]. However, during the growth process, different structures emerge and disrupt the triangular form of the compact city growth.

The slope of the envelope of the diagram, i.e. the boundary of the black cells in Fig.4.2, is denoted by s. s is an indicator of the speed (υ) of urban expansion (or enlargement speed of the commuting field's external limit). In the case of uniform and continuous growth without open-space amenities, an isosceles triangle is formed with s=45° and $\upsilon = cotgs = 1$, i.e. the exogenous growth rate given to both sides of the CBD.

The external envelopes of the diagrams in Fig.4.4.1 are always linear or concave, indicating that the expansion rate of the commuting zone is either constant or decreasing. One can already conclude, therefore, that with open-space amenities only, the city cannot first develop in a compact manner and then in a discontinuous way. If discontinuities are going to emerge, then they will already appear from the first time steps. Moreover, when the greenness amenity (β) is more important, the concavity of the envelope is weaker and thus urban expansion slows down later.

A second important feature of the global envelope is the decrease in the slope (s) with increasing neighbourhood size ($\hat{x}$). When households consider a larger neighbourhood, and when the openness of this local landscape is their sole interest (distance excepted), the commuting zone expands more rapidly. More precisely, as discontinuities are generated by the neighbourhood effect and because the neighbourhood is fixed, there is no benefit for a household to locate at distances from a developed cell that are greater than the neighbourhood extent. The maximum leapfrog distance is determined by $\hat{x}$. With a strong liking for greenness, the city urbanises with maximum leapfrogging until the maximum expansion is reached. In Fig.4.4.1, the maximum speed is reached and remains constant when $\beta = 2$ for the two smaller neighbourhoods. The maximum speed is $\upsilon = \hat{x} + 1$ and thus, in these two cases, s holds a constant value of 26.57° and 18.43° respectively. When the neighbourhood comprises 7 cells ($\hat{x} = 3$), the maximum speed is achieved and remains constant until the end when $\beta = 3$ (with s=14.04°). Such a constant maximum growth is not

[7]This is not strictly correct for the graphs presented in Fig.4.4.1 which have been cropped to avoid border effects. The local density has different values when the number of cells in the neighbourhood is smaller close to the two limits of the segment. Diagrams are therefore presented up to $t = 95$ only

yet achieved at $\beta = 3$ for the largest neighbourhood example (for $\hat{x} = 5$, the envelope is still slightly concave in the bottom figure).

In summary, the speed of urban expansion is a function of the urban growth rate (households migration), but also of the level of preference for open-spaces and spatial neighbourhood size. When the preference for greenness increases, the speed of urban development increases (i.e. expansion rate is higher and the slope of the triangle flattens). However, once the increased in greenness preference has reached a high level, the speed of expansion does no longer increase because the size of the neighbourhood is fixed. When a household wants to avoid the crowding in a given neighbourhood, he decides to settle beyond, but at the limit of this neighbourhood. He has no incentives to be far away from this neighbourhood as he is subject to the commuting cost. In brief, open-space externalities accelerate urban expansion, but the maximum expansion rate is determined by the spatial horizon of households.

Different local density structures can be identified within each global envelope and determine sub-envelopes. The larger the neighbourhood, the greater the diversity of local settlements and the number of these sub-envelopes. As for the slope, this diversity is determined by $\upsilon = \hat{x} + 1$. Therefore, respectively 2, 3, 4 and 6 types of local structures are identified in our examples ($\hat{x} = 1, 2, 3$ or 5). With $\hat{x} = 1$, residents locate either in a compact manner amongst others, or leapfrog one cell. In subsequent examples, they can make jumps of different lengths.

The shape of the sub-envelopes provides information on the expansion speed of each local structure or settlement type. For example, the concavity of low density settlements indicates decelerating expansion of this type of local structure. Conversely, the convexity of black cones indicates the acceleration of the compact (non-mixed) city development. This compact expansion is more rapid in periurban areas since it needs only to fill in agricultural interstices that were left undeveloped in previous steps.

The classic development pattern of a city where rural land at the fringe is converted stepwise, ring by ring, no longer holds when residents have preferences for greenness externalities. This circular expansion pattern is replaced by a discontinuous, leapfrogging, development of the city. A scattered or stripped belt emerges at the periphery of the city and the fragmentation increases with the preference for local open-spaces (see 2D simulations in Caruso et al., 2004b). When agricultural value does not constrain the development of the city, the leapfrogging patterns are shown here to be intermediary. Most importantly this first set of 1D simulations can help to better characterize the expansion speed of mixed periurban configurations. The main results obtained when a local dispersion force is included in the behaviour of households ($\beta > 0$ while $\gamma = 0$)

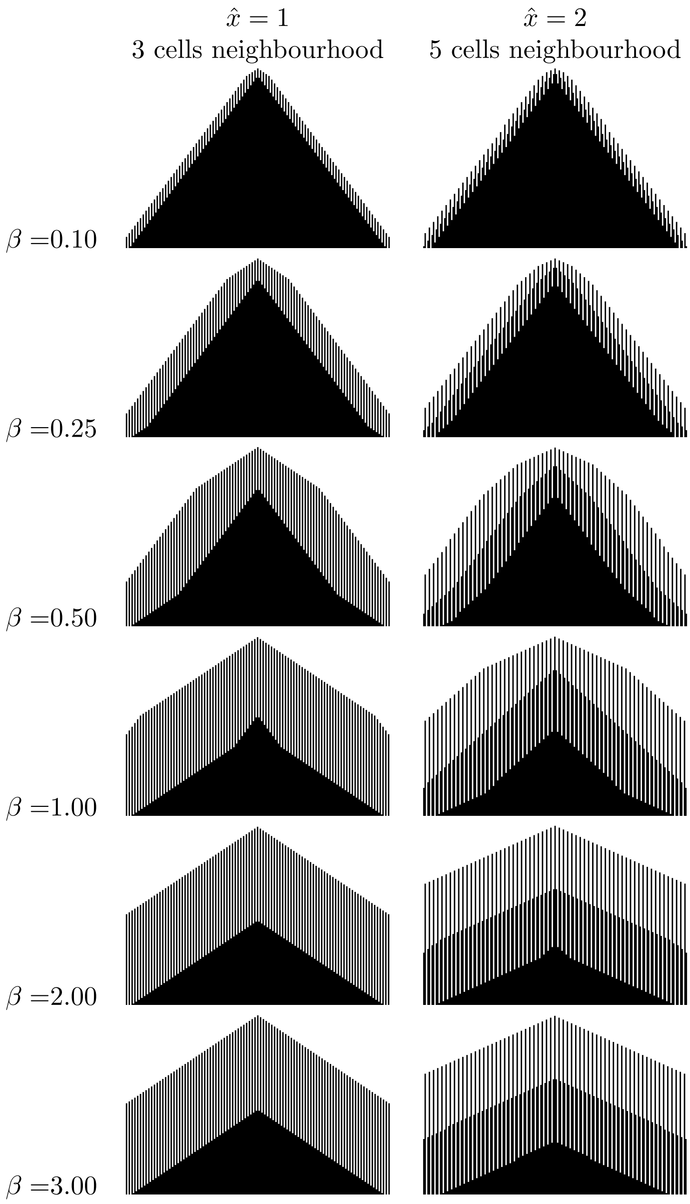


Figure 4.3: Space-time diagrams and increasing preference for low densities (from top to bottom) with increasing neighbourhood size (from left to right). (In each individual tile, time increases from top to bottom and distance is represented horizontally with the CBD at the centre of the segment).

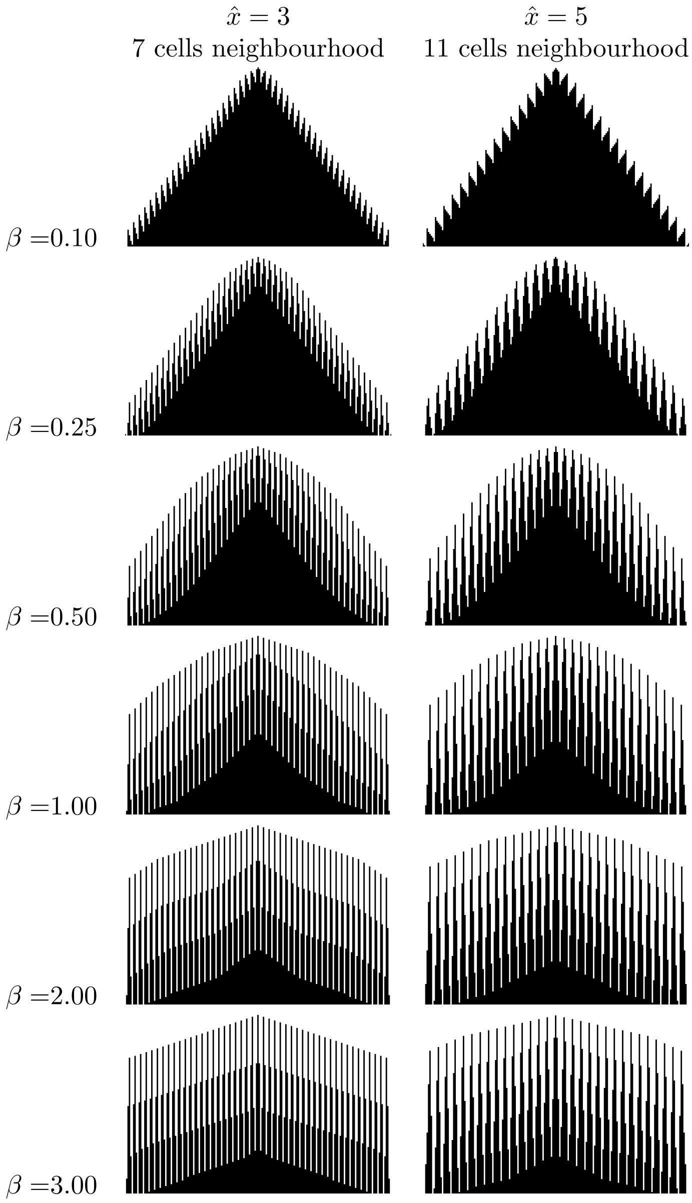


Figure 4.4: *(Fig.4.4.1 continued)* Space-time diagrams and increasing preference for low densities (from top to bottom) with increasing neighbourhood size (from left to right). (In each individual tile, time increases from top to bottom and distance is represented horizontally with the CBD at the centre of the segment).

can be summarized:

Result 4.1. *(4.1.1.) Under the presence of open-space amenities (or crowding dis-amenities), the city develops in a discontinuous manner at the fringe from the outset of migration. (4.1.2.) When greenness is more valued, the discontinuous fringe (mixed area) is wider, the speed of expansion of the commuting field increases, and the compact core of the city emerges later (the commuting field's expansion slows down later). (4.1.3.) When households consider openness over a larger neighbourhood, the commuting field expansion is more rapid, rural interstices are larger, and the local arrangement patterns are more diverse.*

4.4.2 Long-run equilibria and the dynamic profiles of rent and housing consumption

In the previous examples, complete urbanisation of the cellular array was observed because the income of households was very high, the external utility very low or the opportunity rent of land was very low. In the simulations presented in Fig.4.5, parameters are chosen in order to better represent the timing and pattern of a long-run equilibrium. Parameters are determined[8] for which the classic urban fringe without externalities arises at a distance of 50 from the CBD (see Fig.4.5a). The space-time diagram indicates that 50 is also the time of the long-run equilibrium (denoted by t^*) as one cell can be urbanised at each time step on both sides of the CBD. The total urban cells at t^* is 100.

The urban land rent, as shown in Fig.4.5a, decreases with distance whatever the time period. At t^* ($t50$), the residential rent corresponds exactly to the reservation bid of the standard monocentric model of urban economics. As time progresses, the residential rent increases because of the increase in population and the decrease in utility. This utility decrease is due to increasing commuting distance at the urban fringe for the last migrants. The size of housing does the opposite of this. Moreover, as housing lot size decreases through time the density of occupation of each cell increases during the urban growth. These general mechanisms of the dynamic cities are summarized in the following Result proposition. The dynamic features of rent and lot size profiles are respected in the presence of externalities.

Result 4.2. *A dynamic city is characterised by decreasing rent and increasing housing lot with distance. As time progresses, with the addition of urban cells at the urban fringe, residential rents and intra-cellular densities increase while*

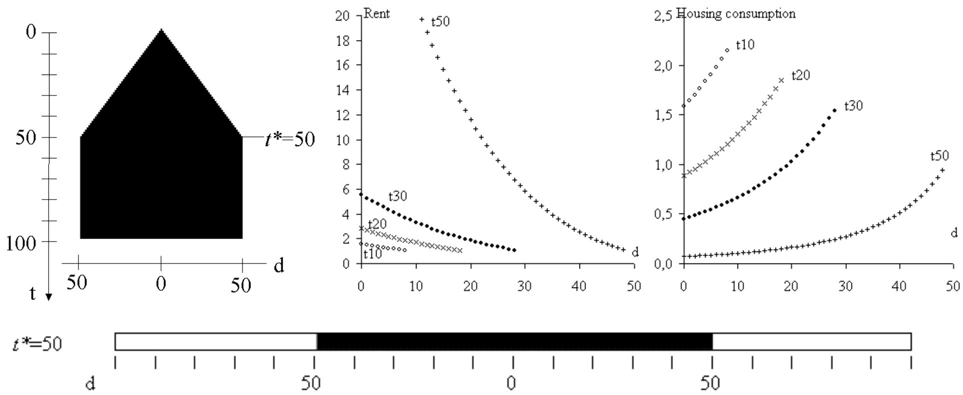
[8] $Y = 10$, $a = 0.12$, $\Phi = 1$, $u = 4$, $\alpha = 0.25$, $\beta = 0.00$(Fig.4.5a) or $= 0.50$(Fig.4.5b and c), $\gamma = 0.00$(Fig.4.5a and b) or $= 0.40$(Fig.4.5c), and $\hat{x} = 5$

housing lot sizes decrease. With no externalities and when the utility is adjusted to the external utility, the city reaches the long-run equilibrium of the static model.

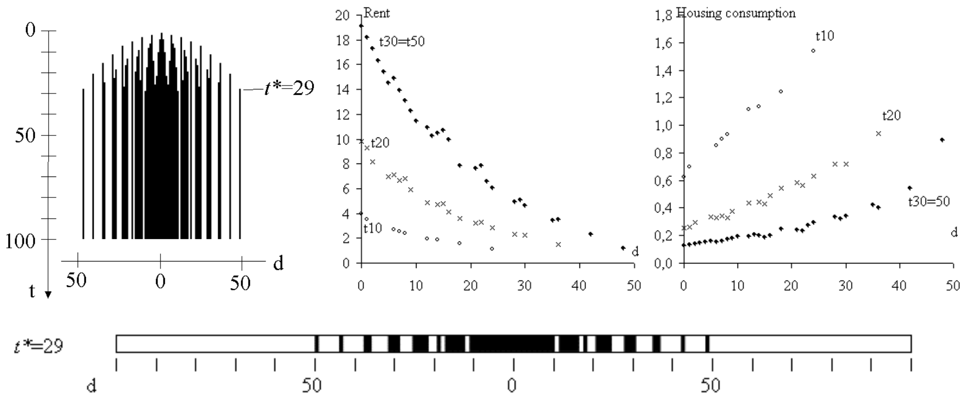
When households have a preference for low neighbourhood densities (or local open-space), the equilibrium is reached earlier, at time $t^* = 29$. The region is less urbanised. More importantly, the equilibrium pattern is not a compact city. Close to the CBD, the built-up area is continuous but, beyond this, disconnected settlements appear. Moreover, one can observe from the slice representing the city at t^* below the graphs in Fig.4.5b, that the size of rural interstices is not constant nor does it immediately increase with distance. In the mixed area, the proximity to the continuously built-up core at the fringe also means proximity to higher levels of intra-cell populations (densities). Individuals who like greenness, can avoid this high neighbourhood density by choosing a location that is more disconnected from the built-up area. They can therefore choose to make a longer jump with the penalty of higher commuting costs. Migrants who arrive later, when the built-up areas or high densities are already more distant, can then choose to arrange themselves in larger clusters, with the size of these clusters eventually decreasing with distance. This result was quite unexpected and was not found when the neighbourhood density considered by future migrants was based on the use of land (as in Caruso et al., 2004b, or chapter 3) rather than on the population living in the neighbourhood (as defined here in Eq.4.3). If two dimensions were to be considered, therefore, emerging spatial patterns would be expected to be more complicated than in Caruso et al. (2004b, chapter 3).

The city structure generates a profile of land values which mixes urban and rural values. The profile is therefore sharply discontinuous (only residential rents are plotted in Fig.4.5). However the pattern corresponds to the first-rank optimum, and arises because of the sequential nature and path-dependency of residential development. The owner of a leapfrogged agricultural cell at equilibrium will always receive agricultural rent. He will not find any household that would accept lower neighbourhood quality and pay a little more than the agricultural rent. No household can afford such a location and achieve at least the external utility level. It is interesting to note therefore how open-spaces are locked-in by the sequential functioning of the land market. Land use planning restrictions are not necessary to maintain the greenness of periurban neighbourhoods at the level required by households. The level of the rural rent plays a very important role in maintaining these green interstices. A sudden drop in the rural rent would imply new residential migration and eventually the infill of leapfrogged locations (like in Fig.4.4.1).

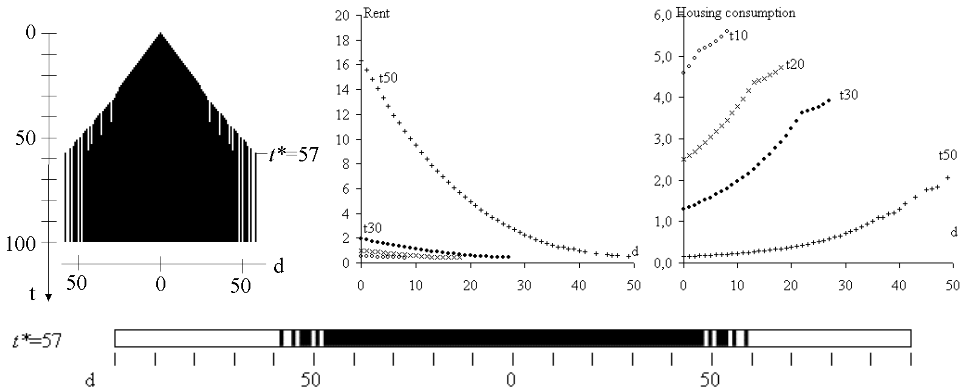
Similarly to the previous case, the general profile of residential rents increases with time because of urban growth. However, in contrast to the no-externalities case, residential rents are also locally adjusted upwards or downwards because of



(a) No externalities



(b) Preference for low local density



(c) Preference for high and low local density

Figure 4.5: Space-time diagrams, long-run equilibrium situations, and the evolution of rent and housing consumption

newly urbanised cells and consequent changes in the neighbourhood population. The land rent profile, therefore, is discontinuous because of the presence of agricultural cells, but there are also local inversions of this profile. Also, in spite of some local downward adjustments of rents through time, the residential rent, even close to the fringe, does not fall below the agricultural rent because housing lot size adjusts automatically. This is not the case when the size of housing is fixed exogenously (see Caruso et al., 2004b).

Finally, by comparison with the previous standard case, the rent is lower for locations at short distances from the CBD. The crowded central part of the city (completely urbanised neighbourhoods) is less attractive to households who value the greenness of the neighbourhood and not agglomeration.

Local agglomeration is introduced in Fig.4.5c. Households have a preference for public goods (S with $\gamma > 0$) that are related to denser neighbourhoods. A slightly weaker preference for this agglomeration amenity was chosen relative to the contrasting open-space amenity. Therefore the more highly urbanised and populated neighbourhoods still induce a total disamenity.

Again, the long-run equilibrium (t^{*}=57) is composed of a compact central part and a discontinuous mixed periphery. The shape of the discontinuity patterns and the discontinuities and dynamics of the land rents are the same as discussed above. However, in contrast to the previous case and to subsequent cases discussed hereafter, the global envelope of the space-time diagram is convex. Thus the city starts to grow in a compact manner with leapfrogging developments arising only later as the expansion of the commuting limit accelerates. This result indicates that under the assumption of the existence of local agglomeration and local dispersion forces, a mixed periurban area emerges only when the concentric compact city has reached a certain size. One could observe at the same time, therefore, both small growing cities with no mixed belt and large cities in an equilibrium state for migration surrounded by a mixed zone. This may be counter-intuitive if leapfrogging patterns are thought of as a characteristic of growing cities. Moreover, if the level of rural rent is sufficiently high, the mixed periphery may not appear at all in long-run equilibrium even though the underlying forces are present.

The housing consumption profile also shows particular behaviour for this simulation. The increase in housing lot size flattens within the mixed periurban area. The peripheral households can afford a relative loss in housing size because they benefit from greater neighbourhood greenness. ‘Suburban’ households, i.e. households completely surrounded by urban cells, would not be able to compensate for such a loss in housing lot size. Finally, everything else being constant, the size of the housing lots is greater with these local agglomeration amenities than without, wherever the location and whatever the period of time (as agglomeration due 'naturally' to commuting costs is here a supplementary gain for households).

Result 4.3. *(4.3.1.) Neighbourhood open-space amenities reduce the timing of the long-run equilibrium and the number of rural land conversions, while the introduction of neighbourhood public goods delays the long-run equilibrium and increases the number of urbanised cells. (4.3.2.) If local open-space amenities are more strongly valued than public goods, the city is composed of a compact core and a discontinuously built periphery at the long-run equilibrium. Rent and housing profiles are then discontinuous and local inversions occur. (4.3.3.) The possibility of trading-off housing size with amenities and distance causes, first, that the size of rural leapfrogs is not directly dependent on distance and, second, that the residential rent cannot fall below the rural rent at any moment. (4.3.4.) With neigbhourhood public goods, a mixed periphery arises when the city has reached a certain threshold size. In the periurban belt, housing lot size grows less rapidly with distance than in the compact part of the city.*

4.4.3 Income, commuting cost and housing lot consumption

In this analysis Y, a, and α were varied exogenously by 30% for the case where only local open-space is valued by households (Fig.4.5b). Unsurprisingly, increasing income or decreasing the cost of commuting leads to increased urban expansion and delays the long-run equilibrium (see the first 2 columns in Fig.4.6). This is the well known suburbanisation effect.

Furthermore, however, the spatial effects of increasing income and decreasing unitary transport cost are dissimilar. By comparing the situation +30% of Y and −30% of a, the maximum extension of the city is achieved earlier in the first case. The slope of the global envelope of the space-time diagram is higher with the increase in income. Increasing Y leads to a larger compact central part while the mixed belt, still of a similar size, occurs further away. Conversely, decreasing a benefits to the mixed peripheral part. The central compact core maintains about the same size while leapfrogs emerge much further away from the CBD.

Using the same reasoning, it can be seen that when the income is low (see Y −30%), the central, continuously built-up part of the city can completely disappear, leaving a completely fragmented area. It cannot be the case when transport cost is high (see a +30%).[9]

[9] A similar exercice can also be made to evaluate the impact of a change in the agricultural

In light of these mechanisms, the observed increase in the spatial extent of mixed areas in Europe (see chapter 2) would indicate a decreasing importance of commuting costs, rather than an increase in income. It is difficult however to rely on spatially aggregated typologies to attest this process. More detailed is the analysis of small housing nodes and share of dispersed population in Belgium in the period 1970-1991 (Halleux et al., 1998). The increasing dispersion of dwellings and the growth of very small residential settlements in the commuting zones also suggest a decrease in commuting costs. An alternative explanation from the model would be an increasing taste for local open-spaces.

Finally, when varying the preference for housing consumption (α), the effect on urban expansion is imperceptible. However the number of urbanised cells is not constant. Increasing α leads to an increase in the size of the compact part of the city, and, therefore, to a reduction in the size of the mixed zone. Conversely, when α is weak, the compact part of the city can disappear. Again, this might appear to be counter-intuitive, but the only local agglomeration forces come from the commuting cost. Therefore, everything else being equal, increasing α gives less importance to the neighbourhood open-space externalities and thus leads to a more compact pattern.

Result 4.4. *(4.4.1.) Increasing income or decreasing commuting costs expands the commuting limit of the city. However, the higher the level of income, the larger the compact central core. The smaller the commuting cost, the larger the mixed peripheral belt. (4.4.2.) Increasing the utility of housing size does not further expand the city*[10]*, but enlarges the compact centre.*

4.4.4 An example of spatial policy: the effect of a green belt

In this simulation experiment, the stress is put on the impact of the neighbourhood externalities on rent values. Therefore, the weight given to neighbourhood externalities in the utility function is increased ($\beta = 5$ and $\gamma = 3$). Consequently as well, rent values in the profiles of Fig.4.7 are not to be compared with previous rent profiles. On purpose, Fig.4.7 emphasises the local variations in rent value. The objective is to show how the model can be used to explore scenarios of spatial planning policies when the quality of the neighbourhood is one of the major location concerns of households.

rent (Φ), which is constant and equal in all locations througout the simulations. Without changing the slope of the agricultural rent, a change in Φ provides the inverse result of a change in Y, i.e. a rise in agricultural rent would lead to a reduced city expansion but also a reduced compact part.

[10] A vertical increase is assumed.

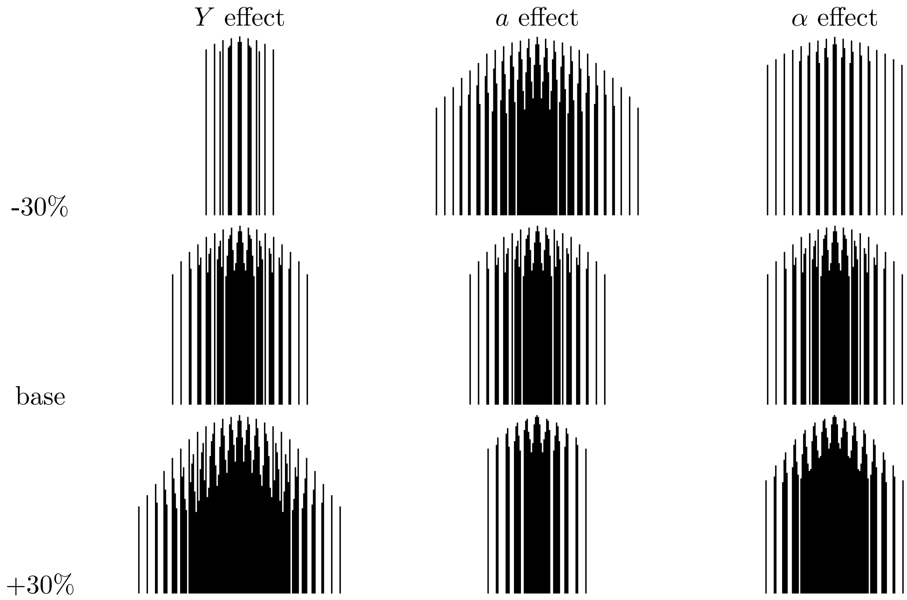


Figure 4.6: Space-time diagrams and the effect of varying income (Y), transport cost (a) and housing preference (α). (In each individual tile, time increases from top to bottom and distance is represented horizontally with the CBD at the centre of the segment).

A green belt is modelled in a very simple way. An area where building is not permitted is defined at a given distance from the CBD (between $d = 30$ and $d = 40$, chosen *a priori* for being within the mixed periurban area, given that long run-equilibrium occurs $t* = 50$). This could for example reflect the impact of the green belt surrounding London, or the 'Green Heart' in the Randstad-Holland. The supply of open-spaces is still advocated today as an effective 'anti-sprawl' policy. The modelled non-build zone can also represent an exogenous natural amenity that benefits only to those who locate in its neighbourhood (e.g. as it its the case for the Forêt de Soignes in the South of Brussels, although this is not a circular protected area).

The results of this experiment are shown in Fig.4.7. The long-run equilibrium is reached at $t^* = 50$ in the non-planned case and earlier with the green belt ($t^* = 43$). The green belt appears to create denser residential settlements in areas close to the inner and outer limits of the green belt zone. However, it has no effect on the neighbourhood density of more central areas. Moreover, we can see that households jump beyond the green belt and accept greater commuting costs only after densification within the inner border. Concentration of developments in the area comprised within the inner boundary of a green belt is also found in empirical studies (e.g. Kline and Alig, 1999)

Such a policy seems able, therefore, to constrain residential development over short time periods. Furthermore, the model suggests that residential development that takes place beyond the protected area tends to be more clustered. The long-run equilibrium shows less fragmentation of the remote periphery. There are also fewer settlements in this remote periphery, but these are of a greater size.

Considering the effect on land rents, the profile is inverted as the green belt is approached. Many empirical studies have shown the enhancement value of various types of open spaces, including greenbelts (see Fausold and Lilieholm, 1996). More particularly, Nelson (1985) examined the influence of greenbelts on urban and exurban land values. Evidence are found that greenbelts increase the value of residential land in proximity. The author also suggests that this effect extend to the area beyond the greenbelt in subsequent periods. Here, the perturbation on both side of the green belt adds to the already discontinuous rent profile generated by the presence of strong neighbourhood externalities and to the variation of neighbourhood population densities. Further, the model shows that the level of residential rent is reduced in the area between the CBD and the green belt. This loss in rent value is particularly severe for the very central locations. The effect on housing lot size is the opposite: larger houses closer to the CBD in the presence of a green belt.

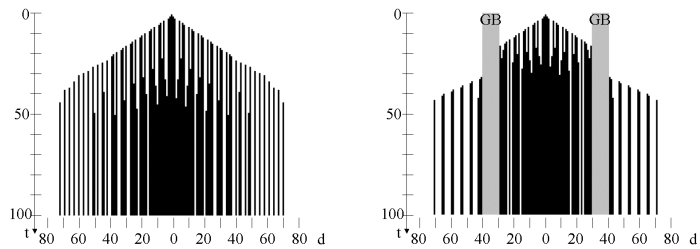
These findings are summarized below:

Result 4.5. *Because it is an attractive open area, a green belt can constrain the development of residents in short-runs. Moreover, remote residential settlements are more clustered beyond the green belt at long-run equilibrium. A green belt also causes inversion of land rents in its surroundings and a fall in rents in the centre.*

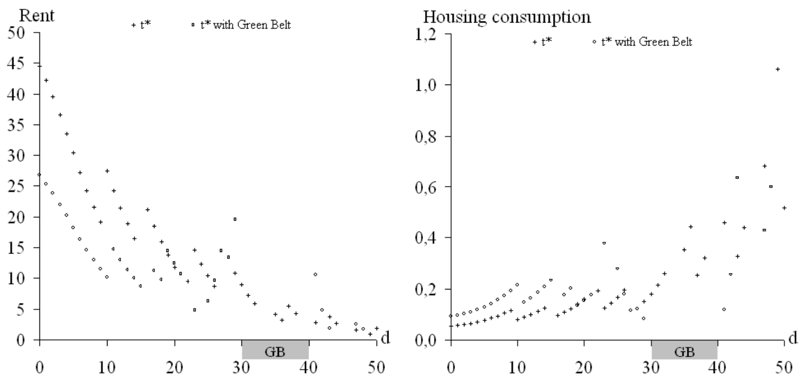
We should however be careful with these results before concluding that green belts are an efficient anti-sprawl policy measure. As suggested by previous modelling works (Lee and Fujita, 1997; Wu and Plantinga, 2003; Brown et al., 2003), it is better when analysing the impact of a green belt to also consider the effect of the size of the green belt and its proximity to the centre with respect to the level of externalities it offers and the level of commuting costs.

4.5 Conclusion

The mechanisms of residential growth within an initially rural space have been explored by means of a simulation tool inspired by cellular automata (CA) and grounded in microeconomics. The analysis emphasised the dynamics of



(a) Space-time diagrams with and without a green belt (GB)



(b) Residential rent and housing consumption profiles

Figure 4.7: The effect of implementing a green belt (GB) at distance 30 to 40 from the CBD in a case where neighbourhood externalities are strongly valued ($\beta = 5$ and $\gamma = 3$).

spatial structures, and in contrast to most CA applications in geography, a single spatial dimension was considered. The dynamics of periurban structures were assessed through space-time diagrams and the analysis of the evolution of rent and housing consumption profiles.

The model assumes that households locate sequentially in an open monocentric city region, and value neighbourhood externalities: either open-space (low densities), or open-space and local services (low and high densities). Both externalities depend on the population density within a given neighbourhood, but act in opposing ways.

The speed and the mode of spatial infill by urbanisation have been described in relation to the preference for greenness (open-space) by households and the size of the neighbourhood that they consider. It is found that the valuation of open-space amenities increases the speed of urban development and delays the emergence of an agglomerated urban core. Moreover, the larger is the neighbourhood that households consider, the more diverse are the mixed spatial patterns.

The emergence of a long-run equilibrium has then been examined where the city is optimally structured in a compact central part and a mixed periphery. The level of discontinuity is not directly related to the distance to the CBD. Moreover, discontinuities and variations in the density of neighbourhoods lead to local inversions of the land rent profile. When households value neighbourhood public goods, it is suggested that only cities of a certain size can be surrounded by a discontinuous periphery. The effect of changing certain parameter values within the model was also tested. Increasing income leads to an enlarged compact central core, while the mixed peripheral belt moves further away from the centre. Conversely, decreasing commuting costs lead to an enlarged discontinuous belt. Also, when households attach more importance to the size of their housing than to neighbourhood quality, the city tends to be more classic, i.e. more compact.

Finally, it was also shown that the method presented here can be used to evaluate the impact of spatial planning policies that seek to reduce the fragmentation of rural space by residential growth. While a green belt policy seems able to constrain urbanisation over short time periods and decrease fragmentation in more remote areas, it was shown to impact negatively on land values in the central part of the city.

Chapter 5

Exploring coexistence and segregation within a scattered morphology: a 2D model with two household types and varying housing lot

Outline

Back in a 2D theoretical space, we complexify the model presented in chapter 3. Like in the previous chapter households can now trade-off the housing good with externalities and commuting. Moreover, two types of households are now considered and differ by their income endowment. The presence of different types of households then leads to a greater variety of social neighbourhood externalities. Finally, the assumption of internal immobility is partly relaxed in order to allow for filtering processes.

The model, therefore, is used to investigate spatial segregation of the two households classes. The city is open and spreading and residents are still assumed to value open-space amenities. The experiment can be seen as an exploration of Schelling's type dynamics within a periurban housing market. The explicit dynamic and spatial nature of the model allows for residential filtering and cumulative processes (neighbourhood tipping) to be analysed jointly with the emergence of a scattered morphology.

Different scenarios are run for four types of social neighbourhood preferences. The scenarios include policy actions that seek to help the poorer household class to locate in the city region.[1]

[1]This chapter is based on a working paper Caruso et al. (2004a). Another paper was previously developed on this periurban segregation issue and presented at the XXXIX Colloquium of the *Association de Science Régionale de Langue Française* in Lyon, September 2003. The paper is available online (see Caruso, 2003b, (in French)) and a revised version has been accepted for publication in a collective book (Caruso, forthcoming). The model is however different than the one presented here as (i) the landlord is considered as a price discriminant monopoly, (ii) simulations have been run with different parameter values, (iii) only the rich households can trade-off housing consumption with amenities and commuting costs, and (iv) the housing lots are increasing with distance but within an exogenously given spatial structure.

5.1 Introduction

Cellular Automata (CA) are discrete spatio-dynamic systems that are widely used nowadays in geography to model land use change and particularly urban expansion (e.g. Batty and Xie, 1994; Batty, 1998; White and Engelen, 1993; Wu, 1998b, 2002; Wu and Martin, 2002). There are also examples of CA that consider the location of different household classes within a city (e.g. Portugali and Benenson, 1995; Benenson, 1998; O'Sullivan, 2002). Furthermore, the cellular modelling approach taken by Schelling (1971, 1978), although not strictly a CA, is still a major reference today in research on urban segregation and neighbourhood change, and more generally on dynamic modelling of social interactions in space.

CA is often seen as a relevant tool for studying urban processes because it can simulate complex dynamics from a bottom-up approach and use geographically detailed information. Researchers have taken interest in CA because micro-spatial (neighbourhood based) interactions generate emergent macro behaviour and spatial patterns. However, micro-spatial interactions in urban CA models have not been connected to residential microeconomics. Two exceptions being Webster and Wu (1999a,b) and Caruso et al. (2004b, see chapter 3). This lack of connection between CA and micro-economics is surprising to us for at least three reasons. First, urban economics has proved ability to explain city structure and change, including suburbanisation, decline, and segregation (see e.g. Anas et al., 1998). Although some lacks can be pointed, this knowledge of urban change mechanisms is essential and cannot be disregarded. Second, there are residential models with externalities that also assume the existence of neighbourhood type interactions between individuals in space (e.g. Kanemoto, 1980). Third, when an external dis-economy is modelled from one household class to another in a city, a dynamic instability or a *cumulative decay process* may occur (Miyao, 1978; Kanemoto, 1980). This process is a generalization to the urban monocentric context of the *tipping* result obtained by Schelling in a cellular setting.

In this chapter, an explicitly spatial and dynamic model is proposed that fully integrates CA and residential microeconomics (with irreversible housing constuction). It is intended to explore the impact of city expansion on the spatial distribution of two income groups. The key of the integration is to implement time-lagged neighbourhood externalities in a cellular and monocentric framework. The model assumes that residents value a twofold neighbourhood externality. They value the proximity to neighbouring open-spaces but they also value the proximity to other households according to their income class. Accordingly, neighbourhood change drives the dynamics of land use like in CA models, but the dynamics is further constrained by the trade-off between commuting costs and housing consumption. Different research streams have explored the issue of urban residential structuring and spatial segregation. Either the role of the preferences for different neighbourhood types is emphasized, either the role of economic factors. Eventually both seems of importance to understand the spatial separation of income groups and are considered in the present chapter.

Several experiments are conducted to understand the emergence of a path-dependent long-run equilibrium that results from the sequential migration to a city of rich and poor households with various neighbourhood preferences. The simulations are chosen in order to present a diverse set of possible dynamic patterns as well as to discuss policy options relative to urban and periurban segregation.

5.2 Background

In a periurban city, households are assumed to value the proximity to open-space amenities or agricultural parcels. The search for green living neighbourhoods within commuting limits is assumed to be an important driver of urban dispersion into rural space. From a theoretical point of view, Cavailhès et al. (2004b) have demonstrated that such an hypothesis can lead to a (static) equilibrium configuration characterized by a mixed residential-agricultural belt at the periphery of a city. Within a CA type framework and under the additional assumption of a growing population, a similar periurban hypothesis can lead to the emergence of a fragmented urban landscape and leapfrogging dynamics (Caruso et al., 2004b).

Empirical studies aimed at measuring the value of low-density environments and distance to open-space for residents also support the periurban hypothesis (e.g. Irwin and Bockstael, 2001; Irwin and Geoghegan, 2001). However, what can be the spatial arrangement of two income classes within such a sprawling morphology with rural-residential externalities is unknown and has not been theoretically modelled (statically nor dynamically). This question might be of importance as, for example, Reginster and Goffette-Nagot (forthcoming) have provided empirical evidence of the impact of environmental quality, including the greenness of the neighbourhood, on the location by income. Moreover, it is known that the income distribution importantly affects the density profile of a city (Anas and Kim, 1992; Goffette-Nagot et al., 2000). Therefore, whether the spatial morphology at a micro level can also be affected by the mix of income groups is to be explored.

From the theoretical monocentric urban model with different income classes, it is known that the equilibrium residential pattern is made of concentric rings with complete segregation. Households locate according to the relative steepness of their bid rent curve. Therefore, if the households groups differ only by income and if all income groups are present in the city, the richer a group, the farther its location from the centre (see Fujita, 1989, ch.4). The comparative statics analysis by Hartwick et al. (1976) has further shown that an increase in income for a given class expands its zone and the zone of the poorer outward (making them better off), while the wealthier households are pushed further out (and their utility is reduced)[2].

However, the income effect that is present within the standard model does not seem to be satisfactory enough in explaining the observed level of spatial segmentation and diversity of location patterns (see e.g. Glaeser et al., 2000). An additional explanation is provided by models where a distance-dependent amenity is assumed. Contrasting spatial patterns can be found when an urban (or suburban) amenity is given exogenously (Cho, 2001). Moreover, multiple equilibria are possible when this amenity causes in turn an endogenous income amenity, with low income neighbourhoods being valued differently than high income neighbourhoods (Brueckner et al., 1999). In monocentric models with neighbourhood externalities, discriminators, i.e. individuals who suffer from a negative externality due to the presence of individuals of another type, tend to move away from the other group and cluster to avoid the externality (Kanemoto, 1980). Complete segregation is the only stable equilibrium pattern as long as one group is indifferent of the neighbourhood composition, or when both groups prefer individuals of its own group. Several empirical analyses show that these community behaviours on the part of White households (e.g. Cutler et al., 1999) or on the part of Black households (e.g. Ihlanfeldt and Scafidi, 2002) can explain a large part of the observed spatial segregation. However, one group can also have preference for integration and being close to the other group, then if this preference is stronger than among the other group, a mixed and stable equilibrium is to be found (Kern, 1981; Kishimoto, 1991).

Beyond the role of income and within a periurban framework, when a household class feels prejudiced by the proximity of another, it is interesting to examine how this disamenity can be traded-off with a lower local density environment (with open-space) and with the distance to the employment centre. The prejudice externality should be modelled through a potential frequency of contacts rather than as a distance to some limit between the two communities like in border models (see e.g. Anas, 2002). Using a CA framework, the externality can be calculated within a defined circular neighbourhood around each location.

The failure to account for the timing of residential location decisions and for the irreversible nature of housing is a second major drawback of the standard

[2] At the condition that the amount of land available increases with distance more rapidly than the commuting cost.

urban model. Dynamic models have therefore been developed where the city at a given point in time is considered as the result of a sequence of short-run equilibria, rather than as an instantaneous long-run equilibrium. These models assume that housing is a non-malleable good. They have first been used to understand the process of urban growth (see the survey by Brueckner (2000a) or Miyao (1987)) and show how inverted rent gradients and leapfrogging patterns arise.

Extensions of the model with irreversible housing to two classes of residents generate spatial patterns which contrast sharply with the perfect segregation pattern induced by the static model. Using a discrete definition of space and starting from the standard separation state, the simulation experiments of Brueckner (1980) show how a mechanism of filtering of houses from one group to another occurs around the initial boundary of the two groups. Furthermore, with redevelopment possibilities, patterns emerge where some rich households are leapfrogged by poor households who therefore occupy the inner part of the city as well as some clusters of different size within the rich area (the outermost location excepted)[3]. Income mixing at the periphery is also found by Vousden (1980) without any filtering process but with new peripheral developments occupied by both poor and rich immigrants. This pattern occurs when utility is constant over time, but the author also suggests that more complicated patterns can arise, including upward and downward filtering, when the utility of one group increases with respect to the other[4]. More recently, Glaeser and Gyourko (2003) have used a similar model. They show that the filtering down of dwellings is also important to explain the decline of cities. Moreover, the authors suggest that filtering down is due to a loss of demand for a location and its amenities[5], rather than the ageing of houses.

In the presence of endogenous socio-economic and green amenities (i.e. the distribution of which vary with the spatial distribution of people and income groups), filtering processes should be even more intricate but arguably crucial to understand income sorting and mixing in cities through time. In fact, by assuming different group preferences, models of irreversible housing with two income groups will be connected to Schelling's type neighbourhood dynamics. In this case, segregation will be the result of both standard economic processes (commuting and housing trade-off) and neighbourhood spillovers. We argue that it might therefore be used to represent different hypotheses including *White Flight*

[3]The concept of leapfrogging is usually used to describe an urbanisation process that leaves agricultural land between the urban centre and the new developments. The concept is somewhat generalised here in order to describe the discontinuous development of an urban class that leaves another urban class between its new and previous developments.

[4]Vousden and Brueckner assume an open-city with a maximizing myopic land developer who choose the density and the income group to occupy any place

[5]The authors do not consider a monocentric city, but the amenity can be related to distance and the model therefore tackle urban *versus* suburban decline

due to racist preferences, *Flight from Blight* and self-reinforcing neighbourhood decline, or mechanism resorting to *Club* formation and *Tiebout* sorting. Finally, the observation of non-linear responses to neighbourhood change (Galster et al., 2000) also indicates the potential importance of using a micro-simulation setting to model the emergent nature of segregation in the housing market (see e.g. Meen and Meen, 2003; Durlauf, 2003).

5.3 The model

Agents compete to settle within a square lattice of I lines and J columns. Each cell represents the location of an agent and is denoted by ij. There are three types of agents: Farmers (A), Black (or poor) households (B), and White (or wealthy) households (W). Only one type of agent can occupy a single cell, i.e. there is no intra-cell mixing. The state of a cell is $C_{ij} \in \{A, B, W\}$. Different density of occupation is however possible for households: household population in a single cell is denoted by w_{ij} or b_{ij}. The surface of a cell is unitary, so that population in ij corresponds to density in ij, and the housing lot size is $H_{ij} = \{1/w_{ij}, 1/b_{ij}\}$.

Assume W and B households choose a location ij by maximizing a utility function under a budget constraint. They work in a unique CBD located at the origin of the grid[6] where they earn a different income, $Y_W > Y_B$. W and B households have a similar utility structure that is expressed through a Cobb-Douglas function.

Agriculture represents the default activity. All cells are in an agricultural state at the initial state ($t = 0$). The bid-rent of the farmer, Φ, is a constant in space and time.

5.3.1 Residential behaviour

Three different cases of inter-group preferences are identified and reflects similar assumptions in the literature. A base case is first presented, where inter-group preferences are neutral and any emergent spatial segregation only due to income differences.

[6]The CBD is at the origin of the lattice and not the centre for two practical reasons: to reduce computing time, and to show a wider range of processes within smaller images. In the last simulations, we however zoom out to an entire 360° city with a central CBD.

5.3.1.1 Neutral behaviour (Case 1)

The utility function is the following for both W and B (for simplicity subscript ij have been removed):

$$U_W = U_B = U(Z, H, E, S) = kZ^{1-\alpha} H^{\alpha} E(\rho)^{\beta} S(\rho)^{\gamma} \tag{5.1}$$

with $\alpha \in [0, 1], \beta \geq 0, \gamma \geq 0$, and $k = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$

In this utility function, there are two market goods: Z is a composite non spatial good, and H is the housing good. Then, two types of non-market interactions follow from location: $E(\rho)$ and $S(\rho)$ representing neighbourhood externalities. Both are defined as a function of the density of residential cells within a finite circular neighbourhood around the chosen location. Denote ρ this neighbourhood density. Each decision is made according to the neighbourhood density that a household can observe before moving. We have $E^t_{ij} = E(\rho^{t-1}_{ij})$ and $S^t_{ij} = S(\rho^{t-1}_{ij})$. ρ_{ij} being a neighbourhood variable, the structure of the externalities is recursive in space and time, and the model dynamics is therefore to be related to CA.

ρ_{ij} is the ratio of the number of residential cells in the neighbourhood of ij to the surface of this neighbourhood. The surface of a cell being unitary, the surface of the neighbourhood is the number, n, of cells kl that fall within a given radius $\hat{x}$ around ij. The structure and surface of the neighbourhood is constant through space and time[7]. Denote $x_{ij,kl}$ the Euclidean distance between ij and kl. Then with $I_{kl} = 1$ if $C_{kl} \in \{W, B\}$ and $I_{kl} = 0$ if $C_{kl} = A$, the neighbourhood density is

$$\rho_{ij} = \frac{\sum_{kl} I_{kl}}{n} \quad \forall kl, \; x_{ij,kl} \leq \hat{x} \tag{5.2}$$

E represents environmental quality externalities, or the intensity of open-space for which households are assumed to have benefits. Households value positively low density environments, greenness of the landscape, the presence of agricultural activities in their vicinity, etc. E contributes to split up residential settlements patterns because it is a 'local dispersion' force (people wants to move away from each other). But E tends also to reduce the total population of a city at equilibrium and therefore reduce the global urban spread compared to a city without externality (Caruso et al., 2004b, see chapter 3). $E(\rho)$ is a decreasing function:

$$E^t_{ij} = e^{-(\rho^{t-1}_{ij})^{\theta}} \tag{5.3}$$

[7]Within the neighbourhood, a distance decay function can be used to weight differently the cells kl that are closer to ij than others. The neighbourhood density would therefore become a local potential. It is not done however in this chapter. Also, we decide that $\hat{x} = 3$ and therefore $n = 28$. Varying $\hat{x}$ impacts on the size of the residential clusters and leapfrogs.

with $\theta \in]0,1]$, so that the function is convex for $\rho_{ij} \in [0,1]$ so that a neighbourhood density change within a low density neighbourhood is more important than when the density is high.

S represents social neighbourhood externalities. Households are assumed to value high density environments because it provides many individual interactions in their vicinity. It can also be seen as a proxy for neighbourhood public goods (buses, schools, ...). S is a contributor to clustering, it is a 'local agglomeration' force (people wants to be close to each other). Conversely to E, the global effect of S is to increase the total population and the extent of a city.$S(\rho)$ is an increasing function:

$$S^t_{ij} = e^{(\rho^{t-1}_{ij})^\phi} \tag{5.4}$$

with $\phi \in]0,0.5]$, so that the function is concave for $\rho_{ij} \in [0,1]$

Households in ij pay a commuting cost which is linear with the Euclidean distance d_{ij} to the CDB. The budget net of commuting cost is used to buy a composite good Z and housing H, respectively at numéraire price and at R_{ij}, the rent per unit of housing. For $c \in \{W, B\}$ indicating the type of households. The budget constraint is

$$Y_c - a_c d_{ij} = Z_{ij} + R_{ij} H_{ij} \tag{5.5}$$

with the unit transport costs $a_c > 0$.

The Marshallian demand functions for the two market goods are obtained from the maximization of utility (Eq.5.1) subject to the budget constraint (Eq.5.5):

$$\hat{Z}_{c,ij} = (1-\alpha)(Y_c - a_c d_{ij}) \tag{5.6}$$

$$\hat{H}_{c,ij} = \alpha(Y_c - a_c d_{ij}) R^{-1}_{ij} \tag{5.7}$$

At the optimum, the level of utility achieved by an household in ij is given by Eq.5.1, 5.6 and 5.7:

$$V_{c,ij} = (Y_c - a_c d_{ij}) R^{-\alpha}_{ij} E^{\beta}_{ij} S^{\gamma}_{ij} \tag{5.8}$$

We assume that all households of a certain type achieve the same level of utility, u^t_c, in the city at a given moment in time (open city)[8]. The bid rent function, i.e. the maximum value a household is ready to pay for obtaining the

[8]Brueckner (1980) considers an increasing function for utility and income, with income increasing more rapidly. Therefore land prices increase with time. By contrast, the evolution of utility is not given *a priori* in our model but, as will be seen later, simply adjust downward to the external utility. Therefore, for an unchanged neighbourhood, land prices also increase with time as income is a constant. Our model is also different because we start from scratch and not from the standard segregation pattern, and because redevelopment are not considered

level of utility of its group is then derived from the indirect utility (Eq.5.8), and includes local externalities calculated from the neighbourhood density at $t-1$:

$$\Psi^t_{c,ij} = (Y_c - a_c d_{ij})^{1/\alpha} u_c^{t^{-1/\alpha}} E(\rho^{t-1}_{ij})^{\beta/\alpha} S(\rho^{t-1}_{ij})^{\gamma/\alpha} \tag{5.9}$$

5.3.1.2 Segregative behaviour

Households can also be assumed to discriminate people within their neighbourhood. The level of social interactions ($S(\rho)$) is no longer a function of the residential neighbourhood density but depends on the density of a given type of household. Denote $\rho_{W,ij}$ and $\rho_{B,ij}$, respectively the density of W and B cells in the neighbourhood of ij. With $I_{W,kl} = 1$ when $C_{kl} = W$ and 0 otherwise, and $I_{B,kl} = 1$ if $C_{kl} = B$ and 0 otherwise, we have

$$\rho_{W,ij} = \frac{\sum_{kl} I_{W,kl}}{n} \quad and \quad \rho_{B,ij} = \frac{\sum_{kl} I_{B,kl}}{n} \qquad \forall kl | x_{ij,kl} \leq \hat{x} \tag{5.10}$$

Three cases are identified, and the utility (Eq.5.1) and bid rent (Eq.5.9) functions changed accordingly. For simplification, the value of the elasticity parameters (α, β, γ) is kept identical for W and B. (Subscripts $_{*B}$ and $_{*W}$ indicate the type of social preference)

Case 2: W prefer W, B are neutral. W and B behaviour are respectively defined by

$$U_{W*W} = U(Z,H,E,S(\rho_W)) \tag{5.11}$$
$$U_B = U(Z,H,E,S(\rho)) \tag{5.12}$$
$$\Psi^t_{W*W,ij} = (Y_W - a_W d_{ij})^{1/\alpha} u_W^{t^{-1/\alpha}} E(\rho^{t-1}_{ij})^{\beta/\alpha} S(\rho^{t-1}_{W,ij})^{\gamma/\alpha} \tag{5.13}$$
$$\Psi^t_{B,ij} = (Y_B - a_B d_{ij})^{1/\alpha} u_B^{t^{-1/\alpha}} E(\rho^{t-1}_{ij})^{\beta/\alpha} S(\rho^{t-1}_{ij})^{\gamma/\alpha} \tag{5.14}$$

Case 3: W prefer W, B prefer B. The behaviour of W is still defined by Eq.5.11 and 5.13, while the behaviour of B is given by

$$U_{B*B} = U(Z,H,E,S(\rho_B)) \tag{5.15}$$
$$\Psi^t_{B*B,ij} = (Y_B - a_B d_{ij})^{1/\alpha} u_B^{t^{-1/\alpha}} E(\rho^{t-1}_{ij})^{\beta/\alpha} S(\rho^{t-1}_{B,ij})^{\gamma/\alpha} \tag{5.16}$$

Case 4: W prefer W, B prefer W. Again, the behaviour of W is defined by Eq.5.11 and 5.13, while the behaviour of B is now given by

$$U_{B*W} = U(Z,H,E,S(\rho_B)) \tag{5.17}$$

$$\Psi^t_{B*W,ij} \quad = \quad (Y_B - a_B d_{ij})^{1/\alpha} u_B^{t^{-1/\alpha}} E(\rho_{ij}^{t-1})^{\beta/\alpha} S(\rho_{W,ij}^{t-1})^{\gamma/\alpha} \qquad (5.18)$$

5.3.2 Land and residential conversion dynamics

The use of a cell at time t is ruled out by the land market where the bids of farmers and households meet. The dynamic functioning of the market is subject to a maximum population growth rate. Moreover all agents are supposed to be myopic. Especially, landowners do not anticipate urban growth, and households do not anticipate changes in their neighbourhood after the time of their location. Their decision at time t is only based on the observation of the city at time $t-1$, as defined through the temporally lagged externalities in the bid-rent functions.

The city is assumed to grow at a given rate, g. The production of residential space follows a 'putty-clay' technology, so that a residential cell cannot return to agriculture. A maximum of g new agricultural cells are therefore converted into an urban use at each time step. More precisely, at time t, a maximum of g_W new W cells and g_B new B cells is allowed, and $g_W + g_B = g \geq 0$. Both population rates g_W and g_B are given but become null as soon as it is not possible for a W or B household to overbid any of the two other agents in any place and enjoy at least the level of utility outside of the city region. The external utility levels are given parameters denoted by $\bar{u_W}$ and $\bar{u_B}$, with $\bar{u_W} > \bar{u_B}$. g_W (or g_B) can also become negative when W (or B) can no longer bid over farmers on undeveloped cells and when, in the meantime, population B (or W) grows at the expense of W (or B) rather than at the expense of agricultural land. g_W and g_B are therefore partly endogenous, depending on the value of competing bids and on the persistence of a utility surplus. In fact, as long as a household is indifferent to occupying at least two agricultural cells in the city-region, landowners are in competition. Therefore, the new migrating household will pay a rent that is below his bid. The difference between the bid rent and the effective rent represents a utility surplus with respect to the utility in the rest of the world, which is a good reason to migrate [9].

Denote Δu^t_W and Δu^t_B the utility surplus of W and B at time t, with $\Delta u^t_W = u^t_W - \bar{u_W}$ and $\Delta u^t_B = u^t_B - \bar{u_B}$. These are the incentives to immigrate, and changes in the land use pattern occur as long as $\Delta u^t_W > 0$ or $\Delta u^t_B > 0$. As soon as $\Delta u^t_W = \Delta u^t_B = 0$, a *long-run equilibrium* is reached and any residential movement stopped. The long-run or migration equilibrium is denoted by t^*. At t^*, the land and residential uses are fixed, as well as population (density) within cells and rent values.

[9] The system is equivalent to what was presented in Caruso et al. (2004b). It is however more complex because in absence of available agricultural cells, one agent type can choose to replace another household type. Internal migration (and filtering) is permitted. See details in the appendix.

At $t0$ all cells are agricultural. At time $t1$ and the next t steps, g_W and g_B new households enter the city and choose a location ij that maximizes their utility in t. Four main situations are identified and can follow each other, whatever the order, during the simulation.

- (I) W and B urbanisation. This is the simplest situation which corresponds to an urbanisation phase by both groups. The g_W and g_B cells that are best suited to W and B households are in agricultural use.
- (II) Filtering down and W urbanisation. In this second situation, one or more of the g_B new cells replace a W resident. The evicted W resident will also have to find a new location, and in turn replace another household or occupy an agricultural cell. Eventually, the consecutive evictions of W households lead to the occupation of one or more additional agricultural cell by W at each time of this phase as long as there is an agricultural cell where $\Delta u_W^t > 0$.
- (III) Filtering up and B urbanisation. This case inverses the second. Some B cells are replaced by W households while the B group develops on agriculture as long as $\Delta u_B^t > 0$ can be found on an agricultural cell.
- (IV) No urbanisation with filtering up and/or down. In this fourth case urbanisation is stopped while either g_W and g_B cells interchange their use[10], either one of the two groups tends to disappear.

The whole process can be simulated by computing reservation bid rents that can replicate the bidding system within a formalism close to a constrained CA model. A cell in B or agricultural use at time t-1 will be converted into W if (i) it belongs to the g_W highest W bid rents which can be found throughout the region (and maximize utility), (ii) the W bid rent is greater than the agricultural one, and (iii) the W bid rent is greater than the B bid rent or if the W bid rent is lower, the cells does not belong to the g_B cells chosen by B. The same holds for B while Farmers are passive actors and their land can only decrease.

All residential bid rents are calculated in the simulation as a function of $\bar{u}$ and are given by Eq.5.9 for W and B in the neutral behaviour case, and Eq.5.13 and Eq.5.16 or Eq.5.18 for the segregative cases, where u_W^t or u_B^t are replaced by $\bar{u}_W$ $\bar{u}_B$. The system approximates a utility surplus maximization when it is assumed that several cells among the agricultural cells can provide the highest level of utility at a given time or that the difference between the first and the second best location can be overlooked (see appendix for details).

[10]The city re-arrange with a constant number of each residential group. However, the city is not closed, but there is an in-migration that is compensated by an out-migration.

5.4 Simulations

5.4.1 Scenarios

Several simulations have been undertaken for each of the four behavioural cases, and shown in Fig.5.2 to 5.7[11]. The same set of parameter values has been used so that the figures can be compared. The aim of these simulations is twofold: (i) analysing the spatial structuring of B and W households, and agricultural activities in space and time when varying the type of social preferences; (ii) analysing the impact on spatial patterns of three possible public policies that aim to reduce segregation by assisting B households in their location decision.

It is assumed that the preference parameters for land consumption (α) and the unitary transport costs (a) are equal for each group. In these conditions and because of the income difference, utility levels must be different to ensure the presence of both household classes. The two groups then sort according to the steepness of their bid rent curve, with B being closer to the CBD. According to the chosen parameter values[12], the B-W border for the classic long-run equilibrium (without externalities) would be at distance 15.00 from the CBD, while the urban fringe would be at distance 23.87. In the cellular setting, this long run equilibrium is achieved at time 279 with 279 W cells and 192 B cells. Up to time 192, a filtering down process occurs at the B-W order, while new W households locate at the periphery of the city. However, the two externalities will lead to changes in the distribution and dynamics. The standard case is presented below for comparison purpose (Fig.5.1, and B.1 in appendix for the reservation bid rent profiles).

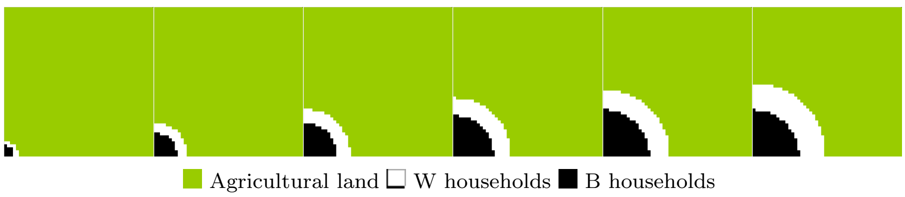


Figure 5.1: Spatio-dynamic patterns in the monocentric case with two household groups and no externality (t10, 50, 100, 150, 200, t^*471)

The first two simulations have been conducted to understand the dynamics of spatial patterns. We start with a simulation where the environmental amenity preference is null ($\beta = 0$) to explore segregation within a concentric growth

[11] A dynamic versions of these figures (or part of them) is available in the enclosed CD

[12] $\Phi = 2.00$, $Y_W = 10.00$, $Y_B = 7.50$, $\bar{u_W} = 6.00$, $\bar{u_B} = 4.17$, $a = 0.12$, $\alpha = 0.25$, $\beta \in \{0.00, 1.25\}$ depending on the case, and $\gamma = 1.00$

without assuming a sprawling morphology. All new developments should be contiguous to the previously built locations and the city compact at least when there is no segregative behaviour. We focus on the relative position of B and W households during the growth process and at long-run equilibrium, and on the shape of the B-W border when the social preference is changed.

Environmental amenities are then introduced. With a single household type, this externality can shape urban developments in a sprawling manner with leapfrogs and stripped settlements. Because the taste for environmental amenity is chosen to be stronger than for social amenity ($\beta = 1.25 > \gamma$ for both groups), the global externality can be negative when the residential neighbourhood density is too high, leading to local dispersion and more fragmentation of the rural area. We particularly analyse the distribution of rural cells within the urban area, because these cells can provide low local densities to households and could also help residents in arranging their social preferences.

Secondly, three public policies have been simulated and reflect observed policies in the USA or in Europe[13]. The first two policies apply on B individuals and increase their income net of transport: B households are either granted a subsidy for income either for transport. These subsidies reduce the effect of income sorting on the housing market with respect to the effect of social neighbourhood preferences. They also represent an incentive toward more suburbanisation of B households. The rationale for a transport subsidy resides in the fact that income sorting has been amplified by extended suburbanisation. Poorer groups, with lower car ownership rate, have been kept within central cities, far from suburbanising jobs (see the spatial mismatch hypothesis (e.g. Kain, 1992; Yinger, 1995; Arnott, 1998) and sometimes within declining environments due to the out-migration of more well-off people. Favouring the mobility of low income groups is considered as a possible solution for reducing a part of this mismatch.

The third policy scenario is a global policy which restricts W households to take over a location from B or farmers depending on the existing level of residential mix within the neighbourhood. Similar inclusion policies have been implemented in US or European cities[14] The literature has also emphasized the negative effect on the socio-economic outcome of low income segregated people of the spatial segregation and the lack of a neighbourhood environment which can offer valuable contacts (e.g. Cutler and Glaser, 1997).

Finally, the integration of our three policy scenarios within a single metropolitan area is considered by the last simulation. It is assumed that these policies

[13]While there is a normative aspect, we do not presuppose any value judgement but observe the spatial impact of policies for given preferences

[14]Moving to Opportunity program, implementation of quota of social housing within communities, the concept of undivided cities, sometimes W are maintained within central B areas, Dutch system.

are not centralised. The city is made of four sub-systems and migrations are possible within these four districts. Therefore, the policies may not have the same impact as in the previous closed systems on each individual district. The districts can also encounter differentiated urbanisation growth as a result of the choice of new migrants. Moreover, while the diversity of agents is small and there is no stochastic perturbation this might be a source of greater spatial heterogeneity that is likely to happen in real cities and needs to be explored.

5.4.2 Resulting spatial patterns

5.4.2.1 Scenario with no open-space amenities: compact city

In the first set of simulations, both types of household benefit from social amenities only ($\gamma = 1.00$, $\beta = 0.00$). They do not wish to have rural cells in their neighbourhood but to agglomerate with other people. The higher the level of local density (ρ), the higher the bid rents (see Eq.5.9 and the appendix for the profile in Fig.B.2). Density can therefore compensate for extra commuting and the city expands more than in the base case. Spatial patterns through time are presented in Fig.5.2 for different variations of social amenities.

The different tiles are not taken from the same time steps from case to case. Therefore, comparisons between cases must be carefully made. The phasing of population and time vary from case to case because of the open-city assumption and the different preferences and parameters chosen. The time steps that are presented have been primarily selected in order to show the main qualitative bifurcations in the spatial patterns of each simulation. However, when possible, similar time steps (e.g. $t = 100$, $t = 500$, $t = 1000$,...) have also been shown to allow for some comparisons between cases. The last tile in a row is always de long-run equilibrium, t^*. For a similar urban extent, the time of the final equilibrium gives an indication of the amount of internal migrations that occurred. For similar time steps (e.g. tile $t = 1000$), comparing the urban extent is also informative of these internal changes.

Case 1: Neutral behaviour. (see Fig.5.2a). The first steps of the growth with neutral behaviour of W and B are similar to the base case, with B filtering W at the B-W border. This pattern lasts until time 192 (the long run equilibrium population of B in the base case). From that time, W households continue to grow similarly to the base case while B, instead of stopping their growth, leapfrog the W ring to settle beyond. There, they can now bid over farmers because of the presence of other residents (firstly W and then B). In the meantime, while the filtering down is stopped at the first B-W border, filtering up occurs at the second border. Finally, the B growth at the periphery is contained by the level of the rural rent. A maximum number of B cells is achieved at time 689 and then B starts to decrease. B households remains only in the central core and the number of B cells at long-run is similar to the base case (192). W residents continue to replace peripheral B cells and in final stages urbanise new cells. The maximum expansion of the city is attained at time 2011. Conversely to a classic concentric growth, the contact zone between W households and rural cells tends to flatten. This spatial arrangement allows for more residents to benefit from social amenities (the stronger the curvature, the more rural cells in a neighbourhood at the fringe).

Case 2: W prefer W, and B are neutral. (see Fig.5.2b). Like in the previous case, the growth is concentric but the long run equilibrium is achieved earlier (at time 1167) indicating less filtering. At t^*, there are 1167 B and 1037 W. The last migrants are therefore B households who push the B-W border further away because they have less restrictive social preferences. Despite the presence of other W, W cannot bid over farmers at longer commuting distance than in the previous case (as there is no B households at the periphery, Eq.5.13 is equivalent to Eq.5.9). The city expansion is therefore the same but the internal structure is different. Moreover, one can see that the B disk is circular until the last moments, where it starts to flatten. Here as well a smaller curvature allows for W to have more contacts with W, they therefore contain the B growth in a more compact pattern. This situation, however, represents some kind of a paradox for B households because the B fringe is not equidistant to the CBD, while the level of externalities is the same for them at this fringe (as they are indifferent to the type of people that reside in their neighbourhood).

Along the whole process, the pattern is always made of two rings. B filter W cells but never leapfrog these cells like in the neutral case[15]. B have no interest here to go in the periphery and endure higher commuting costs because, within the urban environment at the B-W border, they gain more externalities than W households. This gain compensates for the income differential. Finally, at the very beginning of the simulation, the pattern deviates from the classic circular shape because W avoid to be in the presence of the first B households. When a choice is made by B between two similar cells, this choice then orientates the next few B locations. In fact B can more easily bid over W in the direction of the first locations as W disregard agglomeration to B. The first 25 steps in this example are directed to the north. This pattern exemplify a small impact of path-dependency.

[15] This example shows that it is necessary to acknowledge the limitations of numerical simulations in this case as there is no theoretical reason for such a qualitative difference between case 1 and case 2. A much wider set of parameter values should be tested in order to define bifurcation points and therefore provide a more robust description of the patterns. Our analyses are limited to show the direction of changes with respect to the different interplaying

(a) Case 1: Neutral behaviour (t100, 300, 700, 1000, 1500, t^*2011)

(b) Case 2: W prefer W, and B are neutral (t25, 200, 400, 700, 1000, t^*1167)

(c) Case 2bis: W like W and reject B, and B are neutral (t20, 100, 500, 1100, 2150, t^*2204)

(d) Case 3: W prefer W, and B prefer B (t50, 500, 825, 1000, 1300, t^*2081)

(e) Case 4: W prefer W, and B prefer W (t100, 125, 150, 1000, 1150, t^*2011)

■ Agricultural land □ W households ■ B households

Figure 5.2: Spatio-dynamic patterns for different social preferences with no open-space amenities (compact city)

Case 2bis: W **prefer** W **and reject** B**, and** B **are neutral.** (see Fig.5.2c). In case 2, the W preference for being close to other W households is not strong enough to physically separate W and B. The positive preference for a given neighbourhood type cannot generate the leapfrog of a rural cell if there are no open-space amenities. Case 2bis therefore explores this possibility by assuming a stronger segregative behaviour. The aversion of W for B households can be formulated by a function similar to the crowding or open-space externality, i.e. E. Define the aversion externality by $S_{rac} \equiv E(\rho_B)$ and replace Eq.5.11 and 5.13 by

$$U_{W*Wrac} = U(Z, H, S_{rac} \equiv E(\rho_B), S(\rho_W)) \quad (5.19)$$

$$\Psi^t_{W*Wrac,ij} = (Y_W - a_W d_{ij})^{1/\alpha} u_W^{t^{-1/\alpha}} E(\rho^{t-1}_{B,ij})^{\beta/\alpha} S(\rho^{t-1}_{W,ij})^{\gamma/\alpha} \quad (5.20)$$

With these equations, W still prefer W but also reject B. E is now to be interpreted as a disamenity due to the presence of the other group. Results (with $\beta = 1.50 > \gamma = 1.00$) show that leapfrogs can occur but are not stable. One can see from the first two periods presented in Fig.5.2c that W households leapfrog rural cells in order to avoid B people. However, because of the commuting constraint and as soon as W can find more W at the periphery, they regroup themselves, and no new leapfrogs appear. Moreover the filtering down process (as in the previous case) leads to the rapid occupation by B of the W cells that just leapfrogged rural areas. Although commuting might be longer, B prefer to occupy these cells in order to benefit from the urbanisation overvalue. Later in the development, W behave as in the previous cases by occupying new rural cells at the urban fringe until the maximum urban expansion is reached. However the 'racist' behaviour of W residents weakens W on the housing market. Therefore as W reject B households and because social interactions with other W do not compensate enough for this loss, W households disappear from the city in this example. B households eventually fill in the leapfrogged rural cells as they do not care about open-space.

Case 3: W **prefer** W**, and** B **prefer** B**.** (see Fig.5.2d). The long-run equilibrium in this case is very close to the neutral behaviour case with 123 B and 2081 W cells. There are however fewer B people as the equilibrium pattern is marked by the orientation taken in the very first steps. These first steps are characterized by B and W households who grow jointly on rural cells and toward a single direction (north) with B being closer to the CBD. The classic concentric growth pattern is not found because both groups try to avoid the other group and agglomerate with similar households. The local agglomeration externality compensates for greater commuting costs (i.e. the regional agglomeration force).

forces but cannot define threshold values

However, as the time goes on the pattern recovers a concentric shape: W grow eastwards while B occupy a ring at the periphery step by step from the north clockwise (see the second tile in Fig.5.2d). From about time 1000, the pattern and dynamics are very similar to the neutral behaviour case. A filtering up process makes the B peripheral households disappear. However, contrarily to case 1, the second W-B border (e.g. at t=1000) is flattened because each group seeks to avoid the other group.

Case 4: W **prefer** W**, and** B **prefer** W**.** (see Fig.5.2e). The final equilibrium situation corresponds exactly to the neutral behaviour case. This is because the social externality is the same for both household types and, therefore, cannot further differentiate the groups at t^*. Like in the first case, a ring of B cells emerges at the periphery of the city, which gives an intermediary structure made of tree concentric rings. However, contrarily to the first case and because B do not want to agglomerate within themselves, the outer B ring is thinner and does not go as far ahead of the W cells growth. Moreover, the short-run dynamics is quite different in the first steps. The preferences generate a continuous attraction-repulsion process and therefore the land use in entire quarters of the city invert every other time steps (these inversions are shown in the first three tiles using even and odd periods of time). This strong instability is rubbed out by the time, with the increase in commuting costs that forces B households to occupy the whole centre and squeeze out W households. The cyclicity that is generated is highly unrealistic but emphasizes attraction-repulsion processes. We can assume that in the reality these tensions are hidden by an important inertia of locations. Not taking into account this inertia is a weakness of the model. Moving and changing lot size or quality are allowed at no cost. However the model represents a long-run adjustment of the city and the inter-step timing is not defined. Moreover, including a moving (and/or renovation) cost would be able to avoid the cyclicity of patterns, depending on the value of this cost. The lower the cost, the more the situation tends to case 4. The higher the cost, the more difficult is the filtering process for residing people. The case would thus be closer to case 2 because B households do not like to agglomerate with other B.

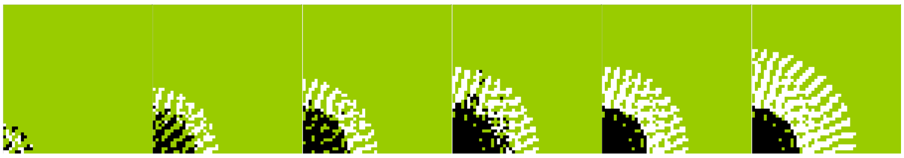
We synthesize the main findings of this set of simulations below:

Result 5.1. *(5.1.1.) Under the presence of social amenities, the city is more expanded and shape into a flattened manner. (5.1.2.) B households always occupy the centre at long-run equilibrium. Whatever the type of social preferences, the maximum extent is the same at long-run equilibrium. (5.1.3.) During the growth process B can additionally occupy a peripheral ring when their preference are as restrictive as the preference of W (case 1 and 3). Conversely* *(case 2 and 2bis) when the preference of B is less restrictive, the B-W border is pushed away from the centre (cumulative decay) and flattens because of W preferences. (5.1.4.) Cities are always compact agglomeration except if households state racist preferences (case 2bis). These leapfrogs are however unstable as they disappear at long-run. (5.1.5.) path-dependency can impact on intermediary patterns (case 2 and case 3) by deeply changing the concentric rings structure. These effects can be taken up to the long-run equilibrium (case 3). (5.1.1.) A strong instability appears at the beginning of a simulation when B households prefer neighbourhoods of W households, while these households discriminate B (case 4). This cyclicity does not affect the long-run equilibrium.*

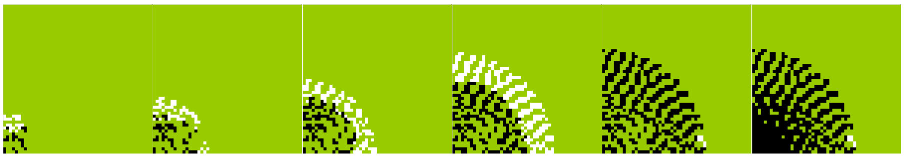
5.4.2.2 Scenario with open-space amenities: scattered city

Both groups now balance social externalities with open-space amenities ($\beta = 1.25$, $\gamma = 1.00$). Therefore, crowding is a disamenity and local agglomeration is limited. Because of this disamenity, the bid rent curve (see Fig.B.3 in appendix) for an optimal local density cuts the rural rent closer to the city centre and therefore city expansion is limited relative to the compact city case. However, for a population similar to the classic monocentric case (Fig.5.1), the city will expand further because of leapfrogged rural cells.

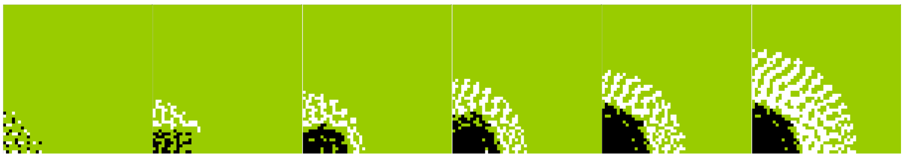
Case 1: Neutral behaviour. (see Fig.5.3a). The long-run equilibrium is close to the classic case with no externalities (Fig.5.1). At t^*, there are 454 W and 178 B, but the maximum commuting distance is 35 instead of 23 because of the scattered aspect of urban developments. B households are confined within the centre and therefore have less benefits from open-space. From a dynamic point of view, both types grow in a diffuse manner in the very first steps and the income stratification is not very marked and depends much on neighbourhood quality. The spatial separation is clearer as the time goes on and superimpose on a scattered morphology. The densification of the centre from about $t = 100$ with B households occupying previously leapfrogged rural cells as they can no longer compete at the periphery. Before $t = 230$, both groups grow and immigrations are made of filtering up and down near the B-W border, and rural cells conversion. From $t = 230$ to $t = 290$, W do not develop new rural cells because of the commuting constraint, but replace B households who have located in the outer ring thanks to environmental amenities. Therefore, the number of B and W cells is identical up to 230, but then diverges with the number of B cells declining stepwise to 178. Until $t = 290$, the external fringe of the city is constant, then, once filtering up in the outer ring has stopped and thus once the B-W border is fixed, W continue to convert cells at the periphery until the long-run equilibrium.



(a) Case 1: Neutral behaviour (t20, 100, 150, 230, 290, t^*454)



(b) Case 2: W prefer W, and B are neutral (t20, 50, 100, 200, 450, t^*611)



(c) Case 3: W prefer W, and B prefer B (t20, 50, 100, 150, 200, t^*432)

(d) Case 4: W prefer W, and B prefer W (t50, 55, 150, 200, 300, t^*449)

Agricultural land ☐ W households ■ B households

Figure 5.3: Spatio-dynamic patterns for different social preferences with open-space amenities (scattered city)

Case 2: W prefer W, and B are neutral. (see Fig.5.3b). Like in the compact case, the W preference for W households benefits to B households on the housing market. Because the B social preference is less restrictive, B gain more cells and therefore increase the repulsion of nearby cells for W. The ratio of W to B cells diminishes. At the final equilibrium W can even disappear[16]. Like in case 1 above, their commuting field is limited as compared to the compact city simulation because of the crowding disamenity. More specifically, t^* is characterized by 6 W and 611 B cells. The presence of some W households in a small cluster shows that we are not too far to having a whole ring of W at t^*. In this example, W households have found, by chance, a morphological niche at the fringe that allows them to resist to B newcomers. This atypical spatial pattern exemplify again the effect of path-dependency. The dynamic is also particular. W flee from B, but B in order to benefit from environmental and social amenities as well as from urbanisation overvalue follow W instead of building on rural cells. B structure themselves by copying the development of W before occupying the centre. A kind of 'green heart' is generated, which is not an optimal local arrangement for B (see the previous case). From time 450, B starts to migrate to the inner core because commuting is too long at the periphery and they cannot bid over the remaining cluster of W. They occupy the core firstly by leaving some rural cells and secondly by densifying the locations at shorter distances.

Case 3: W prefer W, and B prefer B. (see Fig.5.3c). At t^*, the situation is very close to the neutral case as both groups have a similarly restrictive social preference. Three important differences with the neutral case can be emphasized. First, tin the first steps the role of distance in the segregation pattern is less important. It is clear from the second tile for example that the city is organised into two clusters and not as concentric rings. This was also the case in the compact city scenario. Second, it is more difficult for B households to locate into the W periphery as it was the case for intermediate equilibria with neutral behaviour (see Fig.5.3a, fourth tile). This is due to the fact that B are not opportunist households and only agglomerate to other B

[16]The disappearance or not of a household group can be fixed exogenously, e.g. by choosing other utility levels (open-city). The fact that a long-run equilibrium is only composed with one type of households is however not the focus of our analysis. The model is aimed to explore how W (or B) are replaced through time rather than to what spatial extent. In fact, the latter is fixed from exogenous parameters and can be deduced from the reservation bid rents (see graphs in appendix). The only unknown from the graphs is whether or not optimal densities can be achieved at a given distance (i.e. what is the highest bid rent curve), as it depends on the type of local structures and social environments that can be found in each distance. Choosing parameter values in order to keep both population in long-run in any simulations is thus possible although not known exactly. Moreover, the choice of the parameters is also part of a trade-off: if parameters were chosen in order for W to be present in all long-run equilibria, less filtering processes would be shown, or all $t*$ cities would have been more expanded and thus all simulations would have required more computation.

people. W therefore surpass B more easily in a W environment. Third, the spatial separation between B and W is made of a thin rural belt. Both agree for such a spatial arrangement as they need greenness and group only with similar households. A small 'green belt' (not continuous and small because of the size of the neighbourhood and the weight of the greenness preference) appears endogenously. This is an important feature of this simulation that lasts during the whole utility adjustment period and at equilibrium.

Case 4: W prefer W, and B prefer W. (see Fig.5.3d). Like in the compact city case, a strong cyclicity is generated by these behaviours. Again the cyclicity is confined in the central part. Indeed this would mean that central parts of the cities are much more subject to important and repetitive turnovers than the periphery. If this phenomenon is not observed in reality this would also mean that the inertia is stronger in the centre than in the periphery, i.e. internal mobility more difficult. The instability of the centre decreases with time as B households occupy the whole centre. W tends to occupy the periphery but are directly followed by B households. At the third and fourth tile, one can see that B and W follow each other in a clockwise manner within a ring situated between a B centre and a W periphery. This filtering loop (up, down, and up) along an inner perimeter is the main dynamic characteristic of this simulation. From about $t = 300$ this dynamics stops, B housing is constant and W grow on agricultural land. W finally expand the city as in the non-discriminating case. At equilibrium however, the number of B cells is reduced because agglomeration with other B is a disamenity for B newcomers and because there is no W in the centre. Interestingly therefore, the land use pattern along distance perturb the classic density decreasing pattern, with a scattered B centre being followed by a densely urbanised W ring and then scattered W developments.

These findings are summarized below:

Result 5.2. *(5.2.1.) Under the presence of social amenities and open-space amenities, scattered developments arise at the periphery of the city. (5.2.2.) Like in the compact city case, B households always occupy the centre at long-run equilibrium. Whatever the type of social preferences, the maximum extent is the same at long-run equilibrium and this extent is shorter than in the compact city case. (5.2.3.) Contrarily to the compact city case, no intermediate equilibrium can be found where the city is structured in more than two rings. However when the preference of B is less restrictive (case 2), the B-W border is also pushed away from the centre and could lead to the disappearance of W (cumulative decay). (5.2.4.) Non-concentric long-run patterns can arise because of path-dependency and specific local spatial arrangements (case 2). path-dependency can also generate non-concentric intermediate patterns (case 3).(5.2.5.) The* *urban density to distance decreasing function can be disrupted in short-runs because of a slowest urbanisation of the centre (case 2) or in long-runs either because of the presence of a thin green belt at the B-W border (case 3), either because of a scattered arrangement of the centre (case 4). (5.2.6.) Like in the compact city case a strong instability appears at the beginning of a simulation when B households prefer neighbourhoods of W households, while these households discriminate B (case 4). The cyclicity does not affect the long-run equilibrium. During intermediary steps, these behaviours generate a filtering loop within an inner ring.*

5.4.2.3 Transport and income subsidy policies in a scattered city

In these scenarios (see Fig.5.4 and 5.5), B households have a new budget net of transport. A decrease by 16.7% of transport cost ($a = 0.10$ instead of $a = 0.12$) and an increase in income by 13.3% ($Y = 8.50$ instead of $Y = 7.50$) are modelled. The profiles presented in Fig.B.4 and B.5 of the appendix show the changes of the B bid rents while the W curves remains unchanged (i.e. the same as in the scattered city case, see Fig.B.3). Despite a lower income subsidy, the graphs show that W households might be more easily evicted from the city in the income subsidy scenario than with the transport subsidy scenario. In optimal local density conditions (see $\rho = 0.20$, which is quasi optimal if behaviours are neutral) the B curve meets the rural rent at a distance > 35 with income subsidy, and < 35 with transport subsidy. 35 being the maximum commuting distance for a W household in optimal neighbourhood conditions (see ρ^* in chapter 3). This structure will however depend on whether the households will be able to arrange spatially in order to find optimal neighbourhoods, and therefore on the type of social preferences. With the income subsidy, the B bid rent is much higher for locations close to the CBD but is also more rapidly decreasing. With the transport subsidy, the B bid rent remains closer to the level of the W bid for a longer distance interval. The role of the different social amenities is therefore potentially more important in this case for shaping mixed or complex patterns.

Case 1: Neutral behaviour. (see Fig.5.4a and 5.5a). The impact of both policies is similar in terms of patterns and dynamics when B and W do not discriminate. For the reasons explained above, a W ring persist at long-run equilibrium in the case of the transport subsidy. (Of course, it could also persist with an even lower income subsidy). The spatial extent of the city is therefore the same in this case than without any subsidy, but the B-W border is pushed away from the centre. The long-run stage appears later as B filter more of the existing W cells. The number of W cells is reduced as in the meantime, the maximum commuting distance of W is reached. In the income subsidy case, after the filtering down of the whole periphery, B go on a little further on rural cells and therefore the city is more expanded at equilibrium. In both scenarios, but more in the transport case, intermediate equilibria show a mixed area between the B centre and the W periphery. This mixed area lasts during a longer period of time than without any subsidy to B. The two policies therefore appear to be useful as de-segregation policies, but in short-runs only.

Case 2: W **prefer** W**, and** B **are neutral.** (see Fig.5.4b and 5.5b). The patterns resemble strongly the case with no subsidy. The only difference is that no W persist because it is even more difficult for W to find a niche arrangement at the periphery, given the higher bids of B.

Case 3: W **prefer** W**, and** B **prefer** B**.** (see Fig.5.4c and 5.5c). In these situations whit a stronger bidding power of B and segregative behaviour form both groups, the patterns generated are much more intriguing. The long-run pattern under the income subsidy scenario is again the same all black pattern as in case 1 and 2 preferences. However, the dynamics is different. A kind of filtering loop appears at the B-W border, with B households following W. As there is no preference for such a neighbourhood (contrary to case 4), the phenomenon is here explained by the urbanisation advantage and by the cumulative process that filters down the neighbourhood at the B-W border, itself emphasized by the fact that in this zone, B households bid higher than in previous scenarios. The process leads to the progressive disappearance of W but not in a concentric way (in this example, W remains longer in the eastern part).

Under the transport subsidy, the situation this time is very different than under the income subsidy. The emerging long-run pattern is quite independent from commuting distance and marked by path-dependency. A B scattered outer periphery develops stepwise but is not designed as a ring. Starting from the northern periphery, this B development constraint W peripheral developments into a single direction (eastwards). Therefore, the equilibrium at t^* is characterized by a B densely urbanised core, surrounded by a density decreasing W inner periphery, a W outer striped periphery in the eastern part, which is connected to the previous W settlements, and, finally, a B scattered periphery in the north, which is completely disconnected from any previous urban developments. A neighbourhood-wide rural ribbon traces the limit between the W inner periphery and the B outer developments. The rural area is not circular. Moreover, B at the periphery first organize in parallel to the green ribbon in order to benefit from this open-space, while further away B structure into striped radial settlements like in the previous cases. It can also be noticed that the first B-W border, near to the centre, is also mostly made of rural cells.

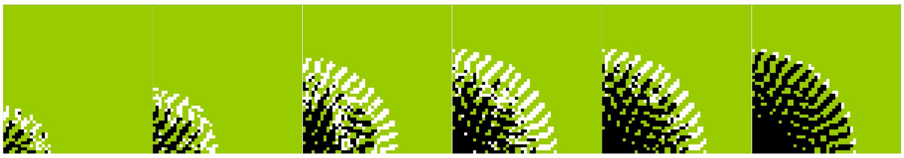
Case 4: W prefer W, and B prefer W. (see Fig.5.4d and 5.5d). Again this case shows a strong cyclicity which in this case, is not limited to the core part of the city anymore. In the income subsidy case, B and W quarters are generated on each side of the axes bisector. Again, a green ribbon emerges but radially this time. Eventually (last two tiles) B occupy the whole city but do not further densify the settlements or urbanise green areas, as density is always a disamenity to B in the absence of W. The region contains therefore less urban cells than in the three other cases. In the transport subsidy case, no stable equilibrium is found. A thin W periphery remains like in the neutral behavioural case, but the rest of the urban cells change from B to W every other step. The last two tiles (t 311 and 312) show this two period equilibrium. In this gain again, the spatial separation of residential groups is made of green cells.

It is clear from these simulations that the two individual policies proposed are not de-segregation policies as they are not able to produce a mix of income groups. However, the simulations emphasize that spatial segregation can also arise in non-monocentric structure although the framework is monocentric. This phenomenon is shown by the emergence of a B and a W outer periphery with a B core and a W 'suburb' in Fig.5.4c. We derive a third Result from these simulations

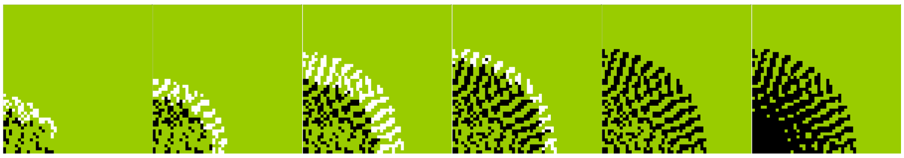
Result 5.3. *(5.3.1.) Increasing the level of income or decreasing the unitary commuting cost of B households pushes the B-W border away from the centre and reduces the number of W cells. (5.3.2.) The spatial separation of W and B is concentric as long as B has no restrictive preference (case 1 and 2). The design of the B-W border at short- and long-run equilibria can be non-concentric when the bidding capacity of B is increased and B also state social neighbourhood preferences (case 3 and 4). (5.3.3.) The impact of decreasing transport cost for B affects more the separation pattern than a proportional increase in income. (5.3.4.) With opposite preferences (case 3) the spatial separation of the two groups is made of green belts but also non-concentric green ribbons that disrupt the decreasing urban density to distance function. (5.3.5.) When the bid rent curve of B is made closer to the bid rent curve of W and when both have a social preference for W (case 4), a long-run equilibrium can be achieved with a stable number of urbanised cells but cyclical segregation patterns.*

5.4.2.4 Social housing scenario

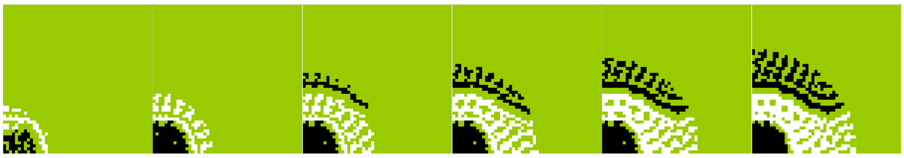
This scenario intends to induce spatial mix of household at a more micro-scale. A 'social' B cell is built as soon as the ratio of W to B urban cells within a defined neighbourhood reaches a given threshold. One can already note as a critique of this model that no account is made of the size and type of social



(a) Case 1: Neutral behaviour (t50, 100, 250, 350, 450, t^*629)



(b) Case 2: W prefer W, and B are neutral (t50, 100, 200, 350, 450, t^*664)

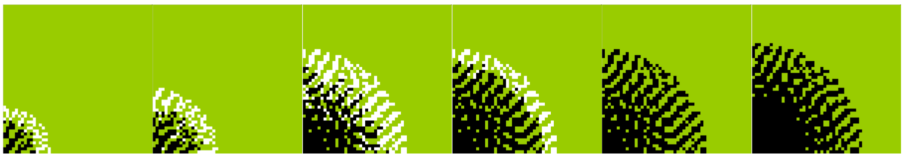


(c) Case 3: W prefer W, and B prefer B (t50, 100, 150, 200, 250, t^*311)

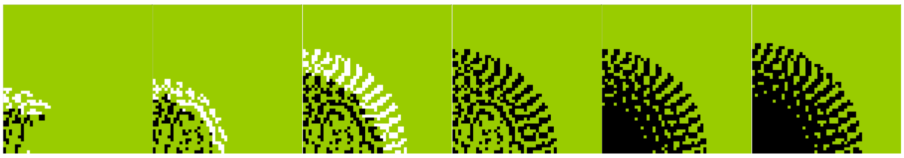
(d) Case 4: W prefer W, and B prefer W (t25, 30, 150, 155, 311, 312 (no t^* but cyclical urban equilibrium from 311))

Agricultural land W households B households

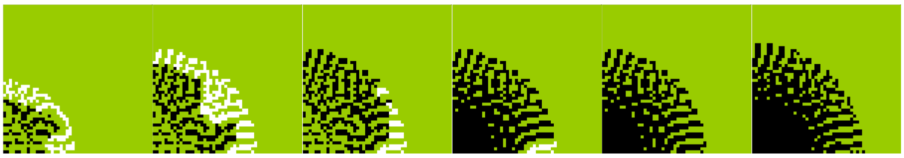
Figure 5.4: Spatio-dynamic patterns for different social preferences and transport subsidy within a scattered city



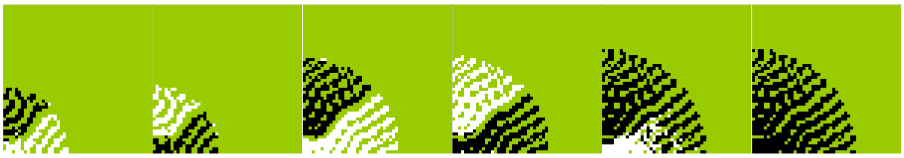
(a) Case 1: Neutral behaviour (t50, 100, 350, 500, 630, t^*816)



(b) Case 2: W prefer W, and B are neutral (t50, 100, 220, 430, 700, t^*819)



(c) Case 3: W prefer W, and B prefer B (t100, 250, 400, 650, 680, t^*819)



(d) Case 4: W prefer W, and B prefer W (t100, 105, 250, 255, 500, t^*616)

Agricultural land W households B households

Figure 5.5: Spatio-dynamic patterns for different social preferences and income subsidy within a scattered city

housing considered. In reality, if some social housing types have relatively few impact on the landscape and the density perceived by the households, others, e.g. with greater height (typical of French suburbs), might have greater impact. This is a drawback of simplifying the model to few processes in a cellular setting. This is also to be related with the assumption of vertical increase in housing when intra-cell population increases (as it is the case for all simulations of chapter 4 and 5).

In the neutral case (see Fig.5.6a), this results in the creation of social cells in a spotted manner within the W stripes all along the city growth. The social housing spots arrange in a very systematic way in order for each neighbourhood to benefit from its presence. The general structure remains unchanged: a core B area more densely urbanised than the W scattered periphery.

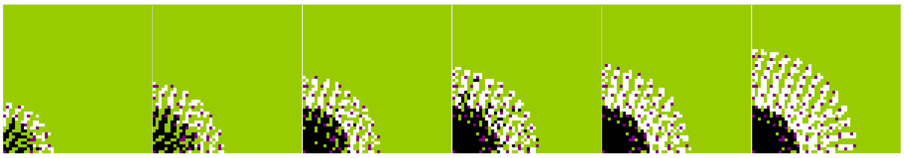
In case 2 where W prefer W while B are neutral (Fig.5.6b), the global dynamics is similar to the previous scenarios. However, the generation of social cells disturb the spatial diffusion pattern. Leapfrogged rural cells still appear but arrange in a different, less systematically organized manner. The same holds for case 3 (Fig.5.6c) when both household types prefer to agglomerate with their own group. The structure of the periphery is disturbed by the social cells. Moreover the shape of the B urban core is also modified. The core tends to a square shape, which is surprising when a radial commuting distance is considered. As seen before, a flattened separation limits the contact between groups, while the presence of social housing in the W periphery nearby the B-W border tends to increase the social amenity for B, who thus can afford longer commuting. Both processes explain the square-like B pattern. In case 2 nor 3, social housing at the periphery is able to induce a local B agglomeration although the social cells represent a local attractor to B.

Finally, one can see that social housing de-structure less the spatial arrangement of W and B in the fourth case (Fig.5.6d). Patterns and dynamics are similar to the previous scenario with no subsidy (Fig.5.3d).

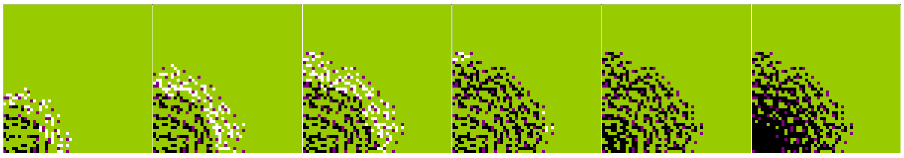
Result 5.4. *(5.4.1.) A policy based on neighbourhood B/W share to construct social housing into W majority areas leads to the emergence of social housing spots mainly at the periphery. (5.4.2.) In presence of differing segregative behaviours on the part of B and W (case 2 and 3), social housing de-structure the local spatial organisation of urban and rural cells.*

5.4.2.5 A metropolitan area with four subsystems and policies

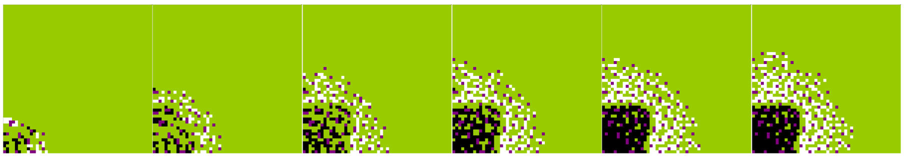
In this scenario, the level of complexity is increased. It is aimed to show how highly heterogeneous patterns can arise through time with only two household



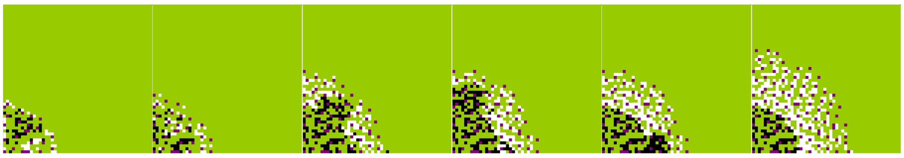
(a) Case 1: Neutral behaviour (t50, 100, 150, 200, 250, t^*389)



(b) Case 2: W prefer W, and B are neutral (t50, 100, 150, 250, 300, t^*437)



(c) Case 3: W prefer W, and B prefer B (t20, 50, 100, 150, 200, t^*224)



(d) Case 4: W prefer W, and B prefer W (t35, 40, 100, 120, 150, t^*275)

■ Agricultural land □ W households ■ B households ■ Social housing

Figure 5.6: Spatio-dynamic patterns for different social preferences and social housing within a scattered city

types, while the initial surface is homogeneous except for distance and public policy. The area is now a 360° metropolitan area of 100x100 cells, with a point CBD in the centre. The area is subdivided into four quadrants, each characterized by one of the policies explored before: no policy in the NW quadrant, social housing in the NE, income subsidy in the SE, and transport subsidy in the SW. Migration is allowed from one quadrant to another, and the urban growth rate is given for the area as a whole. New immigrants settle down in whatever quadrants after observation of the spatial structure of all four quadrants. Therefore, the growth rate of B and W can be different in each quadrant. Conversely to the previous scenarios, only one simulation is presented here and corresponds to the case where W prefer W, and B are neutral (case 2).

Because the area is larger, it is chosen to change the immigration rate to 10 W and 10 B per time steps so that the equilibrium can be computed more rapidly. As mentioned earlier (in chapter 3) this is likely to imply more compactness. However, higher growth rates also reduces the effect of the random tie-breaker, and even more importantly in this case since the CBD is central and thus more cells are likely to be equivalent. For instance, the four cells that are contiguous to the CBD would tie (except the one corresponding to the income subsidy for B households) at $t = 1$ if the growth rate was unitary. The simulation being path-dependent, asynchrony would thus probably impact more strongly on the equilibrium in this 360° metropolitan case. Additional sensitivity analysis is required to assess the impact of changing growth rate in this setting, which bares more complexity than the results obtained in previous simulations and chapters. However, the scope of this simulation is limited to show an example of the level of complexity that can be reached within a framework which, after all, is very simple compared to many CA applications.

Analysing differences between the previous closed quadrant simulations and this one, may provide interesting information for the integration of spatial policies at the metropolitan level. Important differences can be pointed from the comparison of Fig.5.3b, 5.4b, 5.5b and 5.6b respectively with the NW, SW, SE and NE quadrants in Fig.5.7. Certainly the sequence of short-run equilibria differ, but also the long-run equilibrium. These differences are due to the fact that the quadrants are no longer closed systems, which implies that (i) differentiated migration of B and W per quadrant is possible, (ii) inter-quadrant migration is allowed, (iii) the spatial pattern in a given quadrant is gradually influenced by the spatial pattern in another because neighbourhoods cross the borders.

As shown on Fig.5.7, the long-run equilibrium is reached at time 248 with 2478 B urban cells including social housing. The disappearance of W is not surprising given the chosen parameters[17]. It was already the case in each corresponding quadrant taken separately. Nevertheless, there are differences in the

[17]See appendix for reservation bid rents graphs.

morphological local structures of the mixed rural-urban belts. A first example is the blurred shape and the direction of stripes in the periurban belt (NW and SW) although density patterns are similar. Second, the social housing distribution in the NE is more clustered. Both differences, however, might be explained by the fact that asynchrony is partly relaxed here. A third difference, more striking, appears in the morphology of the periphery in the SE quadrant (transport subsidy). A belt of compact B cells arises concentrically after some rural leapfrogs. Then, stripped settlements emerge in the more remote periphery. Density is not continuously decreasing with distance, while it was previously the case with transport subsidy. In fact, this denser part of the periphery at t^* is inherited from previously W settlements where W households agglomerated together in order to benefit from social ('club') amenities.

To understand this pattern, further description of the urban growth process is needed. Short-run equilibria are different in this metropolitan area than for each individual cases. The growth process observed in previous simulations with these preferences was similar for the four quadrants: B occupy the centre in a very dispersed manner while W settle in a striped periphery, then the migration of W is stopped and downward filtering occurs at the periphery until no W remains, and finally, B occupy the centre in a denser manner. Under social housing policy, the process is the same but the morphology of the W periphery is disturbed by social housing sites. An important characteristic of these processes is the cumulative decay of peripheries. It is easier for B households, who have less restrictive preference than W, to replace W households at the periphery than converting agricultural land. This filtering is made easier as W avoid the presence of B.

The process is similar here, but instead of migrating out of the area, W households escape the presence of B by settling at the periphery of another quadrant, where the presence of B households is less important. It is possible because at a given distance and local density, the utility surplus of B is not the same in each quadrant. So B filter W peripheries where they maximise utility. Once these peripheries are fully filtered down, B follow W in the other quadrants.

Even more striking is the way in which W households, 'squeezed out' by B households, migrate towards other locations. Because neighbourhoods crosses quadrant borders, W gradually occupy contiguous quadrant at similar distances. Urban development therefore occurs in a spiral shape. Furthermore, agricultural zones are preserved between the branches of this spiral. W households value these green areas, which acts as a buffer zone between W and B residential areas. The emergence of multiple concentric 'green belts' is the main morphological characteristic of these short-run equilibria. This characteristic disappears in the last time steps of the simulation, as the filtering down process is completed and

B households start converting agricultural cells and fill in previously leapfrogged central locations.

This general functioning is disturbed by social housing and the fact that the utility of B is less important in the NE and NW quadrant, as there is no subsidies. Hence, B settle in these peripheries in the last periods only. The absence of subsidies delays the filtering down process. Remind that this simulation is a particular case where B, with less restrictive preferences, can bid over W in all the periphery. One can run alternative simulations where a W periphery remains in long-run, subsidies allowing B to expand further but not to filter all of the W area.

Result 5.5. *(5.5.1.) Considering subsystems open to internal migrations changes the dynamics of the urban growth process and can potentially affect the expected long-run equilibria. (5.5.2.) If W have more restrictive behaviour than B with respect to their neighbourhood, they can migrate gradually to other parts of the city in short-runs, before migrating out. (5.5.3.) The search for optimal local configurations that combine social preferences and open-space amenities can result in spiral-type urban developments that leave concentric green belts at different distances from the CBD in the long-run. (5.5.4.) The presence of social policies in parts of the cities encourages the location of B in these parts and delays downward filtering in the other parts.*

5.5 Conclusion

A model has been proposed that combines CA and urban economics to explore the mechanisms of spatial segregation within a growing city. An endogenous scattered morphology allows households to benefit from open-space amenities. The method takes advantage of the bid-rent approach to simulate land and residential conversions in a 2D dynamic setting. In this model spatial segregation results from the joint effect of income sorting and neighbourhood effects (including both the demand for social amenities and the demand for open-space). The model can therefore be seen as a combination of Schelling (Schelling, 1971, 1978) and Brueckner-Vousden (Brueckner, 1980; Vousden, 1980) models. Moreover it is characterized by a periurban-type assumption (Cavailhès et al., 2004b; Caruso et al., 2004b) and thus emphasizes the role of rural parcels and local open-space in the development of cities.

The evolution of the city is made of a complex set of various phenomena: upward and downward filtering, cumulative decay, clustering and local concentration, self-creation of green areas with different spatial arrangements (belt,

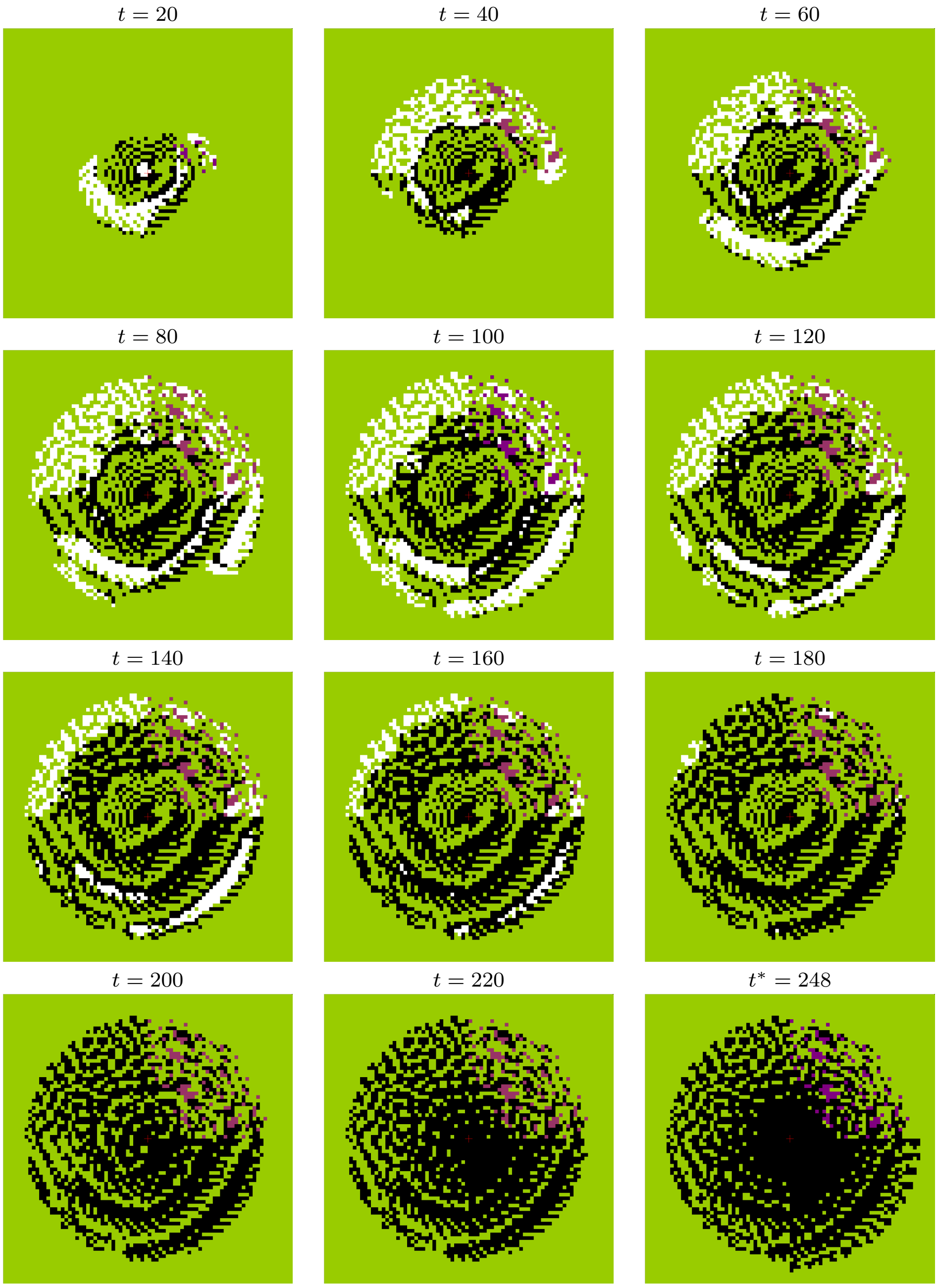


Figure 5.7: Spatio-dynamic patterns when W prefer W and B are neutral, within a scattered metropolitan area with different policies: NW quadrant – market, NE quadrant – social housing, SE quadrant – income subsidy, SW quadrant – transport subsidy

ribbon, spiral), and cyclic 'chassés-croisés' of households groups. Short and long-run equilibria are dependent on the past history of the city (as in non-malleable housing dynamic models). Spatial patterns are therefore complex, with locked-in development directions or local niche arrangements. Moreover, when parts of the city have different characteristics (for one household type at least) and residents are free to move from one part to another, the complexity of long-run and, above all, short-run spatial structures increases even further.

Results derived from our simulations have been summarized for four set of behavioural preferences and different policy scenarios. Most of the emerging spatial patterns are globally structured by the distance to the CBD, with central location of poor people and peripheral location of the more well-off. This result is in adequacy with static urban economics, although within a scattered morphology. However, decreasing density to distance function is often disrupted, showing the impact of multiple household types on land use structure. Moreover, distance cannot render all segregation patterns. Describing segregation as an opposition between the centre and the periphery is therefore too simplistic, certainly at short-run but also at long-run equilibrium.

Except for the impact of considering multiple household types on the density to distance function, only few attempts can be found in the literature that relate spatial morphology and the socio-economic characteristics of the inhabitants. However, De Keersmaecker et al. (2004) found strong correlations between fractal dimensions and socio-economic characteristics of periurban neighbourhoods, including income.

Most empirical descriptions of income and ethnic distribution oppose peripheries and centres and look further at intra-urban differences rather than potential intra-periurban variations. The contrast between Paris and Brussels is an example of reversed centre-periphery opposition: poor centre and a rich close periphery in Brussels, rich centre surrounded by poorer classes and middle classes in the more remote periphery in Paris (see e.g. Thomas and Zénou, 1999; Goffette-Nagot et al., 2000). The role of amenities (Brueckner et al., 1999) or the role of the time and modes of transport (Goffette-Nagot et al., 2000) is emphasized for Paris.

Some empirical work, although rarely dynamic, also show the importance of non-distance based distributions of income groups, i.e. their relative structure within both peripheries and centres. Within the morphological agglomeration of Brussels, for instance, radial structures superimpose on the concentric model. These radial structures expand Eastward and Southward beyond the agglomeration. Segmentation of the periphery is also found around Paris and oppose generally the West and the North-East. In the Netherlands, differences are less obvious as the social sector played an important role until the 1990's and helped the deconcentration of poorer classes (Vandermotten and Vermoesen, 1995). In the example of Lyon described in Dujardin and Goffette-Nagot (2004), most well-off categories locate at the periphery, but pockets of deprived neighbourhoods are found within the close periphery.

Segregation is much lower in Europe than in the USA, but the level of local mix varies a lot from a city to another (Murie and Musterd, 2004). Consequently, the geographic distribution of income groups in Europe is rather fuzzy, although it appears that many close periurban areas are more well-off than their centre. In the case of Belgium, the average income per household in 'banlieue' zones (see spatial typology in table 2.1) is 10% above the average income in the agglomerations. Beyond the 'banlieue', in the commuter zone, income level is about the same than in the agglomeration. In Brussels, average income is higher in the centre compared to other cities, and it is even more important in its 'banlieue' (+13%)[18]. In France, the average income in the suburban part of urban poles is 14% above the centres. The average income in periurban municipalities (see also spatial typology in table 2.1) is weaker, but progresses more rapidly than in poles (Cavailhès and Goffette-Nagot, 2001). Another aspect of the distribution is the larger variation of income in French urban communes and Belgian agglomeration than in periurban municipalities and belgian 'banlieues' (see Cavailhès and Goffette-Nagot, 2001, for France and own calculations for Belgium). In summary, periurban zones are more homogeneous than centres, however differences remain and periurban areas encounter rapid change. Moreover there exist differences between periurban zones. For example, access to housing might be very difficult for less well-off young families in earlier zones of periurbanisation or in zones of high economic growth (Eggerickx, 1999, for Belgium). In the Netherlands, this process has been exacerbated by a shortage in large housing in recent years.

With respect to these observed structures and changes, our model proposed possible income distribution structuring processes through time within periurban areas. Although not tested, resulting dynamic patterns include differentiations of periurban areas mainly when the different social groups state preference for their own group and can equally compete in the housing market (i.e. when they have similar income endowment or subsidies).

Our experiments also point out the difficulty of obtaining spatial mix at micro level, while this could be raised as a public policy objective. Transport and income subsidies can help less well-off and discriminated people to raise their bid to obtain other than central locations. However, this results in greater suburbanisation of these people rather than a stable mix. In a city open to in- and out-migration, where the more well-off discriminate people based on their

[18]Calculations for Belgium are based on income data of 1993 from the Insitut National de Statistiques

income or race, these policies tend to decrease the more well-off population and therefore increase the decay process. Examples have however been found where the periphery is occupied by the different groups, but still in a separated manner (with buffering agricultural areas). An endogenous decision to implement social housing into rich areas seems to be the way to induce less segregated peripheries. However, this affects the utility of those who discriminate and therefore affects the optimal spatial arrangement of residents and agricultural locations in the periurban area.

Policies in Europe have either been oriented to improving deprived zones, either to reducing spatial segregation (Musterd and De Winter, 1998). The former do not consider poverty as a spatial problem, while the latter suppose that there are needs to de-segregate and encourage the mix of households (which is more in accordance with the emphasized role of social contacts in the neighbourhood (e.g. Cutler and Glaser, 1997)). Empirical evidence show few impact of these de-segregation policies. The quota policy in Frankfurt did not achieve its goals because running against the preference of people, or because poorer groups were excluded when quota were reached. The Dutch policy, conversely encouraging the location of more well-off households in poor areas, does not seem to fulfil de-segregation objectives as it covers an actual market effect and is likely to lead to homogeneity of rich in these areas in the long term (see Musterd et al., 1997; Musterd and De Winter, 1998, for a more detailed analysis of segregation patterns and policies in Europe).

Finally, the model presented here showed how distance based and neighbourhood based effects can combine to create diverse urban and periurban segregation patterns through time. Patterns are diverse and can become very complex. Diversity and complexity is also found in Europe. However, testing the mechanisms proposed by the model would require a detailed knowledge (intra-neighbourhood level) of the spatial distribution of socio-economic groups for different time periods, both for urban and periurban areas. Priority should therefore be given to refining this exploration (e.g. gradual change in income, commuting costs and utility differentials, or differing migration rates) in order to identify bifurcation thresholds and make the results more robust.

Chapter 6

Application to the Southern Brussels area: calibrating a 2D model to a periurban morphology

Outline

This chapter proposes a partial calibration procedure of the model in order to assess the capacity of the model to generate global spatial patterns that are similar to real patterns. The model with constant housing consumption (chapter 3) is used so that no information is required on the size of housing or intra-cellular density. It is also a reasonable assumption because the model is applied to a part only of the periurban area of Brussels, the urban agglomeration and the close periphery being masked. The determinants of residential choice and the functioning of the model are presented again in order to offer another perspective that emphasizes the relationship between the model and the literature on constrained urban cellular automata.

As in the previous chapter, the model hypothesizes that households induce the morphology of urban developments since they commute to a single centre and value a pair of neighbourhood externalities. Therefore, a sensitivity analysis is undertaken of the relative weight of greenness and social neighbourhood externalities, in order to explore the relationship between neighbourhood preferences and emerging spatial morphologies. Conversely to the other chapters, the lattice is not theoretical, distance is not Euclidean but accounts for the road network, and planning regulations prevent urbanisation in a large set of cells. The experiment can therefore assess whether the archetypes generated in chapter 3, within an homogeneous and isotropic space, can fit observed spatial patterns when they are more spatially constrained.

The sensitivity analysis is used to fit spatial patterns generated by the model to the observed land use. This partial calibration is based on minimizing the distance between the 'real map' and spatial outputs of the model along four indices that describe the overall spatial structure. The corresponding parameters are assumed to reflect the relative importance of green and social externalities in the location choice of periurban migrants. For the sake of simplicity, the other determinants of residential choice (relative weight of other consumptions in the utility function and commuting costs) are not taken into account, so that rent value cannot result from this experiment. [1]

[1] This chapter is based on Caruso et al. (forthcoming)

6.1 Introduction

Many techniques have been developed to model land use dynamics and these applied to a broad range of research questions (Parker et al., 2003). The analysis of urban land use change has benefited from new types of models that have made the consideration of space and/or time more explicit than in traditional spatial economic and spatial interaction models. The study of urban and regional geographical processes, in particular, have increasingly looked to the theory of dynamic systems and self-organisation (Pumain et al., 1989; Krugman, 1996a; Allen, 1997). For more than a decade, urban simulations based on cellular space have been flourishing, using statistical techniques, Markovian transitions, cellular automata, multi-agent systems or some combination of these (Parker et al., 2003).

Cellular automata (CA) applications emphasize the role of local processes that create spatial structures and dynamics. This underlying principle of CA, as well as its affinity with raster data models explain the development of numerous CA models for hypothetical and real case study applications (Batty et al., 1999). There is a large variety of CA models exploring urban patterns and many empirical applications that simulate the growth of urban settlement or land use transitions in a regional context (e.g. White and Engelen, 1993; Batty and Xie, 1994; Portugali and Benenson, 1995; Cecchini, 1996; Sembolini, 1997; White and Engelen, 1997; White et al., 1997; Batty, 1998; Clarke and Gaydos, 1998; Wu, 1998a,b; Wu and Webster, 1998; Li and O, 2000; Jenerette and Wu, 2001; Wu and Martin, 2002).

Advances in modelling have, however, often been accompanied by an increase in the complexity of the models applied to real case studies. This is sometimes inevitable given that many human systems are themselves complex, but as Couclelis (1997, p.167) stated: 'once complexity degenerates into complication, the game is lost'. This has important implications for the value of models in understanding geographic processes, and simplifying model construction may be a way of avoiding these problems. For example, reducing the number of parameters makes sensitivity analyses more tractable as well as allowing the identification of (non-)linearities and thresholds (bifurcations) in the model response to parameters changes. Taking such a reductionist approach might appear contradictory when considering cities as complex systems (Torrens, 2000), but this is not a problem provided that the model is still able to generate emergent properties from the selected processes. Simplification, therefore, requires a good selection of the processes to be represented. Many CA applications also suffer from a lack of a theoretical foundation, especially in relation to economics (Irwin and Geoghegan, 2001). This statement is supported by: (i) the emergence of agent-based models that seek to model land use change through the decision making and interactions of individuals (Parker et al., 2003); (ii) the criticism of CA for being tools that focus on the modelling exercise rather then enlightening urban theory (Torrens and O'Sullivan, 2001); (iii) the lack of non-economic dynamic models to express preferences and behaviour, as well as in modelling price systems (Anas et al., 1998).

The principal advantage of including economic theory within a spatio-dynamic model is that the simulations have greater explanatory power. Since the parameters in a model are more strongly based on reality, the simulations not only attempt to reproduce spatial patterns, but also provide a real opportunity to relate these patterns to individual behaviour. This clearly is an important goal of CA models. Such models can further contribute to the understanding of land use change, as well as being useful tools for spatial policy making. If robust relationships could be established between form and process it would be possible to devise policy instruments to direct urban growth and form (Batty et al., 1989). With behavioural economic models, spatial metrics reflect the ecological, but also the socio-economic functioning of the landscape (e.g. see Parker and Meretsky, 2004). Furthermore, Wu and Webster (1998), were able to evaluate the relative performance of CA simulations using a range of socio-economic indicators as their model was grounded in economics. Certainly this type of model can be useful in addressing the sustainability of urban environments and contribute to the debate on efficient city size (Capello and Camagni, 2000) and urban form (Anderson et al., 1996).

Because the parameters within a behavioural model are so important, calibration is critical. Moreover, calibration is increasingly important when one considers the problem of 'equi-finality' (see a discussion in Richardson (2002) or Batty and Torrens (2001)). Various techniques have been explored to determine parameter values when applying CA models, but there is no standard procedure, probably because the objectives and constructs of the models are different. Clarke and Gaydos (1998) or Silva and Clarke (2002) explored a very large set of possible parameter values and measured the statistical fit to different spatial metrics. The Markov CA model applied by Jenerette and Wu (2001) was parametrised by a linear interpolation of empirical data and by using a genetic algorithm, the performance being assessed through a set of landscape indices. Yeh and Li (2003) estimated development probabilities by means of neural networks trained on an observed sequence of land use. Wu (2002) used logistic regression and Monte-Carlo simulations to assess the performance of a model on a cell-by-cell basis, as well as comparing both autocorrelation, and development density within distance (time) rings around the city. Verburg et al. (2003) also used logistic regression to reduce the solution space of a CA and assist in calibration. For the types of CA models proposed by Straatman et al. (2003) implemented an algorithmic procedure based on neighbourhood comparisons in order to automate calibration, while the random parameter of the model is fine tuned on radial fractal dimensions (White et al., 1997).

Since behavioural models relate individual decision making to emergent spatial morphologies, macro pattern characterisation can be used to parametrise micro behaviour. CA models should, therefore, aim to generate spatial morphologies that are similar to reality and not to reproduce the exact pixel based location of agents. The focus of this chapter is on the spatial morphologies of periurban development. Several methods have been adopted in the literature to define and measure sprawl, or to describe urban land use structure and change. Of interest for the purpose of calibration are those methods which provide a global characterisation of space by means of indices or functions.

The measurement of urban density gradients and its change has seen many applications and functional adaptations since the determination of Clark's law (Clark, 1958) (see also the discussion in Anas et al., 1998). More recently, however, Galster et al. (2001) have indicated the fuzziness of the sprawl concept and the need for operational quantifications other than density gradients. The diverse range of spatial indicators proposed by different researchers (Galster et al., 2001; Hasse and Lathrop, 2003; Torrens and Alberti, 2001) generally include density measures, measures related to scattered development, isolation, leapfrogging, functional segregation, or discontinuities, landscape composition metrics, ribbon or stripped development, proximity of each land use, and accessibility to services. Herold et al. (2002) used landscape metrics derived from information theory and fractals to describe changes in urban land use from a high resolution dataset. These indices were also used to calibrate CA models (e.g. Herold et al., 2003; Jenerette and Wu, 2001).

Fractal analysis has also proven to be useful at measuring space-filling patterns as well as urban evolution trends. Such analysis is able to reflect the chaotic nature of cities, as well as providing evidence for similarity between cities (Frankhauser, 1994; Batty and Longley, 1994). Global properties of monocentric urban morphologies, evaluated in terms of scaling behaviour, have also been obtained from diffusion-limited aggregation (Batty et al., 1989) or correlated percolation models (Makse et al., 1998). These studies are insightful for the assessment of urban densities and the compactness of global urban forms and clusters. The use of a fractal measure should, therefore, also be a commonly accepted methodology to test or to parametrise urban growth models.

When considering the creation of periurban morphologies, the economics of residential choice can be expressed by two types of agglomeration-dispersion forces. First, at the regional scale, these forces correspond to the monocentric residential model (Alonso, 1964) where households trade-off additional transport cost for space and amenity (see Fujita, 1989; Anas et al., 1998). The second set of forces is local: new migrants within periurban areas search for lower density environments and amenities that are provided by the neighbourhood of a given location. New residents are assumed to value local proximity to schools, shops, network infrastructure,... that is the proximity to urban land use. They also, however, value the openness of the landscape, low density environments, greenness,..., in simple words, the proximity to rural land use. Given these neighbourhood processes, periurbanisation seems to be particularly well suited for experimenting with the integration of micro-economics within a CA framework.

Neighbourhoods are the central element of a CA model as they support the interactions between land use and the dynamics of the system. The relevance of neighbourhood processes and distance decay effects (i.e. spatial autocorrelation,) has long been recognised as an important law in geography (Tobler, 1979), and an early application of the neighbourhood-based concept of space in simulation can be found in the mean information field of Hägerstrand (1967). In most CA models the existence of interaction between two locations is implied by their membership to an a priori defined neighbourhood extent, or the level of interaction is derived from the Euclidean distance between cells. In practice, any other length metric relationship could be used to represent the relative separation of a set of locations (see the axiomatic approach proposed by Beguin and Thisse (1979) and a recent discussion in Miller and Wentz (2003)). The concept of proximal space (Couclelis, 1997) emphasises the relative representation of space that is characteristic of CA, and a notion of a neighbourhood that goes beyond its geographic representation. Basically, any framework that seeks to implement a behavioural model of land use change within a CA dynamic has to consider the formalisation of proximal interactions. Here these are created through the coexistence of two land uses, and represent externalities that are valued by one of the land users.

Thus, two hypotheses underpin the approach presented here: (i) the integration of a raster GIS, a cellular dynamic mechanism, and an economic framework can provide useful information on the decision making of individuals in space, and contribute to the understanding of land use change. (ii) In order to achieve this goal, a calibration procedure is required that is based on a sensitivity analysis of the exogenous inputs and endogenous behavioural interactions for which the results are measured by global geographic indices. The spatial morphology of urban development is taken as an example. A model of residential spread is proposed where a part of the southern periurban area of Brussels is represented as a cellular-dynamic system including two types of located agents: households and farmers. The model focuses on household behaviour and aims to understand the role of residential choice in the emergence of discontinuous urban developments. A simple sensitivity analysis is used to assess the capacity of the model to generate residential location patterns that are similar to observed ones.

6.2 Methods

6.2.1 Theory

6.2.1.1 A general CA framework for simulating land use competition

The framework proposed here is inspired and very similar to the generalised and constrained CA model approach of White and Engelen (1993), White and Engelen (1997) and White et al. (1997). The main differences are: first, no macro model is used to determine the number of cells in each land use class at each time step; second: the model is deterministic so that no (stochastic) heterogeneity is added to the variations which already exist within the raster inputs. Therefore, the differences observed between each run of the model, represent the result of varying parameter values only.

The framework was created to implement models which incorporate neighbourhood dynamic elements as well as other location specific attributes. The objective was to show how, by extending a conventional CA framework, it becomes possible to simulate processes based on urban economics. Takeyama and Couclelis (1997) introduced the 'Geo-Algebra paradigm' for integrating GIS and CA within a general mathematical framework when modelling dynamic spatial processes. Within this framework: (i) there is no a priori structure given for the locations in space (i.e. not only regular cells), (ii) values attributed to locations can be continuous, (iii) any type of neighbourhood definition is possible (by using relational maps), (iv) interactions and transition rules are not necessarily homogeneous in space and time, (v) any interaction can be formalised from a multiplicity of GIS layers, and finally (vi) CA are no longer closed systems as they can take any GIS layer as an additional input.

The framework proposed here also includes some of these extensions to conventional CA, namely: the possibility of dealing with continuous values, the definition of a neighbourhood that is based on a proximal distance and decay functions, an open system that can account for additional raster datasets at any time step and differentiated according to the land use under consideration.

Let the land use be denoted by C^t_{ij}, where ij determines a unique location on a lattice and t refers to a given point in time. A limited number (c) of land use classes is possible $C^t_{ij} = \{1, ...c\}$. A sequence of land use maps can be computed from, at least, a given initial state C^0_{ij} and a set of formalised interactions. A potential function is calculated for all competing uses of land in order to drive the landscape dynamic. The comparison of these potentials leads to land use conversion.

Let $P^t_{c,ij}$ be the potential value of the cell ij to be in state c at time t. Firstly, $P^t_{c,ij}$ is a function of some location specific variables that are given exogenously as raster type inputs, X_{ij}. These 'grid' inputs can vary with time, X^t_{ij} is then a multi-temporal raster dataset, or varying with c, $X_{c,ij}$ is then a multi-state raster dataset. $X^t_{c,ij}$ can for example represent the suitability of a cell ij when considering the possibility of ij being in land use state c. There is potentially a large number of raster data types that can 'constrain' locally the potential of being in use c (topography, soil types and yields, accessibility by road or train, zoning permissions,...). Let therefore q be the number of raster (or multi-raster) inputs to be considered and define the vector of variables $Xq^t_{c,ij}$ with q ranging from 1 to q.

A second type of variable influencing $P^t_{c,ij}$ is neighbourhood based and this represents the core of the CA dynamics. A neighbourhood variable, $\mathcal{N}_{ij}$ is calculated on the basis of the land use in the neighbouring cells of ij at time $t-1$ (the framework is constrained to a first order Markov process). Any $\mathcal{N}_{c,ij}$ can then be defined similarly to $X_{c,ij}$, as a neighbourhood function can also vary according to the land use under consideration. Again, a set of neighbourhood variables can be considered, and so let n be the number of neighbourhood variables used in the model and define the vector of variables $Nn^t_{c,ij}$ with n ranging from 1 to n.

So, a general formulation for any land use potential is

$$P^t_{c,ij} = f\left(Xq^t_{c,ij}, \mathcal{N}^t_{n,c,ij}\right) \tag{6.1}$$

$\mathcal{N}^t_{c,ij}$ itself is a somewhat more general formulation of White and Engelen (1997)'s approach. While their neighbourhood function was based on the state (land use) of surrounding cells, any quantitative measures can be used here (provided that an initial raster input is given). A neighbourhood function is a spatially homogeneous influence function (in the sense of Takeyama and Couclelis (1997)) as it is applied in the same way at every location. It is a part of the transition determination in the CA. An example of such a function would be to calculate the average of the land prices observed at $t-1$ in a neighbourhood when considering location at time t in a certain cell.

A neighbourhood function is then generally defined as a weighted sum of the values of the cells kl belonging to the neighbourhood of ij. X being whatever variable for which a value is available for every cell at $t-1$. I being a binary variable equalling 1 if cell kl is in state c' at time $t-1$, and 0 otherwise.

$$\mathcal{N}_{c,ij}^{t} = \sum_{kl} w_{c,c',kl} I_{kl}^{t-1} X_{kl}^{t-1} \tag{6.2}$$

$w_{c,c',kl}$ is a weighting parameter that allows for every c' land use in the neighbourhood of ij to impact differently on $\mathcal{N}_{c,ij}$ and therefore on the potential of being c. This impact is defined for each pair (c, c') of land uses and is constantly defined across space and time. In addition, $w_{c,c',kl}$ allows the more remote cells kl belonging to the neighbourhood of ij to impact differently on $\mathcal{N}_{c,ij}$ than the contiguous cells. It is defined once and as a distance decay function within the range of a given neighbourhood extent.

$$w_{c,c',kl} = f\left(x_{kl}\right) \ \ and \ x_{kl} \in [0, \hat{x}] \tag{6.3}$$

with x_{kl} being the distance from cell ij to kl, and $\hat{x}$ being the maximum distance considered in the neighbourhood, that is the neighbourhood extent.

Finally, the set of cells ij that can be converted into use c at time t is defined by comparisons of $P_{c,ij}^{t}$ values. If no further constraints are added, the transition is made for every cell ij towards the state c having the highest potential value in ij. A global constraint can also be defined for each land use c in the form of a percentage of the total number of cells in the area. A limited percentage of the total area can for example be converted into an urban use. This percentage can be defined as a function of time and therefore reflects a known growth rate. In the following example, the global constraint is used to determine exogenously the residential growth rate, (e.g. a single resident at each time step). It is then necessary to make a choice between the set of cells where c has the highest $P_{c,ij}^{t}$ value, in order to select only some of them for conversion into class c. The choice is made by ordering $P_{c,ij}^{t}$ values and selecting the highest.

6.2.1.2 Implementing economic theory within a CA framework

The economic model hypothesizes that dynamic spatial configurations in periurban areas are related to economic factors and behavioural aspects of residential choice, including the level of income, transport costs, and preferences for local amenities. A variety of theoretical equilibrium and transitory configurations have already been obtained from this model and discussed (Caruso, 2003a), where the economic theory, parameters conditions and equilibria are presented in more details.

A bidding system is implemented within the transition rules of the CA which determines the conversion from agricultural to residential use. In this example, the time-lagged cellular neighbourhood, is used to calculate periurban location externalities, as well as their change through time and space.

In urban economic theory, residential behaviour is usually formalised by the maximisation of a utility function under a budget constraint. From this maximisation can be derived the bid rent of a resident for each location at any time. In this case the bid rent represents the amount a resident is willing to pay to acquire a particular cell, given the distance to the city and the observation of the neighbouring environment. Residents are assumed to value the neighbourhood environment of a cell in two ways. First, they are interested in having a certain density of residences around them, i.e. the social externalities (S) at a single location. Secondly they value the environmental quality of the landscape around them, i.e. the environmental externalities (E). Both of these neighbourhood preferences, E and S, are expressed as a function of the density of households in the neighbourhood, ρ. E decreases and S increases with ρ, both at a decreasing rate. The neighbourhood density is assumed to be evaluated by newcomers just before making their location decision – the time lag – giving more importance to proximal cells within a neighbourhood.

The preferences have been formalised in a Cobb-Douglas function to be maximised (where Z represents the set of all other consumptions): $Max\ U \equiv ZE^{\beta}S^{\gamma}$

The location choice of a household is then constrained by its income (Y) and a location specific commuting cost ($T(d_{ij})$). Each household is assumed to earn the same income, and $T(d_{ij})$ comes from a cost distance GIS calculation along the main road network. Given a utility level (u) to be achieved at any location, the maximisation produces an expression of the bid-rent for each location and time, where Y and u are given constants. Similarly to equation (6.1), this residential bid rent is interpreted as the potential for a cell ij to be in residential use at time t, $P^t_{resident,ij}$:

$$P^t_{resident,ij} = Y - T(d_{ij}) - \left(E^t_{ij}\right)^{-\beta} \left(S^t_{ij}\right)^{-\gamma} u \tag{6.4}$$

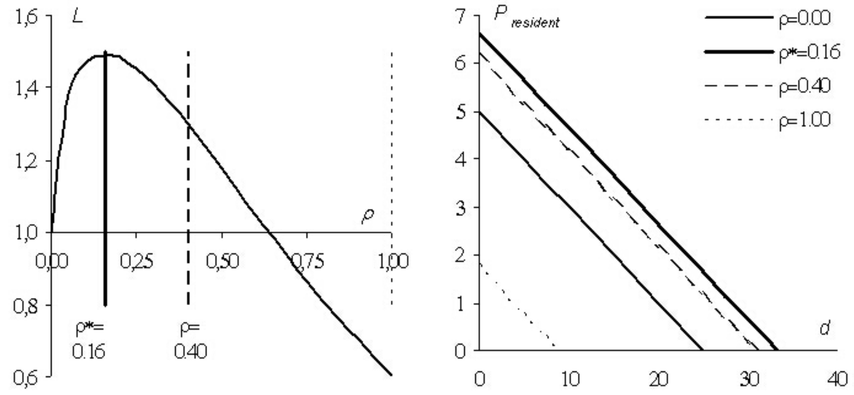
The two contrasting preferences, E and S, can be summed into a single 'local externality variable' (L) which is a function of the (distance decay weighted) neighbourhood density, ρ, in $t-1$: [2]

$$L^{t-1}_{\rho_{ij}} = \left(E^t_{ij}\right)^{\beta} \left(S^t_{ij}\right)^{\gamma} \tag{6.5}$$

The level of local externalities is represented as a function of the neighbourhood

[2] The detailed functional form comes from a simplification of the parameter values in Caruso et al. (2004b), and equals $L^{t-1}_{\rho_{ij}} = \left(E^t_{ij}\right)^{\beta} \left(S^t_{ij}\right)^{\gamma} = e^{\gamma\left(\sqrt{\rho^{t-1}} - \frac{\beta}{\gamma}\rho^{t-1}\right)}$

density in figure 6.1. The function displays a maximum at ρ^* [3] which represents the optimal preference of households in terms of neighbourhood density. Each level of density corresponds to a bid-rent curve (figure 6.1) which decreases linearly with the distance to the city because of the commuting cost $T(d_{ij})$ (in equation 6.4).



(a) Level of externalities (L) as a function of the local neighbourhood density (ρ)

(b) Residential bid-rent curves ($P_{resident}$) for different neighbourhood qualities (ρ) as a function of the distance to the city (d)

Figure 6.1: Level of externalities and bid rent curves (for $\beta/\gamma = 1.25$ and $\rho^* = 0.16$)

The ratio of the two parameters β and γ (equation 6.5) represents the relative importance given by households to landscape amenities relative to social amenities, when making their location decision choice. Higher values of β/γ indicate a preference for greener neighbourhoods. This ratio will be tested in subsequent analyses because it affects the value of the optimal density, ρ^* (figure 6.2) and, therefore, the global residential morphology of the area. Clustering is stronger with low β/γ (high ρ^*), whereas more dispersion occurs with high β/γ (low ρ^*).

Once $P^t_{resident,ij}$ is defined, it is compared with the potential of being in agricultural use, $P^t_{farmer,ij}$. The higher the bid of the farmer, the stronger the competition.

Because of a lack of spatially detailed data on agricultural yields and profits, the bid-rent (i.e. the potential) of the farmer is considered as a constant through space and time. Farmers are thus assumed to be unable to prevent residential growth. The potential refers to the opportunity cost of land, and is assumed to be far below the residential potential.

[3] According to the same simplification, $\rho^* = (\gamma/2\beta)^2$

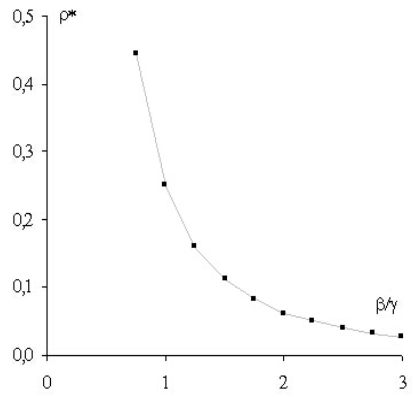


Figure 6.2: Optimal neighbourhood density (ρ^*) as a function of the preference for landscape amenities relative to social amenities (β/γ).

In theory, the bid-rent of the farmer is necessary to limit urban spread and determine a long-run equilibrium city size (see chapter 3 and 4). The present application, however, considers a fixed city size. The number of agricultural cells to be converted in residential use in each model run is constant and given from the observed dataset. Total population being given, spatial patterns are therefore comparable from one run to another and with the observed map. As in the previous chapters, a single cell is converted into residential use per time step (see asynchrony assumption in chapter 3). The selected cell at time t provides the highest potential $P^t_{resident,ij}$. The number of runs for a simulation corresponds, therefore, to the total number of residential cells in the actual map.

This is a simple model given that only one land use class c, the residents, is explicitly modelled. The farmer is considered to be the default land user whose behaviour when faced with urbanisation is not represented here. Moreover the model uses only one neighbourhood variable ($\mathcal{N}_{c,ij}$ see equation 6.2), namely the density of residences, and two simple raster inputs ($X_{c,ij}$), the commuting cost and the planning restrictions on building. These $X_{c,ij}$ inputs provide spatial heterogeneity that was not present in previous chapters and will orientate urban developments.

6.2.1.3 Calibration based on the measurement of macro-structures

The methodology proposed here assumes that micro-behaviour can be partly calibrated from macro geographical indices. The value of the ratio β/γ, being part of the residential choice, will be taken from the simulation that best fits the observed pattern. The correspondence between the modelled pattern and the observed pattern is evaluated using four macro indices: (i) an index of fragmentation; (ii) the radial fractal dimension (iii) a measure of the quality of the adjustment to the whole radial fractal curve, and (iv) to the density-distance plot within the planning zones.

(i) Fragmentation. The fragmentation index, H_{FRAG}, represents the edges of the residential cells that are in contact with farmers as a proportion of the total sum of edges for residential cells. A value of 1 indicates full fragmentation when each residential cell is completely isolated. A value of 0 is not possible if the two land uses are present, but a low value indicates compaction. (The index is similar to the edge density metric used for example by Herold et al. (2003); Jenerette and Wu (2001) or Parker and Meretsky (2004)).

(ii) and (iii) Fractal curve and dimension. Although different measurements of a fractal signature have been used in the literature, the radial fractal dimension was applied here. This dimension has the advantage of reflecting the dilution of the build-up area within space, from the centre to the periphery (Bäck et al., 1996). It is, therefore, well suited to a monocentric approach to urban spread. Furthermore, because the number of cells is rather limited, insufficient variation in the size of residential clusters makes the calculation of a cluster-size index meaningless. The radial fractal dimension, RADIALSLOPE, is obtained from the slope of a linear regression that is fitted onto a double-logarithmic diagram of the number of residential cells within a set of distance radii. This measure is used by White et al. (1997) in order to fine tune the level of stochastic perturbation in a CA simulation. A disadvantage of the radial dimension, however, is the difficulty of applying a single linear regression to a signature that is not a regular fractal. As this is often the case, the observed residential pattern is multi-fractal. From a comprehensive analysis using different fractal metrics, De Keersmaecker et al. (2003) concluded that the urban structure of Brussels corresponds to multi-fractal logic. In order to take this structure into account, and following Bäck et al. (1996), the whole length of the log-log curve is considered. Thus, the capacity of any single simulation to reproduce the observed fractal signature will be assessed by measuring a quadratic distance between the observed and the modelled curves (R^2_{RADIAL}).

(iv) Density *vs* distance. An index based on the relationship between residential density and distance (time) was calculated. The number of modelled and observed residents in each minute commuting ring was divided by the number of cells available for building in the same ring. A determination coefficient was calculated by comparing these values, $R^2_{DENSPLAN}$. As the density/distance function is an output of the urban economic model, it is used in the calibration. Wu (2002) also assessed the performance of a CA model based on such a relationship.

6.2.2 Application

6.2.2.1 study area, target maps and inputs

The study area is located within the Southern commuting area of Brussels. The limits of the commuting periphery is delimited according to the definition of Van der Haegen et al. (1996) (already discussed in chapter 2, see table 2.1). These limits are presented in Fig.6.3. The study area is only an extract of the periurban area because of computing requirements. A 25 x 25 km square has been selected and is enlarged in Fig.6.4 where the additional GIS constraints used as inputs into the model are presented.

Several characteristics of the periurban area of Brussels qualifies it for an exploration of the model in a real space: (i) The land use and road network dataset were readily available at a detailed level. (ii) The road network in the area is orientated towards the Brussels Capital Region where most jobs are located (the development of employment subcentres remains weak as shown in Servais et al., 2004), which is important for a monocentric distance based approach. (iii) Urban development increases in this region mostly at the expense of agricultural land (Jehin and Merenne, 1998). The simple residential-agricultural dichotomy assumed in the model is therefore found in this area where redevelopment and relocation processes can thus be neglected. (iv) The region (and the square study area particularly) is characterised by rapid residential development. It can thus be seen as an open-city type area where immigration occurs. (v) Compared to other European countries, speculation on the housing market is limited because of high transaction costs (Baudewijns, 2002). (vi) Finally, Belgium is often cited as a typical free market and under-planned example (see e.g. Holden and Turner, 2001). Compared to other European countries, the role of social housing in periurbanisation has been weak and planning constraints were unable to curb residential dispersion (see chapter 2). The area therefore fits the demand-oriented nature of the model, and the morphology of urban development is largely endogenous as only weakly constrained (the role of land developers and urban planners on urban form is likely to be much greater in Great Britain or in the Netherlands).

Fig.6.4 presents the urban land use pattern in the study area. Residential land in both maps (red cells) is an aggregation of two classes : residential and, mix of residential and services. This aggregation choice allows to account for all the residential space, while including some of the services does not importantly

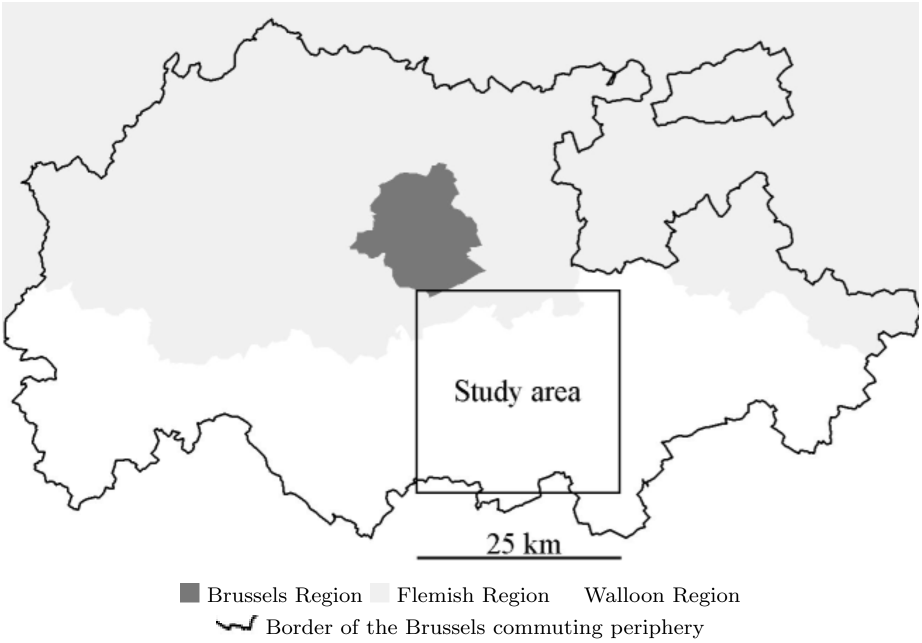
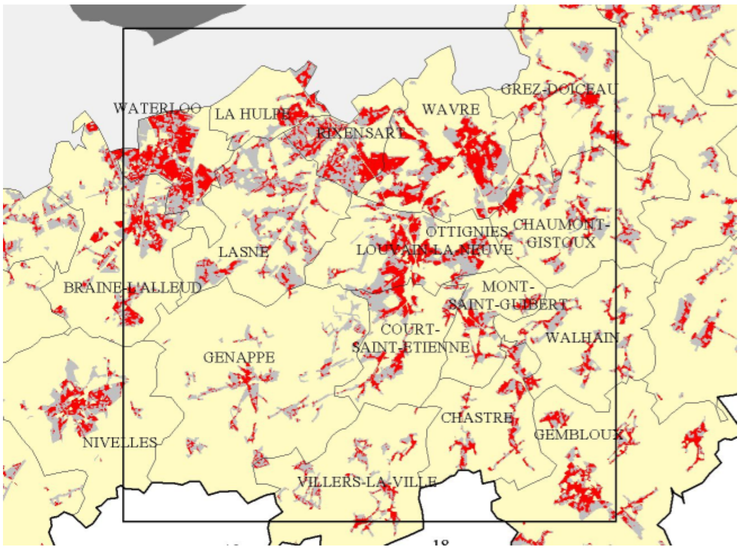


Figure 6.3: Brussels commuting periphery (as defined by Van der Haegen et al., 1996) and study area

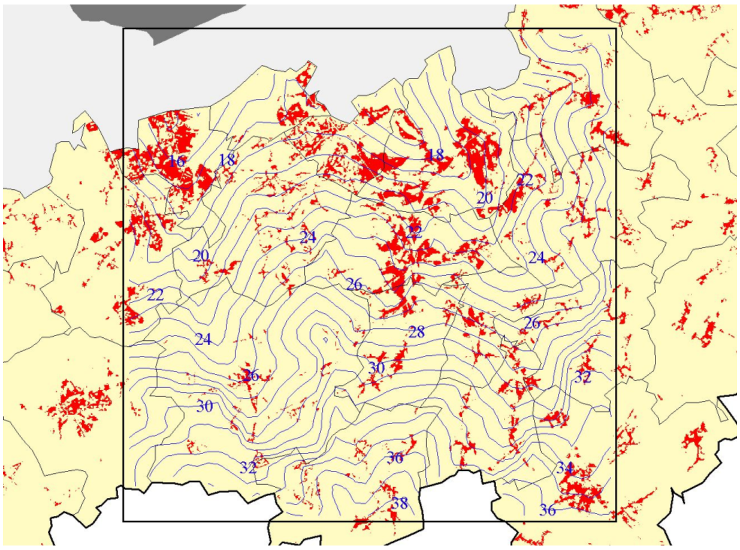
affect the model. On the one hand, households are likely to include service buildings within local density when the greenness of a neighbourhood is assessed. On the other hand, the presence of services as local public goods is a part of the assumed attractiveness of high local densities.

Two target residential maps have been constructed from this observed residential land use. The original dataset, as presented on Fig.6.3, is a fine-grained 20m raster map. The target maps used in the simulation have a coarser spatial resolution because of computing time constraints. In asynchronous setting, when the aim is to generate a given surface of development, the finer is the data, the longer is the computation of one time step, but also the larger is the number of time steps required. The 20m residential land use has been aggregated to 250 and 500m through an averaging procedure. The impact of the aggregation method was assumed to have no impact, although in practice this depends on the level of spatial autocorrelation (Bian and Butler, 1999). At a spatial resolution of 250m, the model needs to allocate a total of 826 residential cells to represent actual developments. At 500m resolution, the model needs to allocate 202 cells. Because asynchrony of residential decisions is assumed, 826 and 202 time steps are needed in these cases. The two (aggregated) target maps are presented in the top-left tiles of Fig.6.6 and Fig.6.7 in order to allow visual



(a) Communes and planning zones

Brussels Region Flemish Region Residential land Non-residential land
Planning zone Communal limits (and Name)



(b) Isochrones to central Brussels (calculation based on road network)

Brussels Region Flemish Region Residential land Non-residential land
1 Minute isochrone (blue figures) Communal limits

Figure 6.4: Observed (target) land use in 25 x 25 km study area and GIS inputs (planning zones and commuting time)

comparisons with the results of the model.

The theoretical processes described above and in the theoretical chapters are not *a priori* related to a specific scale and spatial resolution. That is why the application is undertaken for two spatial resolutions. How sensitive the model is to the spatial definition of the agents is a question to consider. In a CA approach where behaviour is assumed, agents correspond to a cell. Given the chosen resolutions in this example, they correspond more to a group of residents rather than to individuals. Moreover, the model considers two distance attributes, which are influenced by varying the size of the spatial units (Tagashira and Okabe, 2002): the commuting distance and the neighbourhood size. The latter is measured as a number of cells and so, varying the spatial resolution also changes the extent to which neighbourhood externalities are perceived. The leapfrog of a 250m cell might not represent the same process than a 500m. Note that the dependency of neighbourhood characteristics on the spatial resolution has been emphasised by Verburg et al. (2003) and that the results of the optimisation of the parameters of a CA model have been shown to differ substantially when the spatial resolution is changed (Jenerette and Wu, 2001).

An initial land use map is required for running the model. As the land use dataset was not available for two different periods of time, an initial map is created, which is occupied only by farmers. This is a strong assumption as the model must then be able to reproduce the whole urban settlement system. This is unlikely because the assumption are related to periurbanisation processes and the parameters in the model are kept constant throughout a whole model run. A first GIS constraint, the planning zones, will however guide the residential developments towards the observed. In Fig.6.4a, planning restriction zones are shown. The grey areas on the map represent areas where building is allowed. The model cannot generate residential use in any of the yellow parts of the map. Planning zones are thus used as a proxy for other important heterogeneous landscape features, including topographical constraints or existing settlements and villages. It will make the simulation results closer to reality. We know, however, that in Belgium restrictions have not had much impact on residential dispersion and have led to a non parsimonious use of land. The share of residential space filling does not decrease continuously with distance within the planning zones whereas it does, when the whole surface is considered. Therefore, the use of planning zones does not induce the morphological results of the model.

Another raster GIS input layer is used in the simulation that represent the commuting cost in a non-Euclidean way. The commuting cost is calculated in travel time units from a GIS cost distance algorithm based on the main road network. The largest portion of the transport cost is the time (Anas et al., 1998). Local roads are not considered as this detail is not necessary, given the spatial resolution of the simulations. The calculated isochrones are reported on

Fig.6.4b). A minute distance characterises any single cell of the area that can potentially be converted in residential use.

6.2.2.2 Simulations

A set of simulations were run to analyse the sensitivity of the macro-patterns to changes in the parameters, including the preference of individuals for neighbourhood density. Another simulation, independent of the model, was used as a benchmark to interpret the value of the macro-structural indices.

When a single cell represents the location of a decisional entity, it is necessary to define the geographical meaning of a cell and the spatial resolution of the simulations. The choice of the size of the neighbourhood and its shape is then not independent and becomes meaningful. Spatial resolution is no longer just an issue of data availability, quality or computation capacity, but enters the modelling process itself. The spatial resolution issue was tackled explicitly in this analysis: simulations were run on two different spatial resolutions and a sensitivity analysis was undertaken on the size of the neighbourhood where externalities are perceived. Three increments were defined for the two spatial resolutions. The maximum distances considered, i.e. the radius of the neighbourhood, were 1.5 (the Moore neighbourhood), 3 and 5 cells. These radii correspond respectively to a total of 8, 28, and 80 cells. Because of the use of different resolutions, the neighbourhoods considered ranged from 250 to 2500m. This range is simply assumed to cover the possible perception of migrants who want to choose a location in the area.

The sensitivity analysis on the economic parameters was then limited to the parameter to 'optimise': the value of the ratio β/γ was varied from 1 to 3 by increments. A value of 1 indicates an equal preference for social and for environmental externalities. In this case the optimal density (ρ^*) corresponds to the occupation of 1/4 of the neighbourhood by residents.

Running simulations on a real case study in an exploratory or prospective way can benefit from the addition of inputs such as planning zones which constrain the simulated patterns further toward the observation. However, when incorporating these types of data it is necessary to carefully interpret the fit of the simulation to the observed dataset. Specifically, there is a risk of measuring the role of the inputs which constrain the simulation rather than the adequacy of the processes implemented within the model. For example, by randomly choosing the location of the 826 (or 202) residents within the cells available within the planning zones, the measures of global structure might already be similar to the measures made for the target maps (the observation). Thus the contribution of the model must be measured independently from the contribution of the constraining inputs.

In order to solve this problem (at least partly), a random simulation was undertaken for the same land use constraints, and the macro patterns measured. Rather than undertaking a simple random simulation, a distribution of probabilities was used that follows the observed gradient of residential density against distance to the city centre. The probability distribution was constructed from the fit of an exponential curve to the observed data according to the classic law of Clark (1958). This law can be derived from the monocentric model of the urban economy on which the simulation is based. The resulting patterns were then used as a benchmark for the fragmentation and fractal measurements. In this way, the differences between the modelled simulations and the random simulations represent the additional effect of the neighbourhood externalities.

6.3 Results

Figures 6.6 and 6.7 show that where $\beta = \gamma$, the resulting patterns appear to be more compact and most different from the observed patterns. Visually, the model becomes closer to the observed patterns when the preference for rural amenities is increased, and this observation is supported by the values of fragmentation and fractality (see table 6.3). This still occurs in spite of the simple economic assumptions within the model.

For fragmentation (i) the observed map has 74% of the edges of residential cells in contact with farmers at 500m resolution. This increases to 78% at 250m resolution, as in general there is more variation at a lower resolution. The simulations with a value of β/γ of about 2 or 3 (depending on the spatial resolution) have an $\mathrm{H_{FRAG}}$ index that is closest to the observation. The resolution seems to have a strong influence on the results. Whilst visually the size of the neighbourhood (as observed in figures 6.6 and 6.7) seems to affect the size of the clusters, when looking at the three values of the fragmentation index, reported for each β/γ value, such a trend is not apparent.

For the fractality ((ii) and (iii)) the RADIALSLOPE value measured for the target map at 250m resolution equals 1.65, and 1.79 at 500m resolution. The radial dimension and the fit of the curve seem to be less affected by the spatial resolution.

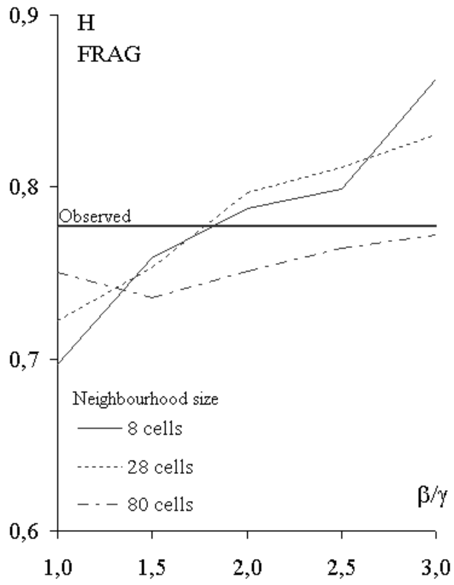
The evolution of the fragmentation level and the radial fractal dimension at 250m resolution are presented in figure 6.5. Although a more precise analysis could be undertaken the figures already display some interesting trends as well as representing the difficulty of calibrating such a model. This is especially true for the fragmentation level. Varying the size of the neighbourhood from 28 to 80 cells increases the range of possible parameter values of β/γ that fit the observed fragmentation. Whereas the best fit points to a value of 3.0 with a neighbourhood composed of 80 cells, a very similar fragmentation level can be obtained with $\beta/\gamma = 2.0$ with an 8 cell neighbourhood. Further analysis could be used to identify precisely the equi-final set of parameters. The fractal dimension points towards values of β/γ ranging from 2.0 to 2.5. Fitting the whole fractal curve, points to a similar, although higher, value for the landscape preferences.

The level of fragmentation generated by the random simulation is too high, but surprisingly quite close to the observation. The radial dimension is however quite weak compared to the observed. In general, the performance of the random simulation is not as good as the micro-economic model. Since the probabilistic distribution accounts only for the decreasing density of settlements with commuting distance, this result indicates the important contribution of the neighbourhood interactions, i.e. the externalities, in simulating good spatial patterns.

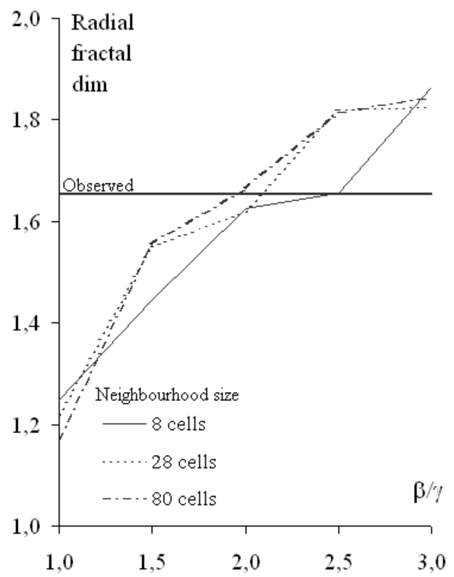
The distribution of density against distance within the planning zones (iv) is far from a continuously decreasing function when measured on the target map. Hypothetically this might represent leapfrogging processes or the role of local subcentres. In contrast to the results presented above, this index suggests much lower values for β/γ. However, compared with the fractal curve, the fit is not good whatever the parameter values. The simulated distributions of density within the zones barely correspond to the observation.

This result demonstrates how strongly the simulation patterns are influenced by the planning constraint. This constraint is very important, therefore, when building predictive models. While the fractality or the fragmentation is quite well replicated by the model, the structure within the planning zone is not. It is still possible, however, to obtain better results than with a random simulation. The neighbourhood effect seems to be important, therefore, in grouping migrants into clusters. It is the location of these clusters which is not well simulated: with a low β/γ value, the compaction is too strong, and the urban expansion too small, whereas with a high β/γ value, the density gradient is extremely flat and not correlated with the observation.

It is possible that this reflects the absence of an initial configuration of small urban nodes that are encroached upon by the urbanisation dynamic of Brussels. However, the addition of small initial residential settlements does not substantially change the results since the dynamic which is represented within the model is essentially monocentric. More precisely, the problems encountered with this criterion reflect the absence in the model of a way to account for local dynamics created by subcentres. The distance to Brussels is 'tyrannic', and even if leapfrogging patterns are possible, no process of counterurbanisation is



(a) Fragmentation level



(b) Radial fractal dimension

Figure 6.5: Evolution of two macro-structural indices when varying the preference for rural amenities (β/γ) for three neighbourhood sizes (at 250m resolution).

	Fragmentation H_{FRAG}		Radial fractal dimension RADIALSLOPE		Radial fractal curve R^2_{RADIAL}		Within planning density *vs* distance $R^2_{DENSPLAN}$	
	500m	250m	500m	250m	500m	250m	500m	250m
Observed	0.7388	0.7771	1.7887	1.6538	1.0000	1.0000	1.0000	1.0000
Random	0.7939	0.8088	1.3732	1.4984	0.8336	0.8229	0.1431	0.1641
β/γ 1.00	0.3886	0.6970	1.1206	1.2488	0.6851	0.7318	0.1677	0.3027
(ρ^* 0.250)	0.4496	0.7222	1.1599	1.2168	0.7180	0.7188	0.1595	0.3096
	0.3798	0.7505	1.1746	1.1718	0.7175	0.7073	0.1695	0.3020
β/γ 1.50	0.6446	0.7590	1.4255	1.4451	0.7666	0.7944	0.2260	0.1717
(ρ^* 0.111)	0.6221	0.7534	1.5508	1.5496	0.7936	0.8029	0.2073	0.1860
	0.5267	0.7356	1.4934	1.5547	0.7801	0.7944	0.2913	0.2124
β/γ 2.00	0.7205	0.7877	1.6022	1.6257	0.7997	0.7953	0.2034	0.1127
(ρ^* 0.063)	0.7231	0.7968	1.7148	1.6174	0.8544	0.8629	0.0793	0.0293
	0.5840	0.7508	1.6422	1.6659	0.8372	0.8587	0.0580	0.0613
β/γ 2.50	0.7599	0.7989	1.6814	1.6566	0.8162	0.7975	0.1682	0.0613
(ρ^* 0.040)	0.7815	0.8114	1.7232	1.8201	0.8808	0.8835	0.0845	0.0866
	0.6598	0.7645	1.7746	1.8145	0.8766	0.8897	0.0477	0.0086
β/γ 3.00	0.8229	0.8630	1.9163	1.8633	0.8882	0.8743	0.0002	0.0225
(ρ^* 0.028)	0.8013	0.8302	1.8237	1.8247	0.8978	0.9018	0.0001	0.0074
	0.7073	0.7721	1.8264	1.8460	0.8970	0.9050	0.0170	0.0008

Table 6.1: Macro-structural indices for two spatial resolutions. The first line corresponds to the observed values. The second presents results from the random simulations based on the probability decreasing with distance. Subsequent lines display the results for simulations with preference for rural amenities increasing from 1 to 3. Within these lines the 3 values correspond to three neighbourhood sizes (respectively 8, 28, and 80 cells, from top to bottom). For indicative purposes, a value is underlined for each criterion which corresponds to the best fit.

simulated. From this perspective, the model appears to be incomplete.

6.4 Discussion and conclusion

The analysis presented here was exploratory and the model simple in its formulation of residential processes. It has, however, developed an economic interpretation of land conversion within a CA framework, provided a methodology for estimating agent behaviour based only on macro-geographical outputs, and demonstrated the usefulness of undertaking sensitivity analysis of the spatial resolution and neighbourhood size when establishing values of behavioural pa-

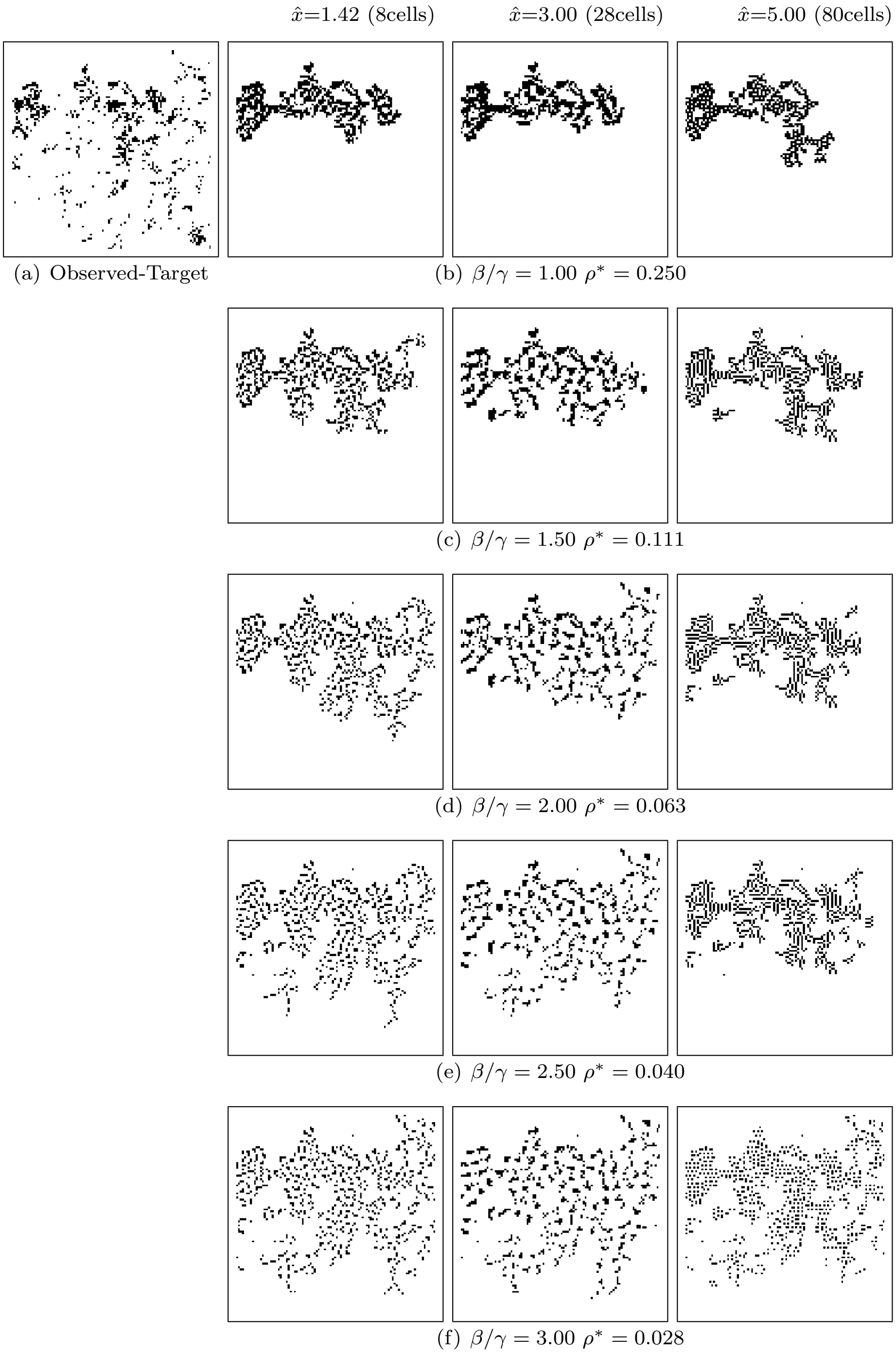
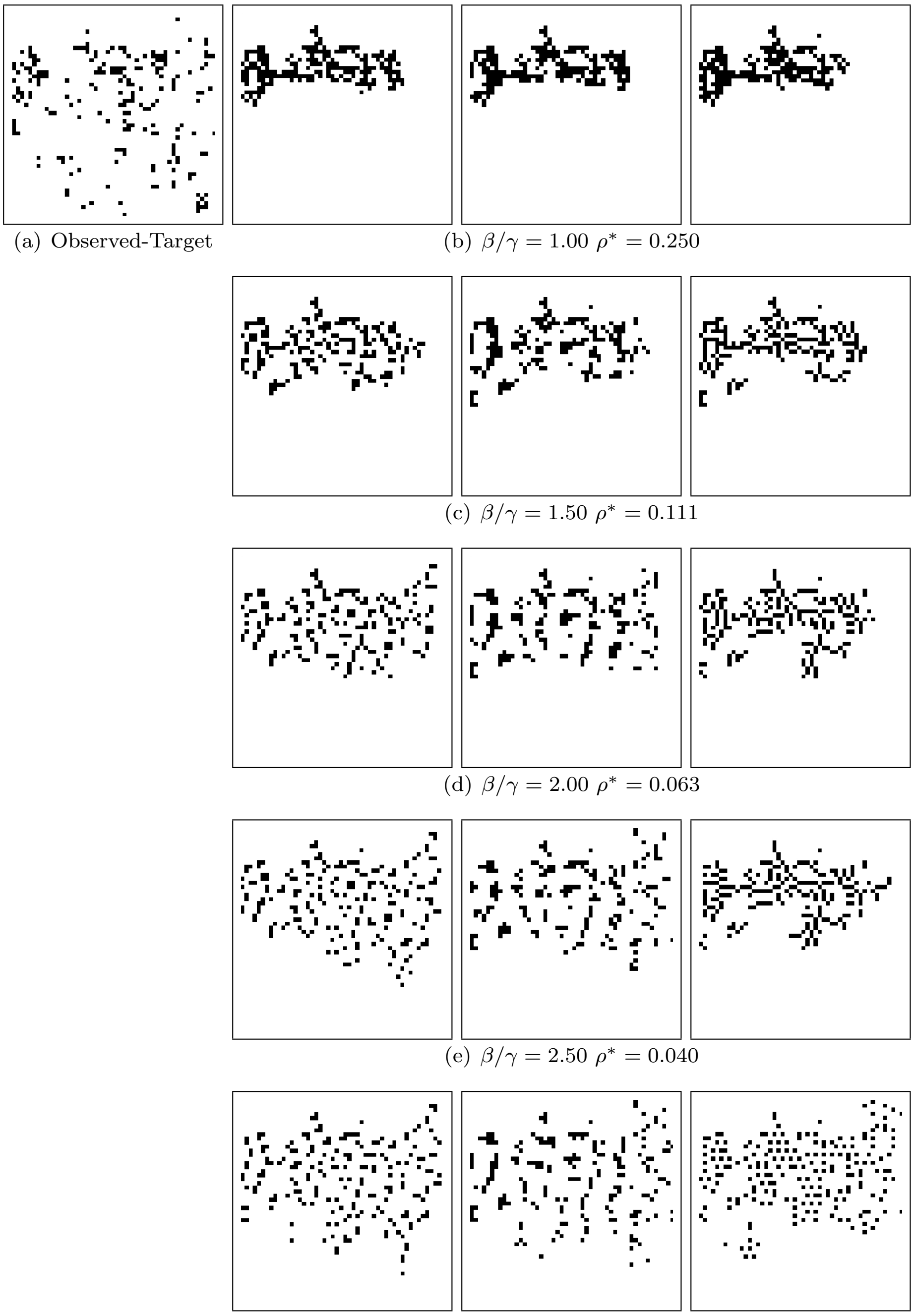


Figure 6.6: Spatial patterns in study area (25 x 25 km) at 250m resolution. From top to bottom: increasing preference for rural amenities (β/γ). From left to right: increasing neighbourhood extent ($\hat{x}$).

$\hat{x}$=1.42 (8cells) $\hat{x}$=3.00 (28cells) $\hat{x}$=5.00 (80cells)



(a) Observed-Target

(b) $\beta/\gamma = 1.00$ $\rho^* = 0.250$

(c) $\beta/\gamma = 1.50$ $\rho^* = 0.111$

(d) $\beta/\gamma = 2.00$ $\rho^* = 0.063$

(e) $\beta/\gamma = 2.50$ $\rho^* = 0.040$

(f) $\beta/\gamma = 3.00$ $\rho^* = 0.028$

Figure 6.7: Spatial patterns in study area (25 x 25 km) at 500m resolution. From top to bottom: increasing preference for rural amenities (β/γ). From left to right: increasing neighbourhood extent ($\hat{x}$).

rameters.

The sensitivity analysis is limited and is not aimed to represent a full calibration and test of the model. However, it has demonstrated the necessity when modelling processes within a CA type framework of varying parameters in order to understand their impacts on the resulting global spatial structures. A limitation of the approach is the lack of automatic methods that would allow for an exploration of a larger set of possible solutions and their ability to reproduce geographical patterns.

From the sensitivity analysis, new residents around Brussels seem to prefer landscape amenities 2.5 times as much as social amenities. Nevertheless this result is quite sensitive to the ways in which the macro structure, i.e. the 'finality', is measured. Different values might be found for this model on the basis of different objective functions. This also underlines the implications of choosing an objective function based on a single criterion, instead of trying to fit multiple criteria. Technical and methodological improvements to tackle this problem should be investigated.

While empirically-based models may be able to reproduce the observed patterns in a more efficient way, fine tuning a deductive model to a geographic output is also possible and useful as it can potentially provide information about underlying processes.

Empirically-based models of urban growth are probably easier to calibrate because the values of the parameters are important for the fit of the model to the observation, rather than as a way of interpreting processes. These models are certainly useful to explore trajectories of change, but it is challenging to calibrate deductive behavioural models because of the potential to interpret the model parameters. This interpretation, in turn, offers an opportunity to test the model with independent sources of information and methods. For example, the spatial distribution of modelled residential potential values (which represent residential bid rents) could be compared to observed distributions of land values. Preferences could also be found from household surveys or hedonistic estimations of land values.

Analysing urban growth patterns by means of an economically-based cellular dynamic model can contribute to further understanding of residential choice. The use of a CA type framework can overcome two limitations in the application of urban economic models: the difficulty in accounting for sequential location dynamics and in dealing with geographically detailed information. The dynamic nature and the role of neighbourhood externalities are important aspects of the urban diffusion process. Both are emphasized in the model presented here. Once properly calibrated, therefore, the model could provide applications that contribute to thinking about spatial management and urban sustainability.

For the moment, however, the experiment is too limited. The calibration procedure showed the ability of the model to represent an important part of the observed diffusion patterns. The strong residential dispersion that is observed around Brussels can be reproduced. The process of increasing dispersion of dwellings in the commuting zone indicated by recent trends (Halleux et al., 1998), and which is likely to continue, suggests that the model is a satisfactory model for the Brussels periphery. However, the level of dispersion is very high in Belgium compared to other countries (see the CORINE map in chapter 2). Whether a more compact development could also be reproduced with this model and thus indicate less preference for the greenness (dispersion) amenities is still an open question and would require a better calibration procedure.

Moreover, the model was unable to show the densification dynamics that have occurred around subcentres and in area of earlier periurbanisation. De Keersmaecker et al. (2004) have shown that in these zones, especially those situated at good communication nodes for accessing Brussels (rail stations), the built-up morphology is denser and contrast with more recent development in more remote areas being more dispersed. The theoretical experiments presented in chapter 3 and 4 have shown that the model can generate a gradual infill of zones that were previously leapfrogged, i.e. densification. The model being monocentric, densification would start with the locations (at constant level of neighbourhood externalities) that are closer to the CBD. Regarding this process, a problem of this calibration experiment is due to the fact that it is stopped after a given number of time steps (independently of the long-run equilibrium) and, either cannot show sufficient local concentration processes, or cannot produce remote development. However, keeping a constant amount of total development is useful for comparing forms.

Finally, the calibration procedure has only taken into account the relative weight of both preferences and does not relate their intensity with the importance of commuting costs in the households budget. Nor the relative weight of the other consumptions of the utility function are considered. Therefore, rent value cannot result from this experiment and the importance of the externalities can therefore be over or underestimated. The calibration is partial, further research should improve the experiment by combining both observed forms and rent values. This is necessary for testing the residential hypothesis that have been made in the model.

Chapter 7

Conclusion

The objective of this thesis was twofold. On the thematic side, the goal was to further understand how mixed patterns can emerge through time at the periphery of a city. On the methodological side, the goal was to integrate urban economics with cellular automata in order to explicitly relate residential decision making and the emergence of spatial forms. It is believed that these two objectives have been fulfilled within this work.

The next section summarizes the main insights given to periurbanisation through the theoretical experiments undertaken in chapter 3, 4 and 5, and the partial calibration implemented in chapter 6. The advantages of the methodological approach are then highlighted. The third section offers a critique of the research and presents some perspectives.

7.1 Understanding periurbanisation patterns

Periurbanisation is general in Europe. Most cities are surrounded by a belt where residents and farmers mix. In these areas, most households are commuters. The population and surface of these periurban belts is important and increasing. European periurban households state preference for green and pleasant living conditions with good accessibility to jobs and services. This suggests the usefulness of modelling local environment preferences within a distance-based analysis framework.

The spatial morphology of periurban areas vary a lot from place to place, depending on local or regional characteristics, including spatial planning. It has been shown through our modelling experiments that spatial morphologies can also vary strongly with respect to the relative weight given by households to neighbourhood preferences, i.e. social amenities (or local public goods) and greenness amenities (or local open-space).

In the thesis, it was assumed that agglomeration-dispersion forces at the scale of a neighbourhood and agglomeration-dispersion forces at the level of the commuting region (trade-off between accessibility and housing consumption) lead to the emergence of a mixed land use pattern in a peripheral belt around the city. These assumptions were implemented in a micro-economic, spatial, dynamic, discrete, and path-dependent simulation framework. The framework highlighted the role of the determinants of residential choice on the emergent morphology of urban development. The main findings are summarized below.

Size and form of the city and periurban areas. From the theoretical experiments in chapter 3, it has been learned that when households have a preference for local public goods or social interactions within their neighbourhood, the equilibrium city is more expanded and populated (with constant growth, the stationary state appears later). When households have stronger preference for a green environment, residential growth starts leapfrogging agricultural parcels. A mixed belt therefore arises at the periphery of a specialized urban core. These are quite intuitive, but important results for our purpose because a periurban area emerges and is shown to be an optimal equilibrium configuration.

What is less intuitive is that the preference for local agglomeration also lead to a flattening of the urban fringe, which departs, therefore, from the classic circular view of urban development. Moreover, increasing the preference for greenness tends to reduce the total expansion and population of the city, which seems counter-intuitive. The internal morphology and level of fragmentation of periurban areas have been found to depend on the relative weight given to local greenness (the more important, the more leapfrogs) and on the extent of the neighbourhood that is considered (the wider, the more clustered are the settlements).

Furthermore, by considering housing lot size in the residential decision (as in chapter 4), the size of rural leapfrogs (or conversely residential settlement clusters) is no longer directly dependent on distance, because of the intra-cell density of previously built settlements.

Income and commuting costs. The role of changing income and commuting costs has been emphasized in both chapter 3 and 4 for a single household type. Contrarily to Cavailhès et al. (2004b), the results of the model suggest that while increasing income benefits to the specialized or compact part of the city, decreasing commuting costs lead to further expansion of the mixed periurban belt.

Land values. Within the periurban zone, the equilibrium rent profile is blurred with local inversions. Residential land values vary not only with distance, but also according to neighbourhood quality. Furthermore, they contrast sharply with remaining agricultural parcels. This suggests that, through the dynamic of urban growth, people arrange themselves in a way that fit their preferences, and that there is no need of planning restriction to keep open-space within a periurban zone. All mixed equilibrium structures obtained from the single household type models in chapter 3 and 4 are example of this. Also, the spatial structure of the 360° metropolitan area in chapter 5, where people have strong segregation preference, is another striking example of 'self-organising green belts'.

Land values change through time as a response to the urban growth effect: rent increases with increasing commuting distance of the last migrant. But land values are also affected by local change in the quality of the neighbourhood. A periurban resident may have several new (unexpected) neighbours as time progresses. These newcomers affect his well-being positively, but more often negatively as previously he choose locations with optimal neighbourhood characteristics (where possible). Because their location is irreversible and because they are myopic, periurban households cannot avoid this process, as they already maximized their utility at the time of their migration. However, as long as landowners adapt the rent, residents compensate for possible losses by other consumptions (composite good in chapter 3 but also housing in chapter 4 and 5).

Timing of urban expansion. Chapter 4 has complemented the results of chapter 3 experiments by showing the dynamics of different coexisting density structures. The expansion speed of the commuting field has been shown to vary with the migration rate as well as according to the relative preference of households for greenness and the neighbourhood size that they consider. Furthermore, introducing a preference for neighbourhood public goods delays the emergence of the periurban belt. This suggests that cities need to achieve a certain threshold size before being surrounded by a mixed belt.

Green belt policy. It was suggested that a green belt policy can delay urbanisation and reduce fragmentation in areas situated beyond a restricted area. A green belt was also shown to decrease land values in the central part of the city (at non constant population) and increase the value of the location that surround the restricted area.

Segregation patterns. Chapter 5 has shown the complexity of spatial organisations and dynamics when households are different in terms of income and preferences. Upward and downward filtering of housing appears at the border between communities or creates additional rings. The scattered urbanisation structure obtained from chapter 3, i.e. the relative position of urban and agricultural cells, can be affected by filtering processes and differing social preferences. Urban growth arranges rural parcels differently in order to accommodate neighbourhood social preferences. Also, scattering can occur in the centre and not in the periphery under certain conditions, where the less well-off households prefer to benefit from urbanisation overvalue rather than building on agricultural land.

While less well-off households occupy the centre at long-run equilibrium, the border between the communities is pushed away from the centre when the preference of the rich households is more restrictive. Moreover, the general core-periphery pattern can be modified. Non concentric structuring of the two communities can occur and be maintained in the long-run because of path-dependency and niche local arrangements. Important turnover forces and instability appear when the poor households search for opportunities by locating in the neighbourhood of rich people who discriminate them.

Social policies. Transport and income subsidies to the poorer group are not de-segregation policies but result in a stronger suburbanisation of this group. Depending on the parameters, i.e on the level of the subsidy, their bid rent can nearly equal the rich ones and, therefore, in presence of social discrimination on the part of both households, lead to segregated patterns that are no longer distance-based. In these patterns, agriculture land use plays the role of a buffer zone between the two communities.

The implementation of social housing within rich areas de-structure the local spatial arrangement of green cells when households have different social preferences. Furthermore, when social policies diverge from one part of the city-region to another, dynamics are even more complex in intermediate steps and might lock-in unexpected spatial structures at equilibrium.

7.2 Integration of urban economics and CA

In terms of the methodology, the thesis was an attempt to fill the gap between applications of cellular models in geography that can simulate the observed complexity of spatial structures (with neighbourhood spatial interactions) and models that propose an economic understanding of urban processes. It is believed that the results summarized in the previous section clearly demonstrate that there are important gains from this integration for the understanding of urban diffusion.

Furthermore, the thesis has shown that, from a quite simple set of assumptions, cities can already be conducted to great morphological and dynamic diversity. It is argued, therefore, that the addition of supplementary processes in urban simulation and model building should be made very carefully as long as one wants to understand why the city can become a complex object.

An economic perspective is important for deriving the residential well-being and global costs of a spatial structure. Arguably, both components are to be taken into account by planners and policy makers to orientate the development of the city and its periphery in a sustainable manner. The costs of residential dispersion and space fragmentation should be compared to the additional well-being afforded by pleasant living conditions, as well as to potential side-effects of restrictive spatial policies. Cellular automata bring the spatial forms and the dynamics in this framework. They can also make the model more operational through the addition of geographical details, provided that suitable raster information can be found.

The interest of taking an economic approach resides also in that the model is grounded on a set of robust urban mechanisms. Findings from the cellular simulations can then be compared to the numerous results brought by static or dynamic urban economics since about forty years. A cellular approach complements these studies by further describing the spatial arrangement of land uses. As it is shown in this thesis a mixed zone can be characterized by a range of fragmentation levels. An additional dimension is thus added to what is a non-compact city expansion. Moreover, the model in this thesis can also be seen as an attempt to consider neighbourhood externalities in a dynamic urban model with irreversible housing.

A final important gain that arises from driving CA with economic processes in the way it has been done here, is related to the simulation of a land market. This is a strong advantage for application purposes. In fact, in a certain way, CA postulates that neighbourhood is the most important feature of land structure. Therefore, if the neighbourhood is taken into account in residential choice, one should be able to measure how important it is. The preference for a neighbourhood environment is not observable in reality. However, it is present within land values. Therefore, simulating a land market, i.e. with land values as an output at each period of time, appears as a strong opportunity for testing cellular dynamic models against an independent source, i.e. observed land rents.

7.3 Weaknesses and perspectives

One can find many weaknesses of this research, that are related to modelling simplifications. For example, neighbourhood quality is summarized by neighbourhood density (of land use or population) while equally dense areas can be very differently appreciated; no physical landscape heterogeneity is modelled while it can impact greatly on residential choice; housing is never degraded or renovated; individuals have similar preferences throughout the course of their life; urban landowners can appreciate the value of their lots at any moment in time and immediately adjust rents;... . These weaknesses results from modelling choices. Improving or relaxing some of these assumptions could represent interesting perspectives for the research. Some prompts for a complete reworking of the model, while others prompts for continuing the research with a similar model or little adaptations.

7.3.1 Towards another framework

From the calibration test to the Brussels area proposed in chapter 6, the addition of neighbourhood processes to distance-based processes has been shown to be important for fitting the patterns to reality. However, the modelled processes were not able to reproduce the emergence, nor the growth, of periurban subcentres. The approach is monocentric, but in reality, periurbanisation processes are mix with counter-urbanisation of settlements within commuting fields, and the emergence of edge-cities. (This brings back to Krugman's view on von Thünen with respect to self-organisation. While von Thünen model qualifies to the notions of complexity and emergence, it is not, however, a self-organizing framework as the existence of the city is given exogenously (Krugman, 1996a, p.12)).

In practice, in the model presented here, as soon as there is some preference for social externalities, there is no need for an exogenously given centre. The first resident will play the role of an attractor, whatever its initial location. However, there is no point to do so, as residential choice is also related to employment location. A mechanism could also be implemented, where new seeds of jobs emerge through time in the periphery. Theoretical mechanisms and empirical evidence can be found for the development (e.g. Garreau, 1991) or endogenous formation of subcentres (see review of Anas et al., 1998). Using a CA framework, Wu (1998a) suggested that polycentric structures are developed because of the unusual location of a firm a resident. This 'niche' location must then be reinforce through time because of local interactions (agglomeration spillovers) to create a subcentre.

A second problem of the model resides in the consideration of an open-city framework. An open-city is adequate for periurban areas because they are growing, and also because the focus is on the long-run equilibrium. However, comparisons of different simulations, for example urban spread with and without a green belt policy, are made difficult as the population is not constant. A close-city framework can be envisaged, although it would be more suited to short-term adjustments, and does not seem to be applicable as long as the system consider land use irreversibility and immobility.

In Schelling (1971) model, mobility of people is assumed. A constant population re-arrange through time according to preferences. In contrast to Schelling, the dynamics is here applied to cells, and cells are assumed to represent the use of land rather than people. Or, more precisely, a decisional agent corresponds exactly to a land use. The irreversibility assumption is therefore reasonable, an urban use cannot be turned back into a rural use. One might think, however, that people are mobile within the urban land. An interesting perspective would thus be to differentiate the land use from the occupants. Some urban land could then be unoccupied. This point has been addressed together with the emergence of the Multi-Agent-Systems (MAS) approach in geography (Benenson et al., 2004; Portugali, 2000; Benenson, 1998; Batty, 1998). It might be useful, therefore, to think the processes presented here in a MAS perspective.

Furthermore, the spatial pattern at a given time, in CA, is a function of the spatial pattern at the previous time and other spatial characteristics (e.g. distance). Therefore, CA are not suited to implement forward-looking behaviour on the part of landowners and residents. A solution might come again from MAS, where behaviour with expectations would be assumed to the agents. Yet, trying to introduce rational expectations in evolutionary type models make them vastly more difficult (Krugman, 1996b).

Finally, urban space is considered as a place of consumption in the model. It is a good assumption when considering open-city and neighbourhood externalities (Glaeser et al., 2001, see the consumer city of). Nevertheless, more attention could be paid to the offer side and the behaviour of land developers. In more operational model, with an applied perspective, their role is usually emphasized. For example, the spatial microsimulation of Waddell and Ulfarsson (2003) implements landowner choice in a multinomial logit formulation. Indeed, the link between the model and microsimulations of urban development could be improved and the approached combined. These models are usually drawn on discrete choice theory and random utility maximisation. A greater diversity of preferences and households, reflecting surveyed population, is represented.

7.3.2 Complementing and adapting the framework

Some of the theoretical findings presented in this thesis are weakened by the fact that they result from a limited set of simulation experiments. However, some of the 'Results' can be directly derived from the assumptions, without resorting to simulation. Findings are probably less obvious for the segregation simulation (chapter 5), where the number of parameters was increased because of the two household characteristics, the different social behaviours and a relaxation of the immobility assumption. However, it is believed that a taste of the increasing complexity of the model has been provided in that case. Future work should handle less processes, but in a more detailed manner.

Refined and additional sensitivity analyses are certainly necessary in this type of exercise in order to clearly identify bifurcation points and to understand the role of different assumptions. For example, the impact of the asynchrony assumption on the spatial patterns has not been explored. Nor has been the role of changing long-run equilibrium utility in the single household cases, and utility and income differentials in the case of two households. Also, a wider set of parameter values could have been tested for the weight of the externalities with respect to the other goods that determine households utility.

The importance of modelling a land market has been mentioned above, with respect to testing the model to observed land values. No such test was implemented in this thesis. In the Brussels example (chapter 6), locations are ranked according to relative bid-rents. However, although a ratio of both externalities has been measured, their global weight in the preferences of periurban households is unknown and thus generated land value cannot be compared to the observed. Furthermore, the exploration is based on the model of chapter 3, which considers constant housing lots. The role of housing consumption in the residential trade-off could be further analysed.

More generally, further research needs to be undertaken on the calibration procedure, including multi-objective (e.g. using both land values and forms) and non-linear methods. It is a challenge to find rigorous way to calibrate a behavioural CA, like the one presented here, as its dynamic is not driven by observed cell-transitions.

In relation with improving calibration, the use of other macro-structural indices should be researched. Fractal measures are known to be able to represent urban realities (Batty and Longley, 1994; Frankhauser, 1994), and their applications to periurban areas are promising (De Keersmaecker et al., 2004). Moreover, an advantage of fractal analysis is its independence to scale effects. Scale or spatial resolution must be handled carefully when applying CA models because neighbourhood variables are computed across the whole space and repeatedly in time. Here, the model itself is resolution-independent. There is no mention of any cell-size or distance unit in the results delivered from the theoretical experiments. At the moment of a full calibration, however, the attention should be paid to the meaning of a cell-size. A one cell leapfrog can be consistent for different periurban realities, but whether it is a 20m leapfrog or a 200m leapfrog might change its interpretation with respect to the commuting cost.

The agenda should then comprise a test of the model to several case study areas in order to invalidate or generalize the idea that periurban morphologies result from sequential residential choices in a monocentric urban structure, where households consider both the greenness and the social interactions taken in a fixed neighbourhood.

A final perspective of this research is to use the framework to model other neighbourhood effects. The externalities have been implemented in a spatial and dynamic framework that can probably be used to reflect other processes where in-(or de-)creasing returns to scale are assumed within a finite spatial area. The model can be usefully transformed to account for other mechanisms, including the location of production or service activities, at the condition that the evolution of the form is a pertinent feature to analyse.

Finally, despite shortcomings in modelling assumptions and despite the weaknesses of the calibration methodology, it is hoped that this thesis was able to show how Geography and Economy can be usefully integrated, within an explicitly spatial and dynamic framework, in order to think about geographical processes. It is believed that the changing pattern of the location of activities and people can be better understood and that interesting solutions to manage territories can eventually be proposed.

Appendix A

Notes to Chapter 3

A.1 Parametric conditions

A.1.1 Parametric conditions on neighbourhood externalities

A.1.1.1 Convexity of $E(\rho)$

$$E(\rho) = e^{-(\rho)^{\theta}} \tag{A.1}$$

with $\theta > 0$ and $\rho \in [0, 1]$

$$E'(\rho) = e^{-(\rho)^{\theta}} \left(-\theta\rho^{\theta-1}\right) \tag{A.2}$$

$E'(\rho) < 0 \quad \forall\rho$, and $E(\rho)$ is continuously decreasing.

$$\begin{aligned}
E''(\rho) &= e^{-(\rho)^{\theta}} \left(\theta\rho^{\theta-1}\right)^2 - e^{-(\rho)^{\theta}} \left(-\theta(\theta-1)\rho^{\theta-2}\right) && \text{(A.3)}\\
&= e^{-(\rho)^{\theta}} \left(\left(\theta\rho^{\theta-1}\right)^2 - \theta(\theta-1)\rho^{\theta-2}\right) && \text{(A.4)}\\
&= e^{-(\rho)^{\theta}} \left(\theta^2\rho^{2\theta-2} - \left(\theta^2-\theta\right)\rho^{\theta-2}\right) && \text{(A.5)}\\
&= e^{-(\rho)^{\theta}} \rho^{\theta-2} \left(\theta^2\rho^{\theta} - \theta^2 + \theta\right) && \text{(A.6)}\\
&= e^{-(\rho)^{\theta}} \rho^{\theta-2}\theta \left(\theta\rho^{\theta} - \theta + 1\right) && \text{(A.7)}
\end{aligned}$$

$E''(\rho) = 0$ when $\left(\theta\rho^{\theta} - \theta + 1\right) = 0$. The curvature of $E(\rho)$ therefore changes at

$$\rho = \rho_c = e^{\left(\theta^{-1}ln\left(\theta^{-1}(\theta-1)\right)\right)} \tag{A.8}$$

The existence of this inflexion point is conditional on $\theta > 1$.

If $\theta \leq 1$	then	$E''(\rho) > 0$	$\forall \rho,$	and	$E(\rho)$ is strictly convex.
If $\theta > 1$	then	$E''(\rho) < 0$	$\forall \rho < \rho_c$	and	$E(\rho < \rho_c)$ is concave,
	and	$E''(\rho) > 0$	$\forall \rho > \rho_c$	and	$E(\rho > \rho_c)$ is convex.

Given $\rho_c \leq 1$ ($\lim_{\theta \to \infty} \rho_c = 1$), $E(\rho)$ is a strictly convex function for $\rho \in [0, 1]$ only when $\theta \leq 1$. The condition for convexity is therefore $\theta \in]0, 1]$.

A.1.1.2 Concavity of $S(\rho)$

$$S(\rho) = e^{(\rho)^{\phi}} \tag{A.9}$$

with $\phi > 0$ and $\rho \in [0, 1]$

$$S'(\rho) = \phi \rho^{\phi - 1} e^{(\rho)^{\phi}} \tag{A.10}$$

$S'(\rho) > 0 \quad \forall \rho$, and $S(\rho)$ is continuously increasing.

$$\begin{aligned} S''(\rho) &= e^{(\rho)^{\phi}} \left(\phi \rho^{\phi-1}\right)^2 + e^{(\rho)^{\phi}} \phi \left(\phi - 1\right) \rho^{\phi - 2} && \text{(A.11)} \\ &= e^{(\rho)^{\phi}} \left(\phi^2 \rho^{2\phi - 2} + \left(\phi^2 - \phi\right) \rho^{\phi - 2}\right) && \text{(A.12)} \\ &= e^{(\rho)^{\phi}} \rho^{\phi - 2} \left(\phi^2 \rho^{\phi} + \phi^2 - \phi\right) && \text{(A.13)} \\ &= e^{(\rho)^{\phi}} \rho^{\phi - 2} \phi \left(\phi \rho^{\phi} + \phi - 1\right) && \text{(A.14)} \end{aligned}$$

$S''(\rho) = 0$ when $\left(\phi \rho^{\phi} + \phi - 1\right) = 0$. The curvature of $S(\rho)$ therefore changes at

$$\rho = \rho_c = e^{\left(\phi^{-1} ln\left(\phi^{-1}(1 - \phi)\right)\right)} \tag{A.15}$$

The existence of this inflexion point is conditional on $\phi < 1$.

If $\phi \geq 1$	then	$S''(\rho) > 0, \forall \rho,$	and	$S(\rho)$ is strictly convex.
If $\phi < 1$,	then	$S''(\rho) < 0, \forall \rho < \rho_c$	and	$S(\rho < \rho_c)$ is concave,
	and	$S''(\rho) > 0, \forall \rho > \rho_c$	and	$S(\rho > \rho_c)$ is convex.

$S(\rho)$ must be concave in the range $[0, 1]$, which is therefore conditonal on $\rho_c \leq 1$, that is

$$\begin{aligned} ln\left(\phi^{-1}\left(1 - \phi\right)\right) &\leq 0 && \text{(A.16)} \\ \phi^{-1}\left(1 - \phi\right) &\leq 1 && \text{(A.17)} \\ \phi &\leq 0.5 && \text{(A.18)} \end{aligned}$$

The condition of concavity is therefore $\phi \in]0, 0.5]$

A.1.2 Conditions for optimal neighbourhood density

A.1.2.1 Characteristics of $L(\rho)$

$$L = E^{\beta} S^{\gamma} = \left(e^{-(\rho)^{\theta}}\right)^{\beta} \left(e^{(\rho)^{\phi}}\right)^{\gamma} = e^{\left(\gamma\rho^{\phi} - \beta\rho^{\theta}\right)} \tag{A.19}$$

$$L(\rho = 0) = 1 \tag{A.20}$$

$$L(\rho = 1) = e^{\gamma - \beta} \tag{A.21}$$

$$L = 1 \quad \Rightarrow \quad \gamma\rho^{\phi} - \beta\rho^{\theta} = \rho^{\theta}\left(\gamma\rho^{\theta-\phi} - \beta\right) = 0 \tag{A.22}$$

$$\Rightarrow \quad \rho = 0 \ or \ \rho = \left(\frac{\beta}{\gamma}\right)^{(\phi-\theta)^{-1}} \tag{A.23}$$

A.1.2.2 Optimum

$$L'(\rho) = e^{\left(\gamma\rho^{\phi} - \beta\rho^{\theta}\right)} \left(\phi\gamma\rho^{\phi-1} - \theta\beta\rho^{\theta-1}\right) \tag{A.24}$$

$L'(\rho) = 0$ when $\phi\gamma\rho^{\phi-1} - \theta\beta\rho^{\theta-1} = 0$, that is

$$\rho^{\theta-\phi} = \frac{\phi\gamma}{\theta\beta} \tag{A.25}$$

$$\rho = \rho^{*} = \left(\frac{\phi\gamma}{\theta\beta}\right)^{1/(\theta-\phi)} \tag{A.26}$$

The existence of the optimum at ρ^{*} is conditional on $\theta \neq \phi$ and $\theta, \phi, \beta, \gamma \neq 0$

A.1.2.3 Characteristics of the optimum

Given the characteristics of $L(\rho)$, ρ^{*} is a maximum when $L(\rho^{*}) > 1$, that is when (see above):

$$\gamma(\rho^{*})^{\phi} - \beta(\rho^{*})^{\theta} = (\rho^{*})^{\theta}\left(\gamma(\rho^{*})^{\phi-\theta} - \beta\right) > 0 \tag{A.27}$$

or, as $(\rho^{*})^{\theta} > 0$, when $\gamma(\rho^{*})^{\phi-\theta} - \beta > 0$

With $\rho^{*} = \left(\frac{\phi\gamma}{\theta\beta}\right)^{1/(\theta-\phi)}$, the condition becomes $\gamma - \beta\left(\frac{\phi\gamma}{\theta\beta}\right) > 0$, that is

$$\gamma\theta\beta > \beta\phi\gamma \tag{A.28}$$

$$\theta > \phi \tag{A.29}$$

Different configurations can then be described according to the position of the optimum, as we also know that $\rho^* = 1$ when $\theta\beta = \phi\gamma$ (see definition of ρ^*).

The configuration corresponding to all hypotheses is $\theta > \gamma$ and $\theta\beta > \phi\gamma$, that implies a maximum at $\rho^* < 1$. The other configurations being

$$\begin{array}{lcll} \theta > \gamma & \text{and} & \theta\beta \leq \phi\gamma & \text{, max at } \rho^* \geq 1 \\ \theta < \gamma & \text{and} & \theta\beta > \phi\gamma & \text{, min at } \rho^* < 1 \\ \theta < \gamma & \text{and} & \theta\beta \leq \phi\gamma & \text{, min at } \rho^* \geq 1 \end{array}$$

A.2 Sensitivity to model parameters

A.2.1 Sensitivity of optimal density and level of externalities to θ and ϕ change

[1]

ρ^*	φ 0,0010	0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000
0,0010	no opt	min	min	min	min	min	min	min	min	min	min
0,0500	0,0000	no opt	min	min	min	min	min	min	min	min	min
0,1000	0,0000	0,0000	no opt	min	min	min	min	min	min	min	min
0,1500	0,0000	0,0000	0,0003	no opt	min	min	min	min	min	min	min
0,2000	0,0000	0,0001	0,0010	0,0032	no opt	min	min	min	min	min	min
0,2500	0,0000	0,0003	0,0022	0,0060	0,0115	no opt	min	min	min	min	min
0,3000	0,0000	0,0008	0,0041	0,0098	0,0173	0,0261	no opt	min	min	min	min
0,3500	0,0000	0,0015	0,0067	0,0145	0,0240	0,0346	0,0458	no opt	min	min	min
0,4000	0,0000	0,0026	0,0098	0,0198	0,0313	0,0436	0,0563	0,0692	no opt	min	min
0,4500	0,0000	0,0041	0,0136	0,0257	0,0390	0,0529	0,0670	0,0810	0,0948	no opt	min
0,5000	0,0000	0,0060	0,0179	0,0321	0,0472	0,0625	0,0778	0,0928	0,1074	0,1216	no opt
0,5500	0,0000	0,0083	0,0226	0,0388	0,0556	0,0722	0,0885	0,1044	0,1197	0,1344	0,1486
0,6000	0,0000	0,0109	0,0278	0,0459	0,0642	0,0820	0,0992	0,1158	0,1317	0,1469	0,1615
0,6500	0,0000	0,0139	0,0333	0,0533	0,0729	0,0917	0,1098	0,1270	0,1434	0,1590	0,1739
0,7000	0,0001	0,0172	0,0390	0,0608	0,0816	0,1015	0,1202	0,1380	0,1548	0,1708	0,1859
0,7500	0,0001	0,0209	0,0451	0,0684	0,0904	0,1111	0,1305	0,1488	0,1660	0,1822	0,1975
0,8000	0,0002	0,0248	0,0513	0,0761	0,0992	0,1207	0,1406	0,1593	0,1768	0,1932	0,2087
0,8500	0,0004	0,0290	0,0576	0,0839	0,1080	0,1301	0,1505	0,1696	0,1873	0,2039	0,2196
0,9000	0,0005	0,0334	0,0642	0,0917	0,1166	0,1394	0,1602	0,1796	0,1975	0,2143	0,2300
0,9500	0,0007	0,0379	0,0708	0,0995	0,1252	0,1485	0,1698	0,1893	0,2075	0,2244	0,2402
1,0000	0,0010	0,0427	0,0774	0,1073	0,1337	0,1575	0,1791	0,1989	0,2172	0,2341	0,2500
θ											

Table A.1: Optimal density ρ^* with $\beta = \gamma = 1.00$

ρ^*	φ 0,0010	0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000
0,0010	no opt	min	min	min	min	min	min	min	min	min	min
0,0500	0,0000	no opt	min	min	min	min	min	min	min	min	min
0,1000	0,0000	0,0000	no opt	min	min	min	min	min	min	min	min
0,1500	0,0000	0,0000	0,0000	no opt	min	min	min	min	min	min	min
0,2000	0,0000	0,0000	0,0001	0,0000	no opt	min	min	min	min	min	min
0,2500	0,0000	0,0001	0,0005	0,0006	0,0001	no opt	min	min	min	min	min
0,3000	0,0000	0,0003	0,0013	0,0022	0,0019	0,0003	no opt	min	min	min	min
0,3500	0,0000	0,0007	0,0027	0,0047	0,0054	0,0037	0,0005	no opt	min	min	min
0,4000	0,0000	0,0014	0,0047	0,0081	0,0102	0,0098	0,0060	0,0008	no opt	min	min
0,4500	0,0000	0,0024	0,0072	0,0122	0,0160	0,0173	0,0151	0,0087	0,0011	no opt	min
0,5000	0,0000	0,0037	0,0102	0,0170	0,0224	0,0256	0,0255	0,0210	0,0115	0,0014	no opt
0,5500	0,0000	0,0053	0,0138	0,0222	0,0294	0,0343	0,0363	0,0342	0,0270	0,0144	0,0017
0,6000	0,0000	0,0073	0,0178	0,0280	0,0367	0,0433	0,0472	0,0474	0,0432	0,0332	0,0173
0,6500	0,0000	0,0096	0,0222	0,0341	0,0444	0,0525	0,0580	0,0604	0,0587	0,0521	0,0393
0,7000	0,0001	0,0122	0,0269	0,0405	0,0522	0,0618	0,0688	0,0730	0,0736	0,0700	0,0609
0,7500	0,0001	0,0152	0,0320	0,0472	0,0603	0,0711	0,0795	0,0852	0,0877	0,0866	0,0809
0,8000	0,0002	0,0184	0,0373	0,0540	0,0684	0,0804	0,0900	0,0970	0,1012	0,1021	0,0992
0,8500	0,0003	0,0219	0,0428	0,0610	0,0766	0,0897	0,1003	0,1085	0,1141	0,1167	0,1161
0,9000	0,0004	0,0257	0,0485	0,0681	0,0848	0,0989	0,1105	0,1197	0,1264	0,1305	0,1317
0,9500	0,0006	0,0296	0,0544	0,0753	0,0930	0,1080	0,1204	0,1305	0,1383	0,1436	0,1463
1,0000	0,0008	0,0338	0,0604	0,0825	0,1012	0,1170	0,1302	0,1411	0,1497	0,1561	0,1600
θ											

Table A.2: Optimal density ρ^* with $\beta = 1.25$ and $\gamma = 1.00$

[1] As indicated in the previous appendix, interesting parameter values should provide a maximum to the $L(\rho)$ function. The value *noopt* or *min* indicate that the corresponding pair of parameters induces no optima or a minimum in the function

$E^{\phi}(\rho^*)$											φ
	0,0000	0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000
0,0000	no opt	min	min	min	min	min	min	min	min	min	min
0,0500	0,9817	no opt	min	min	min	min	min	min	min	min	min
0,1000	0,9905	0,7788	no opt	min	min	min	min	min	min	min	min
0,1500	0,9936	0,8249	0,7436	no opt	min	min	min	min	min	min	min
0,2000	0,9951	0,8543	0,7788	0,7288	no opt	min	min	min	min	min	min
0,2500	0,9961	0,8748	0,8048	0,7566	0,7206	no opt	min	min	min	min	min
0,3000	0,9967	0,8901	0,8249	0,7788	0,7436	0,7154	no opt	min	min	min	min
0,3500	0,9972	0,9019	0,8411	0,7969	0,7626	0,7349	0,7118	no opt	min	min	min
0,4000	0,9975	0,9113	0,8543	0,8121	0,7788	0,7516	0,7288	0,7092	no opt	min	min
0,4500	0,9978	0,9190	0,8654	0,8249	0,7927	0,7661	0,7436	0,7242	0,7072	no opt	min
0,5000	0,9980	0,9255	0,8748	0,8360	0,8048	0,7788	0,7566	0,7375	0,7206	0,7056	no opt
0,5500	0,9982	0,9310	0,8830	0,8457	0,8155	0,7901	0,7683	0,7494	0,7326	0,7177	0,7043
0,6000	0,9984	0,9357	0,8901	0,8543	0,8249	0,8002	0,7788	0,7601	0,7436	0,7288	0,7154
0,6500	0,9985	0,9398	0,8963	0,8619	0,8334	0,8092	0,7883	0,7699	0,7535	0,7388	0,7256
0,7000	0,9986	0,9434	0,9019	0,8687	0,8411	0,8174	0,7969	0,7788	0,7626	0,7481	0,7349
0,7500	0,9987	0,9465	0,9068	0,8748	0,8480	0,8249	0,8048	0,7870	0,7710	0,7566	0,7436
0,8000	0,9988	0,9494	0,9113	0,8804	0,8543	0,8318	0,8121	0,7945	0,7788	0,7646	0,7516
0,8500	0,9988	0,9519	0,9154	0,8854	0,8601	0,8381	0,8187	0,8015	0,7860	0,7719	0,7591
0,9000	0,9989	0,9542	0,9190	0,8901	0,8654	0,8439	0,8249	0,8080	0,7927	0,7788	0,7661
0,9500	0,9990	0,9563	0,9224	0,8943	0,8703	0,8493	0,8307	0,8140	0,7990	0,7852	0,7726
1,0000	0,9990	0,9582	0,9255	0,8982	0,8748	0,8543	0,8360	0,8197	0,8048	0,7913	0,7788
θ											

Table A.3: Environmental externality consumption at optimal density ρ^* with $\beta = \gamma = 1.00$

$E^{\phi}(\rho^*)$											φ
	0,0000	0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000
0,0000	no opt	min	min	min	min	min	min	min	min	min	min
0,0500	0,9818	no opt	min	min	min	min	min	min	min	min	min
0,1000	0,9905	0,8187	no opt	min	min	min	min	min	min	min	min
0,1500	0,9936	0,8419	0,8273	no opt	min	min	min	min	min	min	min
0,2000	0,9951	0,8640	0,8187	0,8504	no opt	min	min	min	min	min	min
0,2500	0,9961	0,8812	0,8293	0,8191	0,8744	no opt	min	min	min	min	min
0,3000	0,9967	0,8946	0,8419	0,8187	0,8273	0,8961	no opt	min	min	min	min
0,3500	0,9972	0,9053	0,8536	0,8253	0,8177	0,8384	0,9147	no opt	min	min	min
0,4000	0,9975	0,9140	0,8640	0,8335	0,8187	0,8213	0,8504	0,9305	no opt	min	min
0,4500	0,9978	0,9212	0,8731	0,8419	0,8234	0,8174	0,8273	0,8626	0,9435	no opt	min
0,5000	0,9980	0,9273	0,8812	0,8498	0,8293	0,8187	0,8191	0,8345	0,8744	0,9543	no opt
0,5500	0,9982	0,9324	0,8883	0,8572	0,8356	0,8223	0,8174	0,8226	0,8423	0,8856	0,9631
0,6000	0,9984	0,9369	0,8946	0,8640	0,8419	0,8269	0,8187	0,8182	0,8273	0,8504	0,8961
0,6500	0,9985	0,9408	0,9002	0,8702	0,8479	0,8318	0,8216	0,8174	0,8203	0,8326	0,8586
0,7000	0,9986	0,9443	0,9053	0,8759	0,8536	0,8369	0,8253	0,8187	0,8177	0,8235	0,8384
0,7500	0,9987	0,9474	0,9098	0,8812	0,8589	0,8419	0,8293	0,8211	0,8175	0,8191	0,8273
0,8000	0,9988	0,9501	0,9140	0,8860	0,8640	0,8467	0,8335	0,8242	0,8187	0,8175	0,8213
0,8500	0,9988	0,9526	0,9177	0,8905	0,8687	0,8513	0,8377	0,8276	0,8208	0,8176	0,8184
0,9000	0,9989	0,9548	0,9212	0,8946	0,8731	0,8558	0,8419	0,8311	0,8234	0,8187	0,8174
0,9500	0,9990	0,9568	0,9243	0,8984	0,8773	0,8600	0,8459	0,8347	0,8263	0,8205	0,8177
1,0000	0,9990	0,9587	0,9273	0,9020	0,8812	0,8640	0,8498	0,8383	0,8293	0,8228	0,8187
θ											

Table A.4: Environmental externality consumption at optimal density with $\beta = 1.25$ and $\gamma = 1.00$

$S^{\gamma}(\rho^*)$											φ
	0,0000	0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000
0,0000	no opt	min	min	min	min	min	min	min	min	min	min
0,0500	2,7183	no opt	min	min	min	min	min	min	min	min	min
0,1000	2,7183	1,6487	no opt	min	min	min	min	min	min	min	min
0,1500	2,7183	1,7813	1,5596	no opt	min	min	min	min	min	min	min
0,2000	2,7183	1,8775	1,6487	1,5248	no opt	min	min	min	min	min	min
0,2500	2,7183	1,9518	1,7210	1,5916	1,5062	no opt	min	min	min	min	min
0,3000	2,7183	2,0114	1,7813	1,6487	1,5596	1,4946	no opt	min	min	min	min
0,3500	2,7183	2,0606	1,8328	1,6984	1,6067	1,5391	1,4867	no opt	min	min	min
0,4000	2,7183	2,1022	1,8775	1,7422	1,6487	1,5791	1,5248	1,4810	no opt	min	min
0,4500	2,7183	2,1379	1,9168	1,7813	1,6866	1,6155	1,5596	1,5143	1,4766	no opt	min
0,5000	2,7183	2,1690	1,9518	1,8165	1,7210	1,6487	1,5916	1,5451	1,5062	1,4732	no opt
0,5500	2,7183	2,1963	1,9831	1,8484	1,7524	1,6793	1,6212	1,5737	1,5338	1,4998	1,4704
0,6000	2,7183	2,2206	2,0114	1,8775	1,7813	1,7076	1,6487	1,6003	1,5596	1,5248	1,4946
0,6500	2,7183	2,2424	2,0371	1,9043	1,8080	1,7339	1,6744	1,6253	1,5839	1,5484	1,5175
0,7000	2,7183	2,2621	2,0606	1,9289	1,8328	1,7584	1,6984	1,6487	1,6067	1,5706	1,5391
0,7500	2,7183	2,2799	2,0823	1,9518	1,8559	1,7813	1,7210	1,6708	1,6283	1,5916	1,5596
0,8000	2,7183	2,2962	2,1022	1,9730	1,8775	1,8028	1,7422	1,6917	1,6487	1,6116	1,5791
0,8500	2,7183	2,3111	2,1207	1,9928	1,8978	1,8231	1,7623	1,7115	1,6681	1,6306	1,5977
0,9000	2,7183	2,3248	2,1379	2,0114	1,9168	1,8423	1,7813	1,7302	1,6866	1,6487	1,6155
0,9500	2,7183	2,3375	2,1540	2,0288	1,9348	1,8604	1,7993	1,7481	1,7042	1,6660	1,6325
1,0000	2,7183	2,3493	2,1690	2,0452	1,9518	1,8775	1,8165	1,7651	1,7210	1,6826	1,6487
θ											

Table A.5: Social externality consumption at optimal density ρ^* with $\beta = \gamma = 1.00$

$S^{\gamma}(\rho^*)$											φ
	0,0000	0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000
0,0000	no opt	min	min	min	min	min	min	min	min	min	min
0,0500	2,7183	no opt	min	min	min	min	min	min	min	min	min
0,1000	2,7183	1,4918	no opt	min	min	min	min	min	min	min	min
0,1500	2,7183	1,6760	1,3290	no opt	min	min	min	min	min	min	min
0,2000	2,7183	1,7946	1,4918	1,2411	no opt	min	min	min	min	min	min
0,2500	2,7183	1,8822	1,5965	1,3945	1,1827	no opt	min	min	min	min	min
0,3000	2,7183	1,9510	1,6760	1,4918	1,3290	1,1408	no opt	min	min	min	min
0,3500	2,7183	2,0070	1,7404	1,5653	1,4221	1,2800	1,1096	no opt	min	min	min
0,4000	2,7183	2,0538	1,7946	1,6251	1,4918	1,3702	1,2411	1,0858	no opt	min	min
0,4500	2,7183	2,0937	1,8413	1,6760	1,5484	1,4375	1,3290	1,2093	1,0676	no opt	min
0,5000	2,7183	2,1283	1,8822	1,7203	1,5965	1,4918	1,3945	1,2950	1,1827	1,0534	no opt
0,5500	2,7183	2,1585	1,9185	1,7595	1,6386	1,5379	1,4473	1,3591	1,2661	1,1601	1,0423
0,6000	2,7183	2,1854	1,9510	1,7946	1,6760	1,5781	1,4918	1,4106	1,3290	1,2411	1,1408
0,6500	2,7183	2,2093	1,9803	1,8265	1,7097	1,6140	1,5307	1,4541	1,3796	1,3029	1,2192
0,7000	2,7183	2,2309	2,0070	1,8555	1,7404	1,6464	1,5653	1,4918	1,4221	1,3527	1,2800
0,7500	2,7183	2,2504	2,0314	1,8822	1,7686	1,6760	1,5965	1,5254	1,4590	1,3945	1,3290
0,8000	2,7183	2,2681	2,0538	1,9069	1,7946	1,7032	1,6251	1,5558	1,4918	1,4308	1,3702
0,8500	2,7183	2,2844	2,0745	1,9297	1,8188	1,7285	1,6515	1,5835	1,5214	1,4629	1,4059
0,9000	2,7183	2,2994	2,0937	1,9510	1,8413	1,7520	1,6760	1,6091	1,5484	1,4918	1,4375
0,9500	2,7183	2,3132	2,1116	1,9708	1,8624	1,7740	1,6988	1,6329	1,5734	1,5183	1,4659
1,0000	2,7183	2,3260	2,1283	1,9895	1,8822	1,7946	1,7203	1,6551	1,5965	1,5426	1,4918
θ											

Table A.6: Social externality consumption at optimal density ρ^* with $\beta = 1.25$ and $\gamma = 1.00$

L(ρ*)											φ
	0,0000	0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000
0,0000	no opt	min	min	min	min	min	min	min	min	min	min
0,0500	2,6685	no opt	min	min	min	min	min	min	min	min	min
0,1000	2,6925	1,2840	no opt	min	min	min	min	min	min	min	min
0,1500	2,7008	1,4695	1,1597	no opt	min	min	min	min	min	min	min
0,2000	2,7051	1,6040	1,2840	1,1112	no opt	min	min	min	min	min	min
0,2500	2,7077	1,7074	1,3850	1,2043	1,0854	no opt	min	min	min	min	min
0,3000	2,7094	1,7903	1,4695	1,2840	1,1597	1,0693	no opt	min	min	min	min
0,3500	2,7107	1,8584	1,5415	1,3535	1,2253	1,1311	1,0583	no opt	min	min	min
0,4000	2,7116	1,9158	1,6040	1,4148	1,2840	1,1869	1,1112	1,0503	no opt	min	min
0,4500	2,7123	1,9648	1,6588	1,4695	1,3369	1,2376	1,1597	1,0966	1,0443	no opt	min
0,5000	2,7129	2,0074	1,7074	1,5187	1,3850	1,2840	1,2043	1,1394	1,0854	1,0395	no opt
0,5500	2,7134	2,0447	1,7510	1,5633	1,4290	1,3268	1,2456	1,1792	1,1237	1,0765	1,0357
0,6000	2,7138	2,0778	1,7903	1,6040	1,4695	1,3663	1,2840	1,2164	1,1597	1,1112	1,0693
0,6500	2,7141	2,1074	1,8259	1,6413	1,5068	1,4031	1,3199	1,2513	1,1935	1,1440	1,1010
0,7000	2,7144	2,1339	1,8584	1,6756	1,5415	1,4374	1,3535	1,2840	1,2253	1,1750	1,1311
0,7500	2,7147	2,1580	1,8883	1,7074	1,5738	1,4695	1,3850	1,3149	1,2555	1,2043	1,1597
0,8000	2,7149	2,1799	1,9158	1,7370	1,6040	1,4996	1,4148	1,3441	1,2840	1,2322	1,1869
0,8500	2,7151	2,2000	1,9412	1,7645	1,6322	1,5279	1,4429	1,3717	1,3111	1,2587	1,2128
0,9000	2,7153	2,2184	1,9648	1,7903	1,6588	1,5547	1,4695	1,3980	1,3369	1,2840	1,2376
0,9500	2,7154	2,2354	1,9868	1,8144	1,6838	1,5800	1,4947	1,4230	1,3615	1,3082	1,2613
1,0000	2,7156	2,2511	2,0074	1,8371	1,7074	1,6040	1,5187	1,4468	1,3850	1,3313	1,2840
θ											

Table A.7: Total neighbourhood externalities consumption at optimal density ρ^* with $\beta = \gamma = 1.00$

L(ρ*)											φ
	0,0000	0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000
0,0000	no opt	min	min	min	min	min	min	min	min	min	min
0,0500	2,6688	no opt	min	min	min	min	min	min	min	min	min
0,1000	2,6925	1,2214	no opt	min	min	min	min	min	min	min	min
0,1500	2,7008	1,4110	1,0995	no opt	min	min	min	min	min	min	min
0,2000	2,7051	1,5505	1,2214	1,0555	no opt	min	min	min	min	min	min
0,2500	2,7077	1,6586	1,3241	1,1423	1,0341	no opt	min	min	min	min	min
0,3000	2,7094	1,7453	1,4110	1,2214	1,0995	1,0222	no opt	min	min	min	min
0,3500	2,7107	1,8169	1,4856	1,2918	1,1629	1,0731	1,0150	no opt	min	min	min
0,4000	2,7116	1,8771	1,5505	1,3546	1,2214	1,1254	1,0555	1,0103	no opt	min	min
0,4500	2,7123	1,9287	1,6077	1,4110	1,2750	1,1750	1,0995	1,0431	1,0073	no opt	min
0,5000	2,7129	1,9734	1,6586	1,4619	1,3241	1,2214	1,1423	1,0806	1,0341	1,0052	no opt
0,5500	2,7134	2,0127	1,7042	1,5082	1,3692	1,2646	1,1830	1,1180	1,0665	1,0274	1,0038
0,6000	2,7138	2,0475	1,7453	1,5505	1,4110	1,3049	1,2214	1,1541	1,0995	1,0555	1,0222
0,6500	2,7141	2,0786	1,7827	1,5894	1,4496	1,3426	1,2576	1,1886	1,1318	1,0848	1,0468
0,7000	2,7144	2,1066	1,8169	1,6253	1,4856	1,3778	1,2918	1,2214	1,1629	1,1139	1,0731
0,7500	2,7147	2,1319	1,8482	1,6586	1,5191	1,4110	1,3241	1,2526	1,1928	1,1423	1,0995
0,8000	2,7149	2,1550	1,8771	1,6895	1,5505	1,4421	1,3546	1,2822	1,2214	1,1697	1,1254
0,8500	2,7151	2,1761	1,9038	1,7183	1,5800	1,4715	1,3835	1,3105	1,2488	1,1960	1,1506
0,9000	2,7153	2,1954	1,9287	1,7453	1,6077	1,4993	1,4110	1,3373	1,2750	1,2214	1,1750
0,9500	2,7154	2,2133	1,9518	1,7706	1,6339	1,5256	1,4370	1,3630	1,3000	1,2458	1,1986
1,0000	2,7156	2,2299	1,9734	1,7944	1,6586	1,5505	1,4619	1,3875	1,3241	1,2692	1,2214
θ											

Table A.8: Total neighbourhood externalities consumption at optimal density ρ^* with $\beta = 1.25$ and $\gamma = 1.00$

A.2.2 Sensitivity of the optimal level of externalities to β and γ change

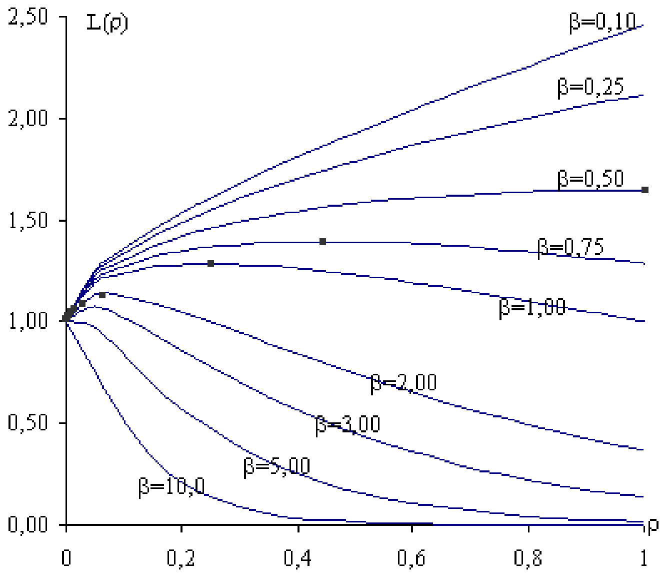


Figure A.1: Sensitivity of total neighbourhood externalities and optimal density (square dots) to change in β, with $\gamma = 1.00$, $\theta = 1.00$ and $\phi = 0.50$

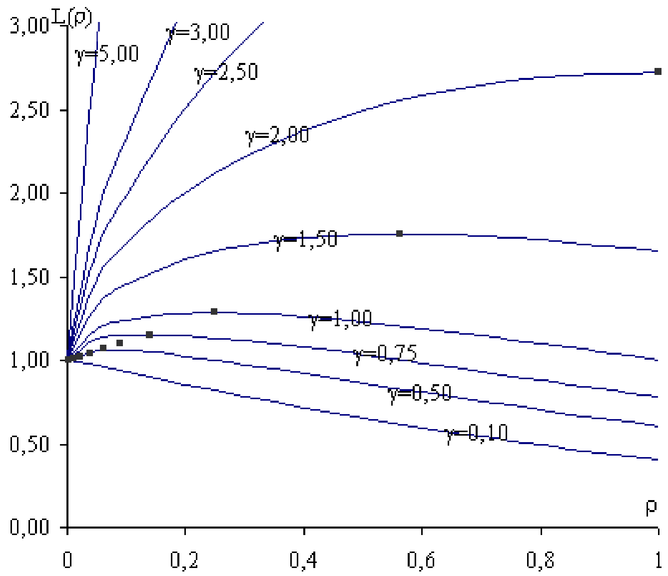


Figure A.2: Sensitivity of total neighbourhood externalities and optimal density (square dots) to change in γ, with $\beta = 1.00$, $\theta = 1.00$ and $\phi = 0.50$

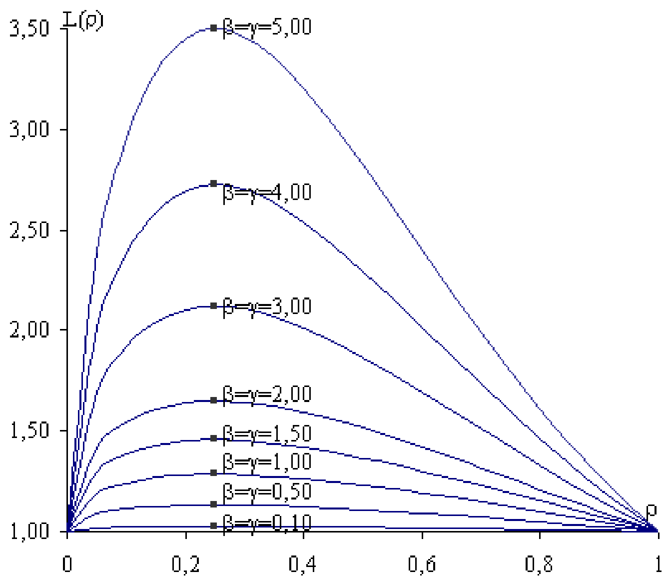


Figure A.3: Sensitivity of total neighbourhood externalities and optimal density (square dots) to change in β and γ, with $\gamma = \beta$, $\theta = 1.00$ and $\phi = 0.50$

A.2.3 Neighbourhood size and sensitivity to a distance decay effect

This appendix first comprises a representation of the three neighbourhoods used in the simulations. Second, the impact of a within neighbourhood distance decay is shown.

A.2.3.1 Neighbourhoods

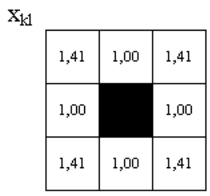


Figure A.4: 8 cells neighbourhood ($\hat{x} = 1.42$)

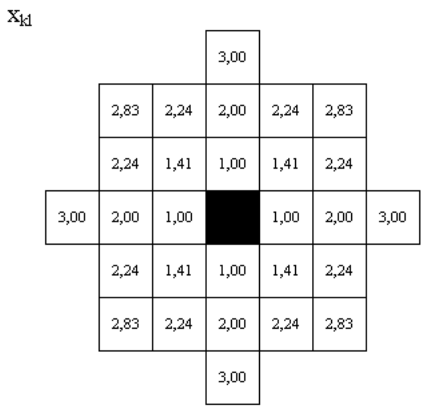


Figure A.5: 28 cells neighbourhood ($\hat{x} = 3.00$)

A.2.3.2 Distance decay effect

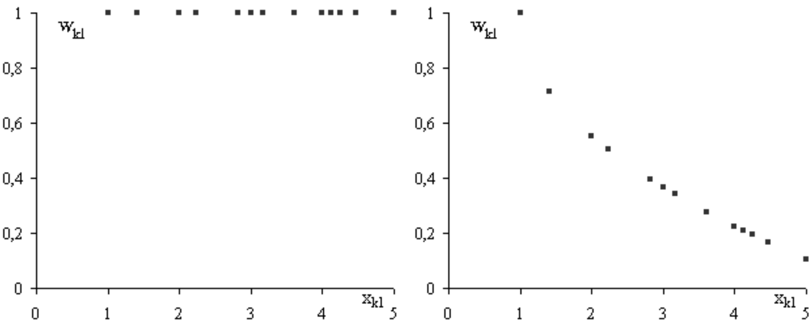
The effect of the distance decay parameter is shown for the largest of these three neighbourhoods. When $\sigma = \infty$ (see the first columns of the two graphics below),

x_{kl}

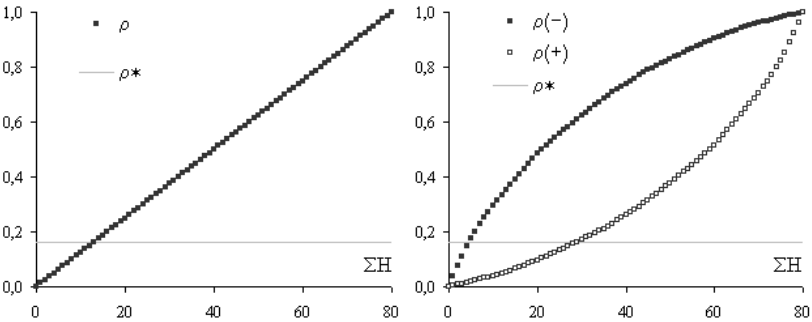
					5,00					
		5,00	4,47	4,12	4,00	4,12	4,47	5,00		
	5,00	4,24	3,61	3,16	3,00	3,16	3,61	4,24	5,00	
	4,47	3,61	2,83	2,24	2,00	2,24	2,83	3,61	4,47	
	4,12	3,16	2,24	1,41	1,00	1,41	2,24	3,16	4,12	
5,00	4,00	3,00	2,00	1,00		1,00	2,00	3,00	4,00	5,00
	4,12	3,16	2,24	1,41	1,00	1,41	2,24	3,16	4,12	
	4,47	3,61	2,83	2,24	2,00	2,24	2,83	3,61	4,47	
	5,00	4,24	3,61	3,16	3,00	3,16	3,61	4,24	5,00	
		5,00	4,47	4,12	4,00	4,12	4,47	5,00		
					5,00					

Figure A.6: 80 cells neighbourhood ($\hat{x} = 5.00$)

there is no distance decay within the neighbourhood, every single cell in the neighbourhood has the same importance on the value of externalities. The level of externalities is thus directly related to the number of households that enter the neighbourhood. When $\sigma = 0.50$ (see the second column of the two graphics below), a non linear decrease is implemented. Therefore, the level of externalities does not only depend on the amount of residents in the neighbourhood (ΣH) but also on the spatial arrangement of households within the neighbourhood. There are two extreme cases: either the neighbourhood is filled in progressively from the central cell toward the external limit (this is noted by (+)), either the neighbourhood is progressively filled in from the external limit toward the central cell (noted by (-)). All other possible cases are comprised between the curves of these two cases. We present the impacts of the effect of σ on ρ, L and Z in the following graphics.

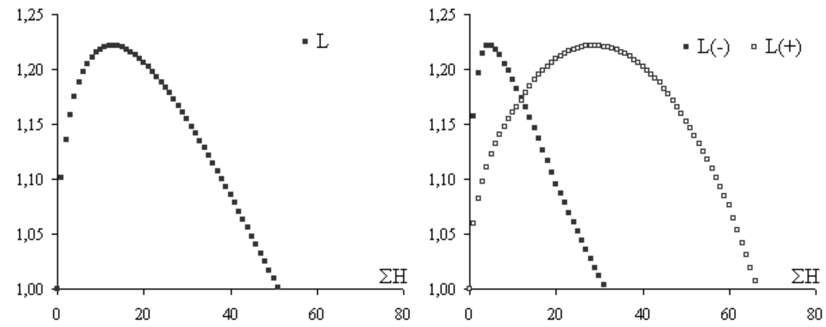


(a) Weight w_{kl} as a function of focal distance x_{kl}

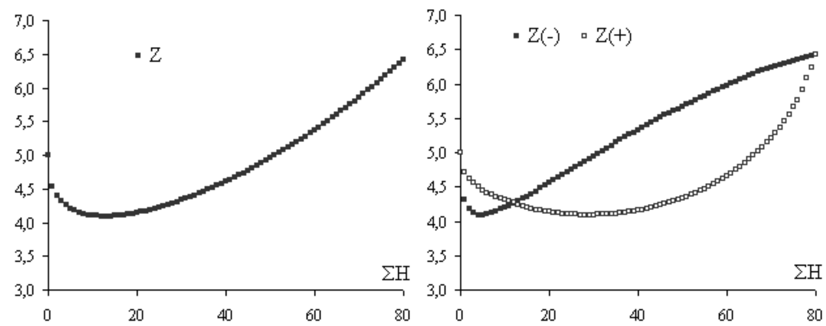


(b) Weighted density (local potential) ρ as a function of the number of households and the way they fill in the neighbourhood

Figure A.7: Within neighbourhood distance decay effect (weight and effect on ρ): $\sigma = \infty$ (left) and $\sigma = 0.50$ (right) ($\beta = 1.25$, $\gamma = 1.00$, $\theta = 1.00$, $\phi = 0.50$, and $\bar{u} = 5.00$



(a) Local externalities as a function of the number of households and the way they fill in the neighbourhood



(b) Composite good consumption as a function of the number of households and the way they fill in the neighbourhood

Figure A.8: Within neighbourhood distance decay effect on L and Z: $\sigma = \infty$ (left) and $\sigma = 0.50$ (right) ($\beta = 1.25$, $\gamma = 1.00$, $\theta = 1.00$, $\phi = 0.50$, and $\bar{u} = 5.00$

Appendix B

Notes to Chapter 5

B.1 Rent dynamics with two household groups and simulation approximation

In this appendix, the calculation of bid rents through time is explained and land rents defined. First we show the situation at time 1, then we generalize the approach to the next t steps. It is finally explained how the rent dynamics is approximated in our simulation by using reservation bid rents.

B.1.1 First time step

Assume g_W and $g_B = 1$. At t1, a new W and a new B enter the region which is completely agricultural. Landowners compete to rent their cell while the demand is limited by the given growth rate. W and B households must at least propose the agricultural bid rent to the landowners if they want to immigrate. Their respective bid rents for ij are given by

$$\Psi^{t1}_{W,ij} = (Y_W - a_W d_{ij})^{1/\alpha} {u^{t1}_W}^{-1/\alpha} E^{\beta/\alpha}_{ij} S^{\gamma/\alpha}_{W,ij} = \Phi \quad \text{(B.1)}$$

$$\Psi^{t1}_{B,ij} = (Y_B - a_B d_{ij})^{1/\alpha} {u^{t1}_B}^{-1/\alpha} E^{\beta/\alpha}_{ij} S^{\gamma/\alpha}_{B,ij} = \Phi \quad \text{(B.2)}$$

Depending on the level of the external utility level, households pocket a utility surplus from their location decision:

$$\Delta u^{t1}_W = u^{t1}_W - \bar{u}_W = (Y_W - a_W d_{ij}) \Phi^{-\alpha} E^{\beta}_{ij} S^{\gamma}_{W,ij} - \bar{u}_W \quad \text{(B.3)}$$

$$\Delta u_B^{t1} = u_B^{t1} - \bar{u}_B = (Y_B - a_B d_{ij})\Phi^{-\alpha} E_{ij}^{\beta} S_{B,ij}^{\gamma} - \bar{u}_B \tag{B.4}$$

Households try to maximize this utility differential. At t1, the differential is maximal for both W and B when the commuting cost is minimized$(d_{ij} = 0)$ because $E_{ij} = 1$ and $S_{ij} = 1$ in any place. This location is contiguous to the CBD and is denoted by d0. The utility surplus in d0 is given by $Y_W \Phi^{-\alpha}$ or $Y_B \Phi^{-\alpha}$.

However, d0 is a unique location and only one type of agent can settle there. If he does not locate in d0, a household will choose the second best location. At t1 the second best location for B and W is d1, contiguous to d0 and where $d_{ij} = 1$[1]. The utility surplus in d1 is

$$\Delta u_{W,d1}^{t1} = (Y_W - a_W)\Phi^{-\alpha} - \bar{u}_W \tag{B.5}$$
$$\Delta u_{B,d1}^{t1} = (Y_B - a_B)\Phi^{-\alpha} - \bar{u}_B \tag{B.6}$$

Denote by δ the difference in utility between the best (d0) and the second best location (d1). This difference is obtained from Eq.B.3 and B.5, or Eq.B.4 and B.6.

$$\delta u_{W,d0d1} = Y_W \Phi^{-\alpha} - (Y_W - a_W)\Phi^{-\alpha} = a_W \Phi^{-\alpha} > 0 \tag{B.7}$$
$$\delta u_{B,d0d1} = Y_B \Phi^{-\alpha} - (Y_B - a_B)\Phi^{-\alpha} = a_B \Phi^{-\alpha} > 0 \tag{B.8}$$

The households are not willing to loose this δu_{d0d1} of utility due to a location in d1. Therefore they will accept to increase their bid rent in d0 up to a certain value in order to obtain at least the same utility as in d1. Their bid-rent in d0 is not Φ but Ψ_{d0}^{t1}, of which the value is defined so that $\delta u_{d0d1} = 0$. By nullifying Eq.B.7 or Eq.B.8 and replacing ϕ in the first term of the difference by Ψ_{d0}^{t1}, the bid rent in d0 is found.

$$\Psi_{W,d0}^{t1} = Y_W^{1/\alpha}(Y_W - a_W)^{-1/\alpha}\Phi \tag{B.9}$$
$$\Psi_{B,d0}^{t1} = Y_B^{1/\alpha}(Y_B - a_B)^{-1/\alpha}\Phi \tag{B.10}$$

Eventually, d0 is occupied by W or B at t1 according to the maximum of $\Psi_{W,d0}^{t1}$ and $\Psi_{B,d0}^{t1}$. d1 is occupied by the other agent. Land rents in d0 and d1

[1]Households are myopic and therefore do not consider the effect of a development of d0 while considering location in d1. At t1, they think that $E_{d1} = S_{d1} = 1$. This assumption is necessary within a CA-type simulation. In fact, the growth constraint makes the simulation partly asynchronous but the decision of W and B is still simultaneous. This probably represents a drawback of CA models as compared to more flexible agent-based approaches.

are

$$R^{t1}_{d0} = max(\Psi^{t1}_{W,d0}, \Psi^{t1}_{B,d0}) \quad \text{(B.11)}$$
$$R^{t1}_{d1} = \Phi \quad \text{(B.12)}$$

B.1.2 Next time steps

While it is possible to know a priori where are the first and second best locations at t1 for both W and B, it is more difficult for the following steps. The heterogeneity of space (some cells are urban and others are agricultural) creates a surface of externalities which does not depend only on distance. We therefore choose to neglect the difference between the first and second best location (as given by Eq.B.7 and B.7 at t1). It is therefore assumed that at any moment there exist at least two similar agricultural cells with landowners in competition. This is not a very strong assumption within a 2D setting where many different cells can have very similar characteristics (e.g. there are two d1 cells). As a result, at the time of a location on an agricultural cell, a household will always pay Φ.

Moreover, while at t1 all new locations are chosen on agricultural cells, it is not necessarily the case for the next steps. One household type may prefer to choose the current location of a household of the other type[2]. In this case (i.e. filtering) the bid rent of a household does no longer depend directly on the level of the agricultural rent but on the level of the bid rent of the other household type, which, in turn , depends on the agricultural bid rent (if these households are not filtering as well).

Generalizing the process for any t time steps is therefore more complex. Four main situations are identified: (I) *W and B urbanisation*, where both W and B grow at the expense of agriculture. (II) *Filtering down and W urbanisation*, where W grows at the expense of agriculture while B newcomers occupy existing W areas. (III) *Filtering up and B urbanisation*, where B grows at the expense of agriculture while W newcomers occupy existing B areas. (IV) *No urbanisation and filtering up and/or down*, where urbanisation is stopped but filtering goes on. Filtering up then decreases the total number of B cells, while filtering down decreases the number of W cells. In these cases, filtering can lead to the outmigration of a whole group . A particular case can also exist where B and W locations interchange, with the number of cells for each group being therefore constant.

[2] The replacement by a similar household is not to be considered as, at the end of consecutive replacements within the group, one household has to find a new location either on agriculture either on the other group

Because the framework of a constrained CA requires simultaneity in the calculation of the different land uses (see the calculation of potentials in White and Engelen, 1993), an approximation is necessary for modelling the dynamics of the bidding system. The importance of this approximation depends on the four situations identified.

B.1.2.1 Situation I: W and B urbanisation

This situation corresponds more closely to the first step. g_W and g_B new W and B cells are developed from agricultural land. This situation is conditional to the existence of a utility surplus on at least one agricultural cell for both households. These conditions are

$$\exists kl | \Delta u^t_{W,kl} = u^t_{W,kl} - \bar{u_W} > 0 \tag{B.13}$$

$$\exists kl | \Delta u^t_{B,kl} = u^t_{B,kl} - \bar{u_B} > 0 \tag{B.14}$$

The W and B newcomers on these kl cells pay the agricultural rent (because we have assumed the existence of at least two similar agricultural cells). The land rent for all kl cells at time t is

$$R^t_{kl} = \Phi \tag{B.15}$$

and the utility of the new migrants is therefore

$$u^t_{W,kl} = (Y_W - a_W d_{kl})\Phi^{-\alpha} E^{\beta}_{kl} S^{\gamma}_{W,kl} \tag{B.16}$$

$$u^t_{B,kl} = (Y_B - a_B d_{kl})\Phi^{-\alpha} E^{\beta}_{kl} S^{\gamma}_{B,kl} \tag{B.17}$$

The city is open so that the residents who have settled previously must have the same utility as the newcomers. At time t, landowners adapt the rents, R^t_{ij} for all ij cells previously developed so that they correspond to the bid rents of the households given $u^t_{W,kl}$ and $u^t_{B,kl}$. The land rent at time t depends therefore on the household type at t-1 and on the new utility level , which are fixed by the new kl locations and Φ.

$$R^t_{ij} \in \left\{ \Psi^t_{W,ij}\left(kl,\Phi\right), \Psi^t_{B,ij}\left(kl,\Phi\right) \right\} \tag{B.18}$$

according to the presence of a W or a B household, with

$$\Psi^t_{W,ij}\left(kl,\Phi\right) = (Y_W - a_W d_{ij})^{1/\alpha} u^{t^{-1/\alpha}}_{W,kl} E^{\beta/\alpha}_{ij} S^{\gamma/\alpha}_{W,ij} \tag{B.19}$$

$$= \Phi \left(\frac{Y_W - a_W d_{ij}}{Y_W - a_W d_{kl}}\right)^{1/\alpha} \left(\frac{E_{ij}}{E_{kl}}\right)^{\beta/\alpha} \left(\frac{S_{W,ij}}{S_{W,kl}}\right)^{\gamma/\alpha} \tag{B.20}$$

$$\Psi^t_{B,ij}(kl,\Phi) = (Y_B - a_B d_{ij})^{1/\alpha} u^{t-1/\alpha}_{B,kl} E^{\beta/\alpha}_{ij} S^{\gamma/\alpha}_{W,ij} \quad (B.21)$$

$$= \Phi \left(\frac{Y_B - a_B d_{ij}}{Y_B - a_B d_{kl}}\right)^{1/\alpha} \left(\frac{E_{ij}}{E_{kl}}\right)^{\beta/\alpha} \left(\frac{S_{B,ij}}{S_{B,kl}}\right)^{\gamma/\alpha} \quad (B.22)$$

B.1.2.2 Situation II: Filtering down and W urbanisation

Although it is possible for both W and B to settle in an agricultural place (Eq.B.13 and B.14 are verified), the utility surplus of the B households can also be maximized by a location on an existing W cell. For this cell ij, B accepts to increase its bid in order to obtain at least the level of utility he would obtain from a location on an agricultural cell kl[3]. By imposing $\Delta u^t_{B,ij} = \Delta u^t_{B,kl}$, the bid rent is the same as the updated rent in the former case. The land rent on cells previously urbanised by W is therefore

$$R^t_{ij} = max\left(\Psi^t_{W,ij}(kl,\Phi), \Psi^t_{B,ij}(kl,\Phi)\right) \quad (B.23)$$

and the values are taken from Eq.B.20 and Eq.B.22

However, at a certain moment of the development of the city, a B household may not be able to bid over farmers on agricultural cells any more and obtain at least the external utility level. The condition Eq.B.14 is not respected. Therefore, the B immigration can only be made on an existing W cell where B can bid over W. The bid rent of W on urban cells is still fixed in order to obtain at least the utility of a location on an agricultural cell kl, following Eq.B.20. If B pays this minimum rent to the landowner, its utility would be given by

$$u^t_{B,ij} = (Y_B - a_B d_{ij}) \Psi^{t-\alpha}_{W,ij} E^{\beta}_{ij} S^{\gamma}_{B,ij} \quad (B.24)$$

where $\Psi^t_{W,ij}$ is given by Eq.B.20.

Because B wants to maximize his utility surplus, he accepts to bid higher for a better cell and offer a bid that allows him to obtain at least the utility he would obtain on the worst ij location. Denote by i'j' this worst urban location, which is defined by

$$i'j'|u^t_{B,i'j'} = min\ u_{B,ij}(\Psi^t_{W,ij}) \quad (B.25)$$

By imposing $\Delta u_B(\Psi^t_{B,ij}) = \Delta u_B(\Psi^t_{W,i'j'})$, we find $\Psi^t_{B,ij}$, the bid rent of B for any ij cell currently occupied by W.

$$(Y_B - a_B d_{ij}) \Psi^{t-\alpha}_{B,ij} E^{\beta}_{ij} S^{\gamma}_{W,ij} = (Y_B - a_B d_{i'j'}) \Psi^{t-\alpha}_{W,i'j'} E^{\beta}_{i'j'} S^{\gamma}_{W,i'j'} \quad (B.26)$$

[3]Using this comparison further supposes that the best location on an agricultural cell kl is always less attractive that a location on any of the existing urban cells ij. This additional assumption could be related to the existence of development costs that are needed to convert agricultural cells

$$\Psi^t_{B,ij} = \Psi^t_{W,i'j'} \left(\frac{Y_B - a_B d_{ij}}{Y_B - a_B d_{i'j'}}\right)^{1/\alpha} \left(\frac{E_{ij}}{E_{i'j'}}\right)^{\beta/\alpha} \left(\frac{S_{B,ij}}{S_{B,i'j'}}\right)^{\gamma/\alpha} \quad \text{(B.27)}$$

$\Psi^t_{W,i'j'}$ can be replaced by its value in Eq.B.20 (i'j' replacing ij). Therefore we can see that the B bid rent at time t, $\Psi^t_{B,ij}$, is a function of the characteristics of the location itself, of the less interesting W location for B, but also of the agricultural location kl chosen by the last W immigrant, and the level of the agricultural rent:

$$\Psi^t_{B,ij} = \Phi \left(\frac{(Y_W - a_W d_{i'j'})(Y_B - a_B d_{ij})}{(Y_W - a_W d_{kl})(Y_B - a_B d_{i'j'})}\right)^{1/\alpha} \left(\frac{E_{ij}}{E_{kl}}\right)^{\beta/\alpha} \left(\frac{S_{W,i'j'} S_{B,ij}}{S_{W,kl} S_{B,i'j'}}\right)^{\gamma/\alpha} \quad \text{(B.28)}$$

There is a competition in ij between W and B. The B bid rent is compared to the bid of W. The land rent for ij occupied at the previous step by W when B cannot grow on agricultural cells is therefore given by

$$R^t_{ij} = max\left(\Psi^t_{W,ij}\left(kl, \Phi\right), \Psi^t_{B,ij}\left(i'j', kl, \Phi\right)\right) \quad \text{(B.29)}$$

with $\Psi^t_{W,ij}$ given by Eq.B.20 and $\Psi^t_{B,ij}$ given by B.28. In i'j', the worst W location that a B would choose, the land rent is

$$R^t_{i'j'} = \Psi^t_{W,i'j'}\left(kl, \Phi\right) \quad \text{(B.30)}$$

which is also given by Eq.B.20 (i'j' replacing ij).

B.1.2.3 Situation III: Filtering up and B urbanisation

Similarly to the previous analysis, the utility surplus of a W household can be maximized by a location on a previously B cell while B grows at the expense of agriculture. When the condition Eq.B.13 is respected, a W household is able to bid over farmers in agricultural parcels. Its bid for existing B cells therefore increases so that $\Delta u^t_{W,ij} = \Delta u^t_{W,kl}$ and is given by Eq.B.20. The land rent on cells previously urbanised by B is then given by B.23 as in the previous case.

When the condition Eq.B.13 is no longer verified, while it is still verified for B (Eq.B.14), the bid rent is then given by

$$\Psi^t_{W,ij} = \Phi \left(\frac{(Y_B - a_B d_{i'j'})(Y_W - a_W d_{ij})}{(Y_B - a_B d_{kl})(Y_W - a_W d_{i'j'})}\right)^{1/\alpha} \left(\frac{E_{ij}}{E_{kl}}\right)^{\beta/\alpha} \left(\frac{S_{B,i'j'} S_{W,ij}}{S_{B,kl} S_{W,i'j'}}\right)^{\gamma/\alpha} \quad \text{(B.31)}$$

Thus, when W migrants have to filter B cells because they cannot bid over farmers anymore, the land rent for ij occupied by B at t-1 is given by

$$R^t_{ij} = max\left(\Psi^t_{W,ij}\left(kl,\Phi\right),\Psi^t_{B,ij}\left(kl,\Phi\right)\right) \tag{B.32}$$

where $\Psi^t_{B,ij}$ is taken from Eq.B.22 and $\Psi^t_{W,ij}$ from Eq.B.31.

In i'j', the last urban location that W would choose, the rent is given by

$$R^t_{i'j'} = \Psi^t_{B,i'j'}\left(kl,\Phi\right) \tag{B.33}$$

which is given by Eq.B.22 (i'j' replacing ij).

B.1.2.4 Situation IV: No urbanisation and filtering up and/or down

In a particular case of phase II, the downward filtering is stopped as soon as B cannot bid over W on existing W cells ij. If B can nor bid over farmers in any kl cells, the immigration of B is stopped. Moreover, if in the meantime it is more advantageous for W to grow on existing B cells, filtering up arises instead of urbanisation and the number of B locations decreases. In this particular case, if W is still able to expand on agricultural cells, land rents are defined by Eq.B.29. However, if W is not able to expand on agricultural cells (Eq.B.13 is neither verified), its bid for any urban cell is therefore a function the worst location he could find on existing B cells. But whether W is able to bid over B in this place is not known because the bid rent of B itself relates to the worst position B can find on W, and therefore on the bid rents of W. The system cannot be determined simultaneously.

In a particular case of phase III, the upward filtering and/or the immigration of W can also be stopped. Then, following analogous conditions, urbanisation is stopped when B prefer to immigrate on existing W cells. Land rents are defined by Eq.B.32 or the system is undetermined depending on whether Eq.B.14 is verified or not.

Finally, another particular case combines both filtering up and down, where B and W locations interchange, with the number of cells for each group and therefore the total being constant. Again, rent values can be determined as long as one of the two conditions (Eq.B.13 and Eq.B.14) at least is verified.

B.1.3 Simulation

In this model, the level of utility and rent across space and time is defined by reference to the utility at the margin of the city, i.e the utility of the last agent (B or W) who locates on an agricultural cell. Sometimes however it is impossible for any of two households to locate on an agricultural cell. Both has therefore to filter the other group or disappear. In this case it is not possible to resolve the bid rent equations simultaneously for B and W. In most CA-type models, even with a growth constraint, all cells update at the same time and therefore the calculation for a cell can neither depend on the result in another cell at the same time.

The model presented here can be simulated without necessarily calculating all intermediary utilities and rents. If we accept an additional assumption for some of the particular cases, the system is equivalent to using reservation bid-rents in order to rule out all land conversions. At time t, for each agent, location decisions are only based on the characteristics of each cells at t-1 (distance, own state and state of the neighbours) and on the reservation bid rents, which themselves depend on a parametric external utility level. The land rents and the evolution of utilities can then be calculated *ex-post*.

The two reservation bid rents are defined by

$$\bar{\Psi_{W,ij}} = (Y_W - a_W d_{ij})^{1/\alpha} \bar{u_W}^{-1/\alpha} E_{ij}^{\beta/\alpha} S_{W,ij}^{\gamma/\alpha} \quad (B.34)$$

$$\bar{\Psi_{B,ij}} = (Y_B - a_B d_{ij})^{1/\alpha} \bar{u_B}^{-1/\alpha} E_{ij}^{\beta/\alpha} S_{B,ij}^{\gamma/\alpha} \quad (B.35)$$

At any time t, residents occupy the gt[4] cells where their reservation bid rents are the highest. If the two bid rents for a given cell are among the highest for both households, the cell goes to the higher bidder. The other household locates in the next better cell available.

When both household types can bid over farmers in the agricultural space, the competing bid rents on urban land are given by Eq.B.20 and Eq.B.22 as shown in Eq.B.23. If these bid rents are replaced by the reservation bid rents known *ex-ante* from Eq.B.34 and Eq.B.35 the competition is identical. In fact, by equaling Eq.B.20 and Eq.B.34 or Eq.B.22 and Eq.B.35, we find $\Phi = \Psi_{kl}(\bar{u})$, i.e. the bid rent of households for the best agricultural cell equals the agricultural bid rent. This assumption has already been made before when it was decided to overlook the difference between the best and second best agricultural location. Moreover, this assumption was necessary to make a decision on the urbanised land, but the decisions made on agricultural land (kl cells) using reservation bids correspond to the decision made in the long-run equilibrium static model without any further assumption.

An additional assumption is necessary however when at least one of the households cannot bid over farmers anymore, i.e when the urban market is represented by Eq.B.29 or Eq.B.32. Therefore, the approximation is found by

[4]the number of urbanised cells is gt as long as urbanisation has not been stopped

equaling Eq.B.31 and Eq.B.34 for W, and Eq.B.28 and B.35 for B:

$$\frac{\Phi}{\Psi^t_{B,kl}} \frac{\Psi^t_{B,i'j'}}{\Psi^t_{W,i'j'}} = \left(\frac{u^t_W}{\bar{u}_W}\right)^{1/\alpha} \tag{B.36}$$

$$\frac{\Phi}{\Psi^t_{W,kl}} \frac{\Psi^t_{W,i'j'}}{\Psi^t_{B,i'j'}} = \left(\frac{u^t_B}{\bar{u}_B}\right)^{1/\alpha} \tag{B.37}$$

In these equations, the first fraction of the left term is unitary given the assumption previously discussed. The remaining part is the additional postulate on the value of the bid-rent of the group who can still grow on agricultural land for the least interesting urban cell (i'j') for the other group. The assumption is re-written below first when W migrants have to filter B cells and second when B migrants have to filter W cells to go on with their own growth.

$$\Psi^t_{W,i'j'} = \Psi^t_{B,i'j'} \left(\frac{\bar{u}_W}{u^t_W}\right)^{1/\alpha} \tag{B.38}$$

$$\Psi^t_{B,i'j'} = \Psi^t_{W,i'j'} \left(\frac{\bar{u}_B}{u^t_B}\right)^{1/\alpha} \tag{B.39}$$

In the previous analysis the assumption has been made that the group which cannot grow on agriculture could gain the urban cell i'j' if its bid rent equals the bid rent of the other group on that cell. This is represented in Eq.B.36 or B.37 by assuming that $\Psi_{W,i'j'} = \Psi_{B,i'j'}$. In analogy with the assumption made for agricultural cells, this assumption means that there are at least two of i'j' cells that are similar (or with a difference which can be overlooked). However, when using the reservation bid rents to simulate the process, the term within parentheses has also to be considered. It equals 1 only when a long-run equilibrium t^* is achieved, and is < 1 before. Therefore although the group who has no other choice than growing on urban cells offers less than the other group, he is assumed to be able to take this cell i'j'. The assumption is undesirable, but necessary to define simultaneously the bid rents in any cell. Moreover this inconvenient will appear quite late during the development of the city (when commuting distances are long enough so that a group cannot bid over farmers) and therefore the value of the parenthesis can already be close to 1 at that time. We choose therefore to neglect this approximation and conduct the whole dynamics of land conversions by the reservation bid rents defined in Eq.B.34 and Eq.B.34[5]

[5]The approach can also be justified if it is assumed that there is a monopolistic land developer whose objective is to maximize the total rent at any time in a myopic manner.

B.2 Graphical representation of reservation bid rents for two household groups

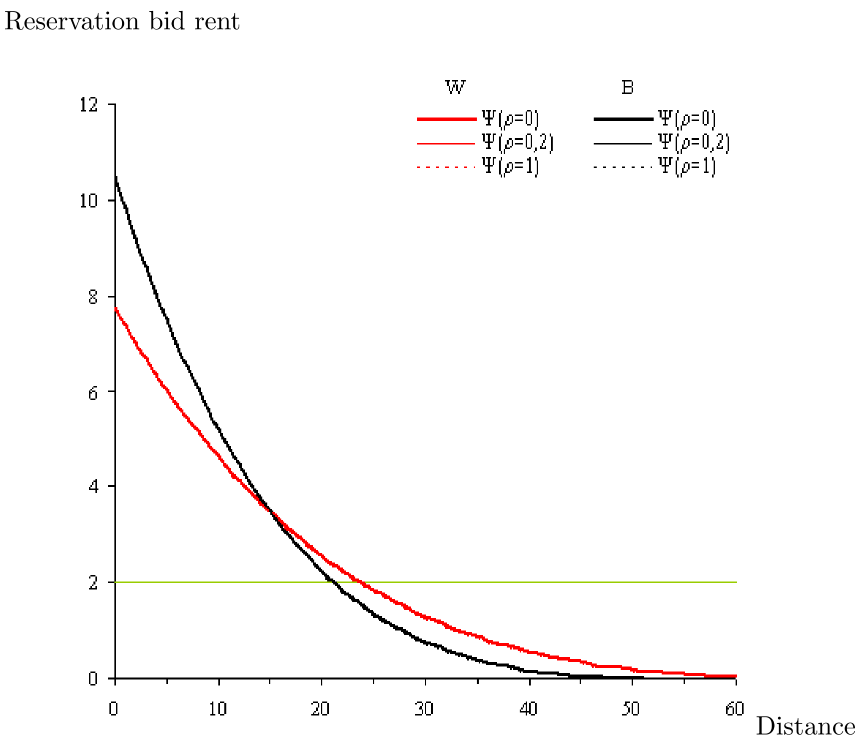
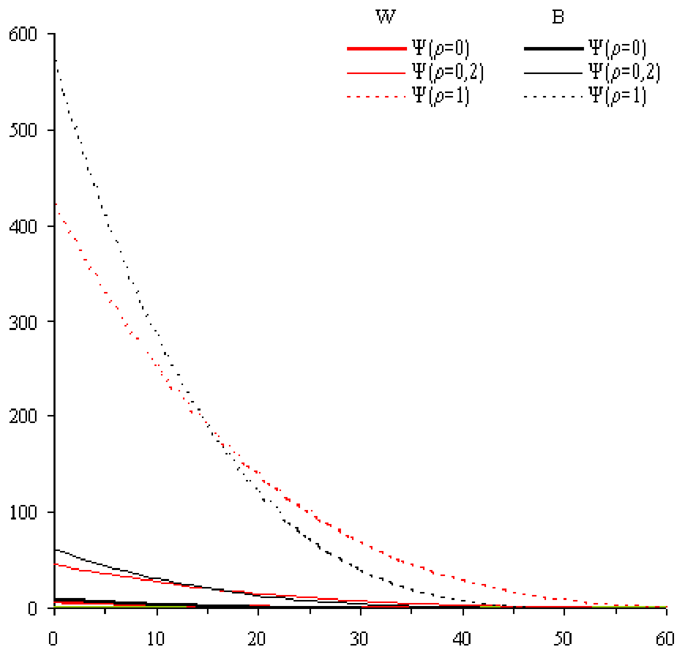


Figure B.1: Bid rent profiles of W and B households for different local densities (ρ). Base case (standard monocentric): $\beta = \gamma = 0.00$

Reservation bid rent



Distance

Figure B.2: Bid rent profiles of W and B households for different local densities (ρ). Compact city case ($\beta = 0.00$ and $\gamma = 1.00$)

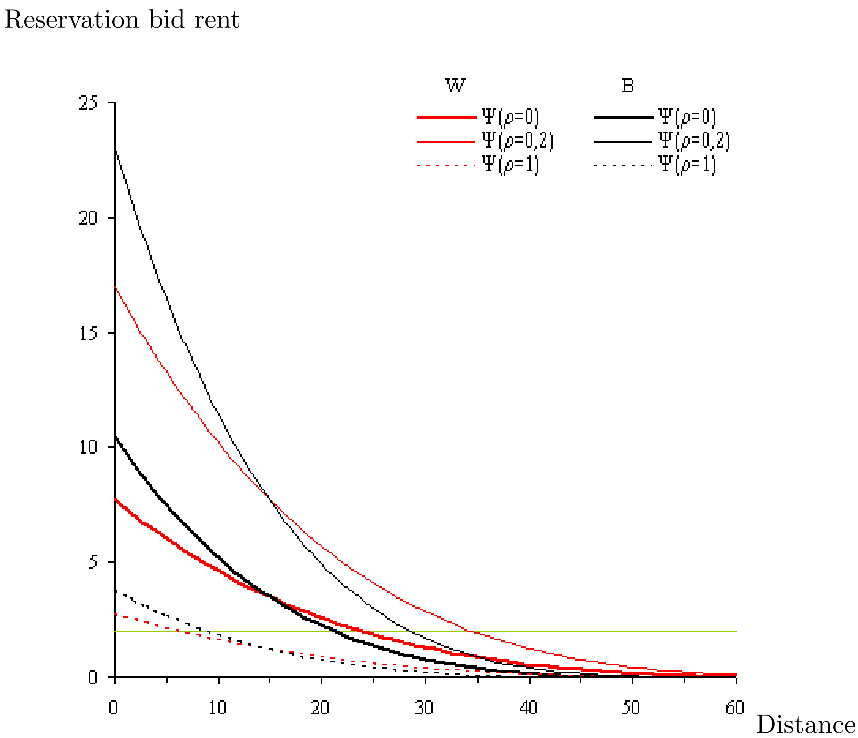


Figure B.3: Bid rent profiles of W and B households for different local densities (ρ). Scattered city case ($\beta = 1.25$ and $\gamma = 1.00$)

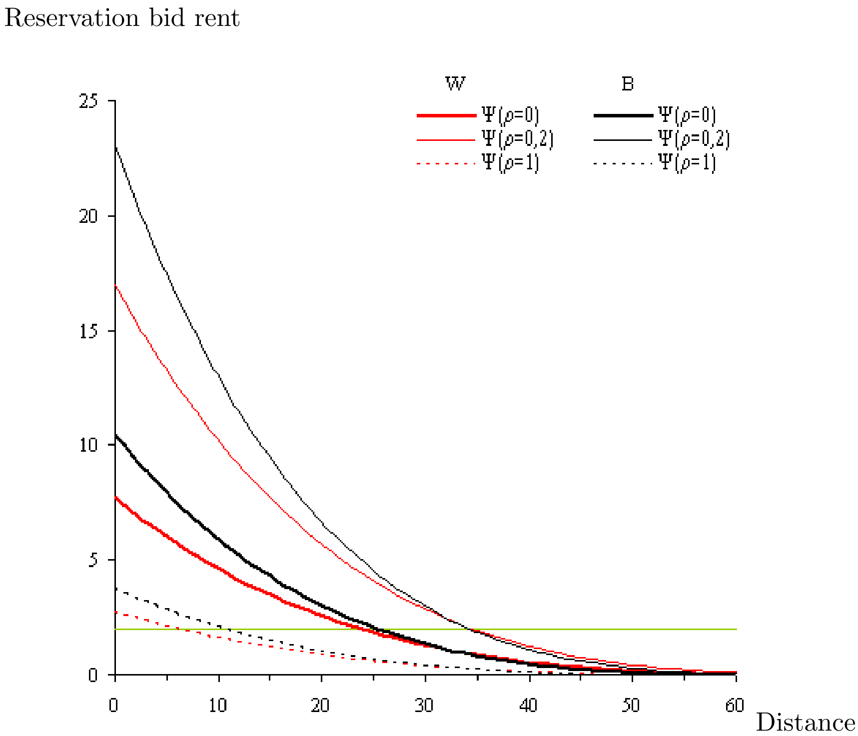


Figure B.4: Bid rent profiles of W and B households for different local densities (ρ). Scattered city case ($\beta = 1.25$ and $\gamma = 1.00$) and transport subsidy ($a_B = 0.10$)

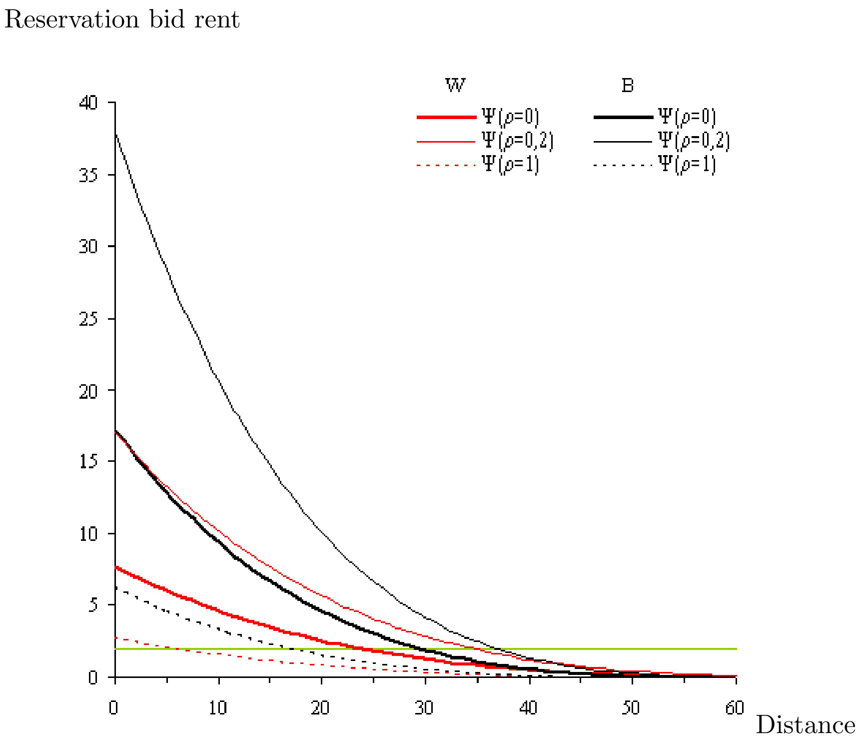


Figure B.5: Bid rent profiles of W and B households for different local densities (ρ). Scattered city case ($\beta = 1.25$ and $\gamma = 1.00$) and income subsidy ($Y_B = 8.5$)

Appendix C

List of symbols

α	Weighting parameter applied to housing consumption in residential preferences (elasticity of U with respect to H)
β	Weighting parameter applied to environmental externality in residential preferences (elasticity of U with respect to E)
γ	Weighting parameter applied to social externality in residential preferences (elasticity of U with respect to S)
Φ	Agricultural bid rent
ϕ	Parameter of the Social externality function (S)
Ψ	Residential bid rent
ρ	Neighbourhood density (local potential)
ρ^*	Optimal neighbourhood density
σ	Parameter of the neighbourhood distance decay function
θ	Parameter of the Environmental externality function (E)
υ	Speed of urban expansion
A	Agricultural state of a cell
A_G	Number of cells in state A in the lattice
a	Commuting cost per unit of distance
B	Black (Poor) state of a cell. Also used in the text instead of Black households
$_{B*B}$	Index indicating the preference of Black households for Black neighbourhoods
$_{B*W}$	Index indicating the preference of Black households for White neighbourhoods
b	Slope of agricultural bid rent function against distance
C	State of a cell (land use)
c or (c')	Index of cell states

D_c	Upper limit of the simulated mixed periurban area (maximum distance of a resident)
D_u	Lower limit of the simulated mixed periurban area (distance of the first agricultural cell)
d	Distance to the CBD
$\tilde{d}$	Classic (no externality) urban-rural fringe
d_c	Theoretical commuting fringe (maximum commuting distance at long-run equilibrium
d_u	Theoretical limit of the specialized urban area (minimum distance of agricultural location at long-run equilibrium
E	Environmental neighbourhood externality (local greenness, open-space)
G	Number of cells (agents) in theoretical lattice
g	Urbanisation rate (number of new urban cells per step)
H (chap. 3)	Household state of a cell
H (chap. 4,5)	Housing consumption
$\mathrm{H}_{\mathrm{FRAG}}$	Index of residential space fragmentation
H_G (chap. 3)	Number of cells in state H in the lattice
I	Number of columns in theoretical lattice
ij or $(i'j')$	Longitudinal (column) and latitudinal (line) coordinate of a cell (location)
J	Number of lines in theoretical lattice
kl	Longitudinal and latitudinal coordinate of a cell belonging to a neighbourhood
k	Utility function simplification parameter
L	Total amount of local externality
$\mathcal{N}$	Neighbourhood (set of cells)
n (chap. 6)	Number of neighbourhood variables
n	Number of cells in a neighbourhood
n_A (chap. 3)	Number of agricultural cells in a neighbourhood
n_H (chap. 3)	Number of residential cells in a neighbourhood
P	Width of the simulated mixed periurban area
P_c (chap. 6)	Conversion potential to state c
$\mathrm{P}_{\mathrm{DENS}}$	Density of residential cells in the mixed periurban area
p	Width (distance extent) of the theoretical (long-run) periurban belt
q	Number of exogenous raster inputs
R	Land rent
$\mathrm{R}^2_{\mathrm{DENSPLAN}}$	Fit of target and generated density-distance curve
$\mathrm{R}^2_{\mathrm{RADIAL}}$	Fit of target and generated fractal curve
RADIALSLOPE	Radial fractal dimension

$_{rac}$	index indicating discriminating preference of White households for Black households
S	Social neighbourhood externality (local public goods, social interactions)
s	Slope of the 1D space-time diagram envelope
T	Commuting cost
TDR^{t^*}	Total differential rent at long-run equilibrium
TDR_{co}	Total differential rent of an equivalent compact circular city
TTC^{t^*}	Total transport cost at long-run equilibrium
TTC_{co}	Total transport cost of an equivalent compact circular city
t	Time
ΔTDR	Surplus of differential rent due to dispersion dispersion (non compact circular city)
ΔTTC	Surplus of transport cost due to dispersion (non compact circular city)
U	Residential utility function
$\bar{u}$	Residential utility level in the rest of the world (long run equilibrium utility)
Δu^t	Residential utility surplus at time t
V	Indirect utility function
W	White (Rich) state of a cell. Also used in the text instead of White households
w	Weighting of a cell within neighbourhood (distance decay function)
$_{W*W}$	Index indicating the preference of White households for White neighbourhoods
X	Exogenous raster input
x	Inter-cell distance within a neighbourhood (focal distance)
$\hat{x}$	Neighbourhood extent (= maximum inter-cell distance within a neighbourhood or maximum focal distance)
Y	Residential income
Z	Composite good consumption

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