Prisoner's dilemma on dynamic networks under perfect rationality

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Abstract

We consider the prisoner's dilemma being played repeatedly on a dynamic network, where agents may choose their actions as well as their co-players. In the course of the evolution of the system, agents act *fully rationally* and base their decisions only on *local information*. Individual decisions are made such that links to defecting agents are resolved and that cooperating agents build up links, as new interrelations are established via a process of recommendation. The dynamics introduced thereby leads to periods of growing cooperation and growing total linkage, as well as to periods of increasing defection and decreasing total linkage, quickly following each other if the players are perfectly synchronized. The cyclical behavior is lost and the system is stabilized when agents react 'slower' to new information. Our results show, that within a fully rational setting in a licentious society, the prisoner's dilemma leads to overall cooperation and thus loses much of its fatality when a larger range of dynamics of social interaction is taken into account. We also comment on the emergent network structures.

Key words: Cooperation, Evolutionary Games, Networks

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1 Introduction

One of the most impressing ways of illustrating situations where mutual trust is beneficial, but egoism leads to a breach of promise is the classical prisoner's dilemma (PD) game, discussed in detail by Axelrod (1984). The game is played by two persons, further denoted by i and j. Representing the game by positive valued payoffs for the two individuals, it can be described by a matrix of the form given in Table 1.

If there is only one round of the game, i, who chooses between cooperation and defection, will select to defect, independent of j's behaviour. Similarly, j, who has the same choices, will decide to defect. The result of these individual decisions is mutual defection and the 'solution' lies in the cell marked I (the cell with a payoff of 0, respectively). However, as the payoffs indicate, both players would find the mutual optimum if they both chose to cooperate.

Broaden the game to a large scale it is often argued (e.g. Buchanan (1975)), that unless there is some convention that dictates altruistic behaviour in the game, rational and utility-maximizing individuals end up in a scheme of overall defection. These arguments refer to problems of constitutional economy, like the respect of property between individuals or cooperative behaviour in bargaining and have been discussed by Buchanan, Brennan and Buchanan (1985) and Tullock (2005). To circumvent the dilemma, the need for a rule is suggested, a binding norm to acquire an acceptable state which overcomes

		Player j	
		C	D
Player i	C	R	S
	D	T	I

$$P_{ij} = \begin{pmatrix} 3 & -1 \\ 4 & 0 \end{pmatrix}$$

Table 1
Table of payoff matrix for two players in the prisoner's dilemma game. The abbreviations stand for Reward (R), Sucker's payoff (S), Temptation (T) and Inefficiency (I). For the prisoner's dilemma the entries have to obey the restrictions $S < I < R < T$ and $(T + S)/2 < R$. On the right hand side, the payoff matrix we used as a typical starting point of our simulations is given.

the social dilemma. Breaking the rule is penalized via external sanctions, effectively leading to a variation of the payoff-matrix and thus resolving the dilemma.

Apart from constitutional economics, applications of the PD range from biological networks, as discussed by Oborny et al. (2000), and the analysis of internet congestion (Huberman and Lukose (1997)) to economic communication, introced via an analysis of a single-shot Prisoner's dilemma by Miller et al. (2002).

Axelrod (1984) explored an extension to the classical PD, where the game is repeated and participants have to choose their mutual strategies again and again - the iterated prisoner's dilemma (IPD). If the number of iterations of the game is a known finite number, a simple argument proves that the only equilibrium is mutual defection in every round. However, if the game is played an infinite or a finite - but unknown - number of times and one considers the players to act according to strategies, the folk theorem implies that cooperation may emerge, if the payoffs are sufficiently little discounted.

Recent research has concentrated on aspects of the iterated prisoner's dilemma based on a spatial version of the game, introduced in the pioneering work of Nowak and May (1992). In their survey, cooperation is made possible by the assumption, that every agent imitates his neighbours in such a way, that he chooses the action of the neighbour who got the highest payoff in the last turn. The imitation rule leads to complex spatio-temporal dynamics and the emergence of cooperation for strategy spaces confined to cooperation and defection. The spatio-temporal dynamics have been studied under the aspect of different topologies of the underlying interaction networks by different authors: Abramson and Kuperman (2001) have studied the case of a small world network, later Kim et al. (2002) have introduced and discussed the role of an 'influential node'. Holme et al. (2003) have investigated Nowak and May's spatial prisoner's dilemma driven by mutations on empirical social networks. On the other hand, also the role of a larger strategy space has been examined extensively in various articles. The well-known computer tournaments of Axelrod showed that the simple tit-for-tat strategy did outstandingly well. Alternative behavioral mechanisms such as 'win-stay' 'lose-shift' have been discussed by Nowak and Sigmund (1993). An overview of the developments which have taken place is given by Weibull (1996). More recently, Ebel and Bornholdt (2002) have discussed co-evolutionary dynamics of different strategies on networks. Less work concerning the prisoner's dilemma has been done on dynamic networks, where also the links between the agents are dynamical variables in the total system. In this regard, Zimmermann et al. (2004) have recently presented results where the evolution of the system is based on imitation within a dynamic network. Literature has also concentrated on learning schemes paired with mechanisms of partner choice and refusal (Ashlock et al. (1996); Hauk (2001)). In this context, Sato et al. (2002) have recently shown that learning may lead to chaos even in the simple two-person rock-paper-scissors game.

To our best knowledge, in the variety of research conducted about the PD, there is no model resembling what we regard as the necessary properties when discussing the iterated prisoner's dilemma game based on a strict interpretation of *homo oeconomicus*: Pure rationality, local information-horizons, as well as access to the full range of social interactions - including both, network dynamics and freedom of choice of actions. Our present model aims at closing this gap to show that neither external rules nor imitative behavior, nor the introduction of strategies are necessary to obtain cooperative behaviour, even within a population of selfish agents maximizing their payoffs expected for the very next turn. The model is based on a society without regulations and without an institution acquiring and providing global information - circumstances which have been long considered as to leading to a 'solitary, poor, nasty, brutish, and short' life, as Hobbes (1943) formulated more than three centuries ago.

If there are no rules in society, there are also no norms and laws demanding and controlling the liability to continue the interrelations one maintains. In other words, the players are not compelled to keep their links and may cancel them whenever they like. Conversely, they may build up new links whenever they want. Then, the variation of the links between the individuals has to be taken into account and implicit rewards as well as internal sanctions originating from the variability of the network start to influence individual decisions. This paper explores the implications of this idea and is structured as follows: A general outline and a detailed description of the model embodying the above mentioned thoughts is given in Section 2. In Section 3, results based on computer-simulations of the model are presented: On the one hand, the characteristics of the global time-series and the influence of the relevant model parameters are discussed (Sections 3.1-3.3). On the other hand, the structure of the networks obtained in the simulations is examined more closely in Section 3.4. Finally, a conclusion is reached and a short discussion of the main results is given in Section 4.

2 The Model

2.1 Basic assumptions

We impose full rationality of each of the N players i and allow them to obtain only local information. Furthermore, they have no memory ('zero-intelligence') and base their decisions on a strictly egotistic evaluation of the expected payoffs in the next timestep.

At each update at time t, each agent i maximizes her expected payoff in the next round, given by

$$\bar{P}_i(t+1) = \sum_{j \epsilon \tilde{N}_i(t+1)} \bar{a}_i(t+1) P_{ij} \bar{a}_j(t+1) \quad . \tag{1}$$

Here, P_{ij} denotes the (given) payoff-matrix (see Table 1). The actions of the agents at timestep t are given by a two-dimensional unit-vector,

$$a_i^c(t) = \begin{pmatrix} 1 \\ \\ 0 \end{pmatrix} \quad \text{or} \quad a_i^d(t) = \begin{pmatrix} 0 \\ \\ 1 \end{pmatrix} \quad , \tag{2}$$

if agent i is cooperating or defecting at timestep t, respectively.

Each agent tries to maximize her payoff by adjusting her future action $a_i(t+1)$ and direct neighbourhood $\tilde{N}_i(t+1)$, which is defined as the set of next neighbours to i (a next neighbour is a node connected with i by one link). As he has no knowledge about future actions of her neighbours, agent i takes their preceding actions as a reasonable expectation value:

$$\bar{a}_j(t+1) = a_j(t) \quad . \tag{3}$$

Furthermore, agent i has full knowledge about the payoff $P_j(t)$ of each of her neighbours j. He needs this information to conduct an evaluation of the payoff expected from establishing new links to other players (see below).

For the update of the neighbourhood $\tilde{N}_i(t) \rightarrow \tilde{N}_i(t+1)$ we conceive that agents cancel a link if the payoff with this co-player is smaller than, or equal to zero. The maximum number of links agents may cancel in one period is limited by a model-parameter α and the neighbourhood after cancellation of α' links is depicted by $\tilde{N}_i^{\alpha'}(t)$.

We conceive that new links may only be established to players next to nearest neighbours by a process of being made acquainted to them by one's nearest neighbours. The reason for this assumption lies in the rationality of the players, which makes them aim at a minimization of potential losses by accepting only 'recommended' co-players. Therefore, only agents who have chosen to cooperate have the possibility to establish new links in our model. To model friction within the system we limit the maximum number of links which may be established in an update. The maximum number of new links is given by the model-parameter β; the neighbourhood after establishment of β' links is depicted by $\tilde{N}_i^{\beta'}(t)$. Parameters α and β can be seen as 'agility', as willingness to change partners upon new information.

Now, to determine their action in the next timestep, agents have to estimate the additional expected payoff $\bar{P}_i^{add}(t+1)$, which they could acquire due to new links. This can in principle be achieved in two ways: On the one hand, agents can be informed by their nearest neighbours about cooperating next-to-nearest neighbours, which would sacrifice the locality of our model. On the other hand, a detailed evaluation of the neighbourhood only using information about

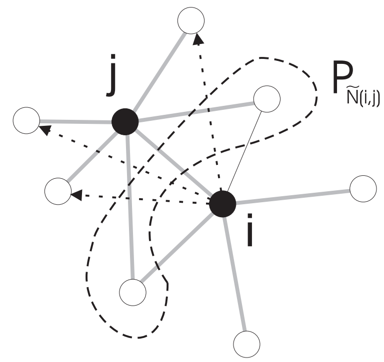

Fig. 1. Illustration for the notation of variables characterizing the neighbourhood of players i and j. The players have two neighbours in common - where the corresponding set is denoted $\tilde{N}_{(i,j)}$ - and a number of $N_u = 5$ unequal neighbours. The payoff player i obtains from the set of neighbours $\tilde{N}_{(i,j)}$ is denoted $P_{\tilde{N}_{(i,j)}}$.

nearest neighbours may be performed, as will be described in detail below. As we have confined ourselves to strictly local information, we have implemented the second alternative. This always leads to a lower expected payoff than the first alternative because the actions of next-to-nearest neighbours have to be estimated . With this choice we therefore model the 'rational worst case scenario' within the presented framework.

2.2 The 'rational worst case scenario'

Figure 1 illustrates how the expected payoff is estimated within a framework of strictly local information: Agent i can first evaluate her payoff obtained from the set of neighbours he and j have in common, denoted by $\tilde{N}_{(i,j)}$:

$$P_{\tilde{N}_{(i,j)}}(t) = \sum_{k \epsilon \tilde{N}_{(i,j)}} a_i(t) P_{ik} a_k(t) \tag{4}$$

Then, he can subtract this payoff from j's total payoff $P_j(t)$ to obtain an approximation of the profit j gained from the number of neighbours they do not have in common, denoted by N_u. Weighting this estimate with the fraction β/N_u, agent i obtains the expected additional payoff he may receive when establishing β random links to next-to-nearest neighbours:

$$\bar{P}_i^{add}(t+1) = \left(P_j(t) - P_{\tilde{N}_{(i,j)}}(t)\right)\frac{\beta}{N_u} \tag{5}$$

Equation (5) gives the highest possible value only if all second next nearest neighbours cooperate. If all of them defect, a negative payoff is obtained. Generally speaking, the procedure always gives a lower estimated payoff then would be obtained when exactly knowing the number and identity of cooperating agents in the vicinity.

The dynamics of our model may be summarized as follows: At timestep t, agent i performs an update of his action and local neighbourhood with probability p_u. This parameter is introduced to capture the synchronisation of the agents' individual decisions (i.e. the 'coupling' of the agents). For $p_u = 1$, the decisions of the agents are fully synchronized and are made simultaneously. Clearly, the more independently the agents conduct their evaluation (the lower the update-probability p_u), the better becomes the estimate given in equation (3). Once chosen for update, each agent calculates the expected payoff in case of cooperation, given by

$$\bar{P}_i^c(t+1) = \sum_{j \epsilon \tilde{N}_i^c(t)} a_i^c(t) P_{ij} \bar{a}_j(t+1) + \bar{P}_i^{add}(t) \tag{6}$$

and the expected payoff in case of defection,

$$\bar{P}_i^d(t+1) = \sum_{j \epsilon \tilde{N}_i^d(t)} a_i^d(t) P_{ij} \bar{a}_j(t+1) \tag{7}$$

Here, $\tilde{N}_i^c(t)$ and $\tilde{N}_i^d(t)$ denote the 'preliminary' neighbourhoods for the two cases, which can be written as

$$\tilde{N}_i^c(t) = \left(\tilde{N}_i(t) \setminus \tilde{N}_i^{\alpha'}(t)\right) \cup \tilde{N}_i^{\beta'}(t) \tag{8}$$

and

$$\tilde{N}_i^d(t) = \tilde{N}_i(t) \setminus \tilde{N}_i^{\alpha'}(t) \tag{9}$$

In equations (8) and (9) $A \cup B$ denotes the union of sets A and B, $A \setminus B$ stands for excluding set B from set A. Based on the estimations given in equation (6) and (7), agent i chooses the action, which gives him the higher expected payoff in the next turn and performs the corresponding update of his neighbourhood. If the expected payoffs are equal, the action is chosen at random. This is done simultaneously by all agents who have been selected for update at timestep t ($N^{sel} \approx p_u N_{tot}$).

The decisions are evaluated at the end of each turn and the whole procedure is repeated in the next timestep.

3 Results

Within the network-dynamics implied above defectors are effectively sanctionized in two ways: By implicitly being effected by link-cancellations and by explicitly not being able to establish new links. The influence of these two aspects is controlled by the two parameters α and β. The third parameter of the model - the update probability p_u - also affects the nature of the resulting time-series. In the following, the influence of these parameters is discussed in some detail, beginning with an overview of the basic properties at given values of α and β for different values of p_u. The discussion is based on simulations with a C++ implementation of the model.

As a starting point, we generated random networks of a size of $N = 10^3$ agents. The random networks have been implemented in a way widely used in literature, initially introduced by Erdös and Renyi (1960). A detailed discussion has recently been given by Barabasi (2002). We have chosen a probability of $p_l = 0.01$ for the establishment of each link in the initial network. As far as the role of different starting configurations of the random networks is concerned, our simulations have clearly shown that the development of the network is independent of these configurations and converges relatively fast towards its attractors (repulsors). If not stated otherwise, simulations have been performed for 10^5 timesteps, providing very accurate statistics.

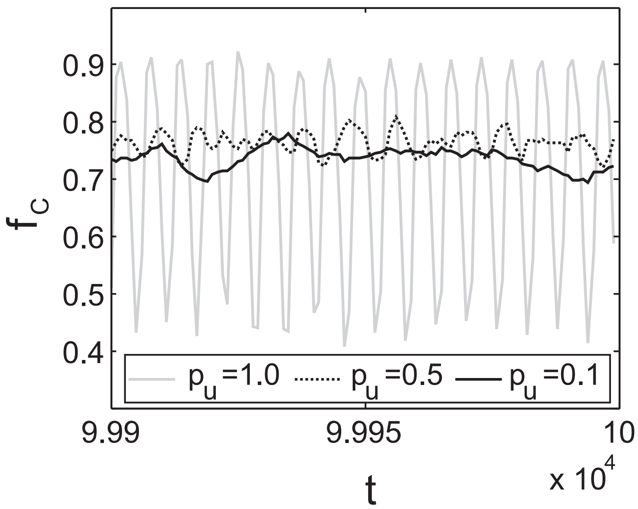

Fig. 2. Time-series of the fraction of cooperating agents f_C for different values of the update-probability p_u ($p_u = 1.0$, $p_u = 0.5$, $p_u = 0.1$). The curves clearly indicate that the regularity in the time-series decreases by lowering p_u.

3.1 Basic properties of time-series

Figure 2 depicts the time series of the number of cooperating agents (denoted by N_c) for the last 1000 steps of a particular simulation. The different lines correspond to different values of p_u with the other two parameters held fixed ($\alpha = \beta = 6$). The payoff-matrix used is the one given in Table 1. For $p_u = 1.0$, where each agent can change up to $\alpha + \beta$ link configurations in each turn, oscillations with a comparably high range are observed: Quantitatively, the number of cooperating agents undergoes rapid changes of about 50% of the overall system size within only 3 timesteps. As shown in Figure 3, the average number of links per agent (denoted by $\langle l_i(t) \rangle = l_{tot}(t)/N$) shows the same behavior, oscillating between a minimum of about 4 and a maximum of about 13 links per agent.

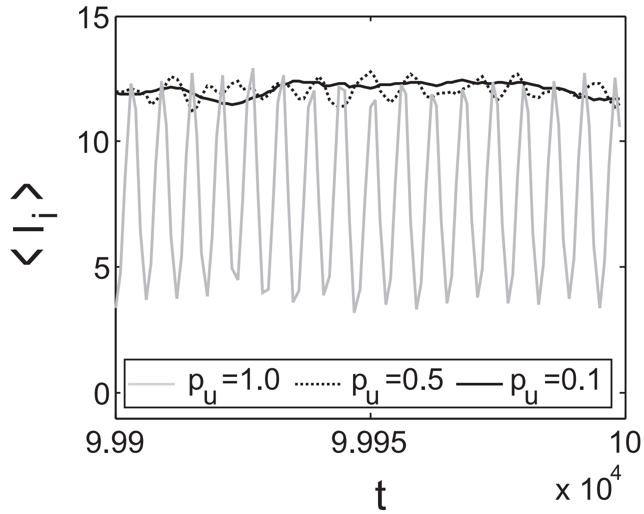

Fig. 3. Time-series of average number of links $\langle l_i \rangle$ per agent for different values of the update-probability p_u ($p_u = 1.0$, $p_u = 0.5$, $p_u = 0.1$). Again, the values become more erratic when loweringp_u. Clearly, the time average $\langle l_i \rangle$ for $p_u = 0.1$ and $p_u = 0.5$ is considerably above the case for $p_u = 1.0$, indicating the stabilisation of the corresponding network.

The reason for this cyclical behavior of the system can be easily understood: In the states corresponding to low N_c, linkage has been reduced to an extent motivating the agents to build up links again, where a considerable amount (in our simulations for $p_u = 1.0$, $\beta = 6$ and $\alpha = 6$ a maximum of 10% of the overall population) of the agents lost all of their links and is relinked to a single agent chosen at random. In configurations with high N_c, the majority of agents has collectively acquired a state of maximum linkage, where there is no more motivation to cooperate in our rational setting and where the classical situation of the PD is reached. This can be understood by the following argument: If all $l_i(t)$ neighbours of agent i cooperate, he starts to defect as soon as

$$\beta R_{eff} + l_i(t) R < l_i(t) T \tag{10}$$

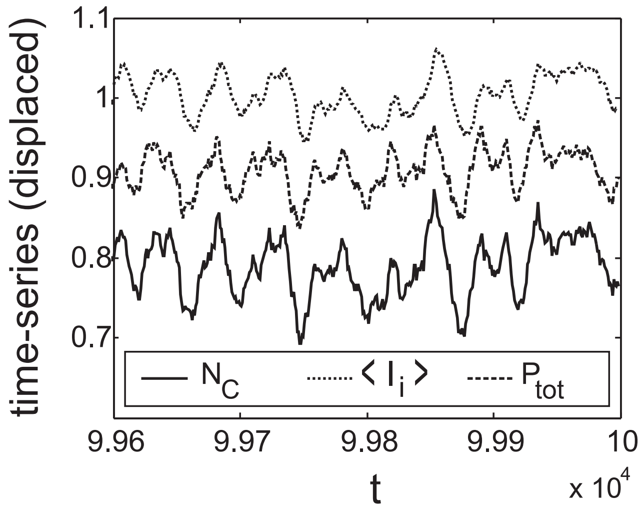

Fig. 4. Time-series of the fraction of number of cooperating agents (solid line), average number of links per agent (dotted line) and total payoff (broken line) in the system for $p_u = 0.1$ and $\alpha = \beta = 6$. The individual time-series have been shifted for better comparison.

is realized. Here, R_{eff} is the averaged 'effective' reward imanent in the system, arising from equation (5) via

$$R_{eff} = \frac{1}{N} \sum_i \frac{\bar{P}_i^{add}(t+1)}{\beta} \quad . \tag{11}$$

Of course, βR_{eff} always gives an estimated payoff lower then βR. The observed rapidity of the oscillations becomes clear, when one considers that the agents may build up $\beta = 6$ links per move and therefore reach $\langle l_i^{max} \rangle$ comparatively fast. By lowering α and β the amplitudes reduce (not shown).

3.2 Dependence on update-probability p_u

In reality, agents are not infinitely fast in assessing new information in their surrounding. Further, they need time to adopt and employ decisions based on

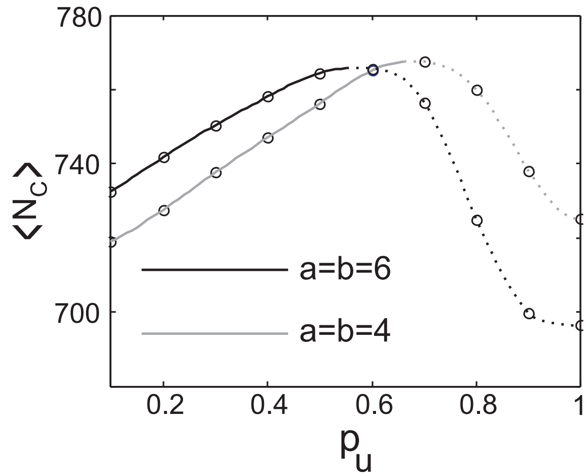

Fig. 5. Average number of cooperating agents $\langle N_c \rangle$ as a function of update-probability p_u for $\alpha = \beta = 6$ and $\alpha = \beta = 4$. When p_u gets larger, correlations between the individual values grow. Taking averages of highly correlated timeseries is to a certain extent problematic, which is why the line is drawn broken in the corresponding regime. To guide the eye, actual values (circles) have been interpolated by a cubic spline.

new information. We model these time-delays by the update probability p_u.

By lowering p_u, the oscillations of the overall population are increasinlgy damped. The time-series corresponding to $p_u = 0.5$ in Figure 2 allows one to examine this issue roughly. In contrast to the rapid update mode at $p_u = 1.0$, the range of the number of cooperating agents exhibits a reduced span of about 12% of the overall population and the average number of links varies between 5.4 and 6.5. Only 1% of the agents have lost all their links in a certain timestep, indicating that the network is in this sense stabilized in comparison to the $p_u = 1.0$ case.

For values of $p_u < 0.5$, the time-series continues to reduce in regularity, but the range of the trajectories does not decrease accordingly. In either case, the time-series of total payoff, average number of links and number of cooperating agents are strongly coupled, as shown in Figure 4 for $p_u = 0.1$. Periods of growing cooperation and growing average linkage $\langle l_i \rangle$ are found, followed by periods of decreasing cooperation and average linkage.

Moreover, decreasing p_u allows to approximate N_c based on equation (10): As soon as the number of cooperating agents does not oscillate too strongly, the effective reward can be estimated via $R_{eff} \approx RN_c/N_{tot}$. Rewriting equation (10) as

$$\langle l_i^{max} \rangle \approx \beta \frac{R_{eff}}{T - R} \tag{12}$$

allows one for the approximation $\langle l_i^{max} \rangle \approx 13$ for $p_u = 0.1$. The average of $\langle l_i \rangle \approx 12$ observed in the simulations implies that the agents have arranged quite efficiently as the average linkage per agent is just one link below the 'rational maximum'. Below (Section 3.4) we show that the efficient arrangement can be characterized by hierarchical structures in the network. The efficiency of the system is also confirmed by noting that the average number of links is considerably increasing for lower values of p_u, as can be seen in Figure 3.

As far as the overall dependence of N_c on p_u is concerned, Figure 5 shows N_c for different values of p_u. For $\alpha = \beta = 6$, the curve exhibits a maximum at $p_u \approx 0.6$, which can be understood when considering that decreasing values of p_u result in higher randomness and therefore destroy cooperative structures in the network. On the other hand, increasing values of p_u lead to a growing amplitude of oscillations which destabilizes cooperative behavior in the system.

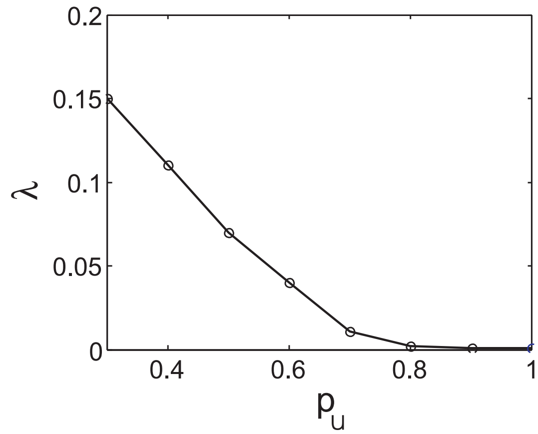

Fig. 6. Correlation length λ determined by an exponential fit to the auto-correlation-function of $\Delta N_C(t)$, given in equation (14). For small p_u the exponential fit becomes problematic because the process becomes practically uncorrelated, i.e. the correlation function turns into a Dira-delta function. This is why the correlation length is not shown for $p_u < 0.3$.

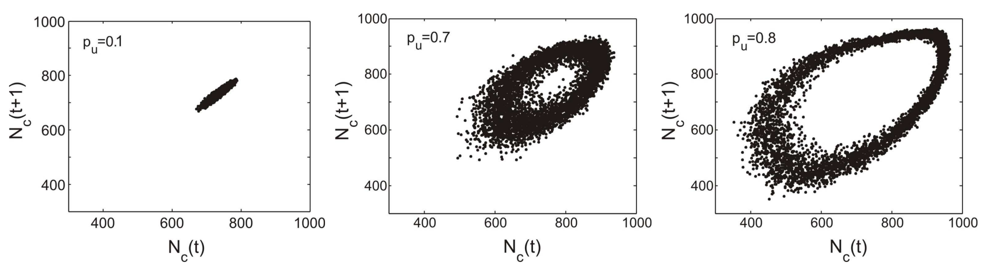

Fig. 7. Visualization of 'attractors' in the space $\{N_C(t), N_C(t+1)\}$ for different values of p_u ($p_u = 0.1$, $p_u = 0.7$, $p_u = 0.8$) and $\alpha = \beta = 6$.

These oscillations also lead to growing correlations, which is why the spline-interpolation connecting the points is drawn as broken line in Figure 5. The gray line corresponding to $\alpha = \beta = 4$ has its maximum at $p_u \approx 0.7$. Obviously, for decreasing α and β, the system is stabilized at a higer value of p_u as the link-dynamics is reduced.

To allow a closer investigation of the qualitative change in the systems-dynamics between $p_u = 1.0$ and $p_u = 0.1$, we have plotted $N_C(t+1)$ against $N_C(t)$ in

Figure 7. This allows to point out that the respective attractor of the system has a quite regular limit-cycle for $p_u = 0.8$. Decreasing p_u to 0.7 leads to a smaller gap in the attractor and at $p_u = 0.6$ the gap has vanished (not shown). Further decrease in p_u narrows the space filled by the trajectory of the system (see Figure 7, $p_u = 0.1$). Furthermore, we have taken a closer look at the correlations in the system by the means of the autocorrelation function of the first differences of N_C(t)

$$\Delta N_C(t) = N_C(t) - N_C(t-1) \quad . \tag{13}$$

The envelope of the autocorrelation function is fitted by an exponential function with correlation length λ, i.e.

$$\langle \Delta N_C(t+\tau) \Delta N_C(t) \rangle \sim e^{-\lambda \tau} \tag{14}$$

for $\tau > 0$. Values of λ for different update-probabilities are summarized in Figure 6. As expected, between $p_u = 1.0$ and $p_u = 0.8$ the time-series correlations in the system are very strong. Lowering p_u below 0.8 leads to an decrease in the correlation, where the exponential fit becomes more and more problematic. For $p_u < 0.3$, the correlation function resembles the shape of a dirac-delta-function and an exponential fit loses sensibility.

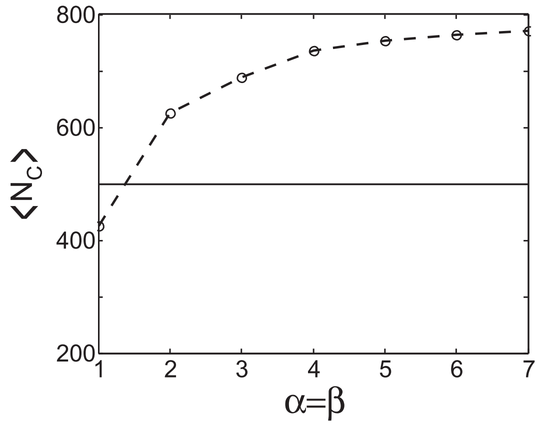

Fig. 8. Influence of the parameters α and β on the number of cooperating agents in a population of 10^3 players. The two parameters have been kept equal. Simulations were done for $p_u = 0.5$.

3.3 Impact of 'agility' α and β and the influence of the payoff-matrix elements

To quantitatively describe the influence of the parameters α and β on N_c, we kept both parameters equal and performed simulations for values ranging from 1 to 7. Results are summarized in Figure 8, for $p_u = 0.5$ and for the payoff-matrix given in Table 1. Only for a parametrization of $\alpha = \beta = 1$, the majority of agents is defecting, which leads to very unstable networks of very low average connectivity (approximately 1.3 links per agent). For higher values of $\alpha = \beta$, the system gets initially stabilized and the increase of N_c flattens as α increases. This can be understood as the parameter α has reached a value, where cooperating agents are able to cancel virtually all the links they have with defecting agents. In other words, further increase of α does not improve the ability of the players to protect themselves from defecting agents - the internal sanction-potential of the system has reached a maximum.

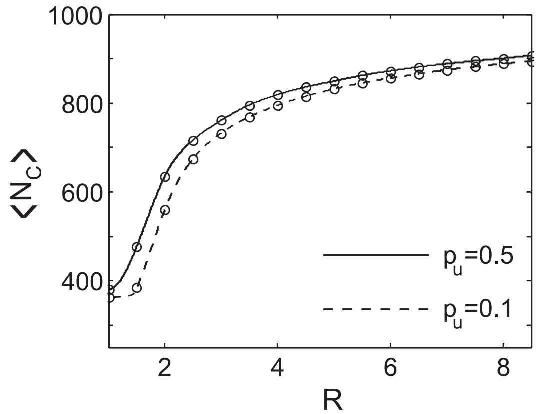

Fig. 9. Influence of the parameters α and β on the number of cooperating agents in a population of 10^3 players. Here, the two parameters have been kept equal and simulations have been performed for an update-probability $p_u = 0.5$

Clearly, not only the parameters α, β and p_u, but also the entries in the payoff-matrix P_{ij} heavily influence the dynamics obtained within the presented model. In this context, the entry for the 'inefficient' constellation I (Table 1) is of fundamental importance: When chosen larger than zero, defecting agents keep the links between one and another, which results in a decrease of the network's stability and the number of cooperating agents: Once defecting players 'organize' themselves and cooperation collapses almost entirely. On the other hand, increasing the values for temptation T and reward R, while holding their difference $T - R$ constant, allows one to examine the corresponding increase in the average number of cooperating agents. This interrelation is quantitatively captured in Figure 9 for two values of p_u ($p_u = 0.5$ and $p_u = 0.1$), $\alpha = \beta = 6$ and $T = 1 + R$. Apparently, for 'low' values of R, the update-probability has comparatively larger influence on $\langle N_c \rangle$, whereas in the flattening regime, the increase caused by switching between $p_u = 0.1$ and $p_u = 0.6$ is decreasing more and more.

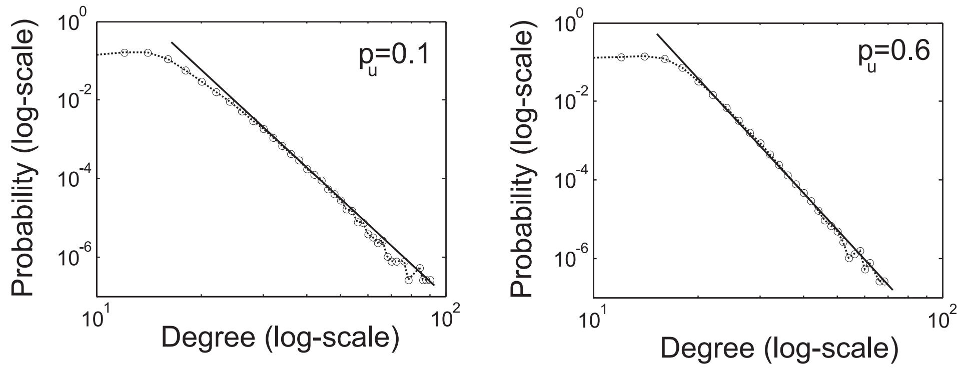

Fig. 10. Degree distributions averaged over time series with $T = 10^5$, $N = 10^3$, $\alpha = 6$ and $\beta = 6$ for two different values of update probability p_u (0.1 and 0.6).

3.4 Emerging Network structure

So far, we have discussed properties of time-series generated by means of simulation. However, also the networks obtained in these simulations show interesting properties resembling some features of real-world networks. We confine ourselves to the discussion of the two most widely used quantities in the analysis of networks: The degree distribution and the cluster-coefficient. For a detailed discussion of these concepts see e.g. the recent discussions of Barabasi (2002) and Dorogovtsev and Mendes (2003).

Figure 10 shows the degree-distribution $P(k)$ in a double-logarithmic plot for two values of p_u. To improve the accuracy of the plot, degree distributions of networks at different times have been averaged, where the correlation in the time-series has been regarded by using appropriate time-differences between the individual network configurations. Also, we have monitored the results via comparison of degree-distributions for larger networks with the same parameters. Although the results were the same, to remain consequent, Figure 10 depicts the degree-distribution for the $N = 10^3$ case, for $p_u = 0.1$ and $p_u = 0.6$. For $p_u = 0.1$, the linear fit shown is apparently inadequate - the network is certainly not scale-free, but somewhere between a power-law and an exponential regime. However, for $p_u = 0.6$, the distribution is more narrow and the linear approximation in the double-logarithmic plot indicates that the tail of the distribution can be given by a scaling law $P(k) \approx k^{-\gamma}$. This shows that the network is clearly not random, but possesses self-similar structure. Further, by increasing the irregularity introduced into the system by lowering p_u, the network obviously loses structure and becomes more random. For $p_u > 0.6$ correlations sharply increase, as does the variation in the total number of links of the individual networks at different timesteps. This hinders averaging the degree-distributions. At $p_u = 1.0$, no 'global' degree-distribution exists, but different $P(k)$'s for each of the possible states (high N_c and low N_c) of the system have to be considered.

The cluster coefficient C, defined as the average of all individual cluster coefficients C_i, provides a quantitative measure for cliques (i.e. circles of acquaintances in the network in which every member knows every other member) in the network. The individual cluster coefficient of a node i is defined as

$$C_i = \frac{2E_i}{k_i(k_i - 1)} \tag{15}$$

where E_i are the number of existing edges between i's neighbours and $k_i(k_i - 1)/2$ gives the highest possible value of edges between the neighbours. The

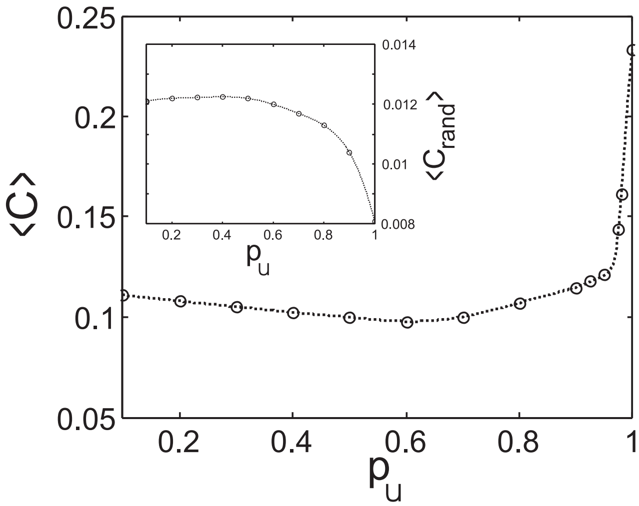

Fig. 11. Time averages $\langle C \rangle$ of the cluster coefficient for different values of p_u and $\alpha = \beta = 6$. The insert shows the cluster coefficients of equivalent random graphs, denoted by $\langle C_{rand} \rangle$. $\langle C_{rand} \rangle$ is decreasing strongly for $p_u > 0.7$ because the average number of links in the system is dropping considerably in this regime.

clustering coefficient of a random graph is given by $C_{rand} = p = \langle k \rangle / N$ (see e.g. Barabasi (2002)). As the expected total number of edges in a random graph can be obtained via $\bar{l}_{tot} = p(N(N-1))/2$, one can compare the cluster coefficient obtained from given networks to those of equivalent random networks. Figure 11 shows the average cluster coefficients from simulations at different values of p_u and the other parameters fixed ($\alpha = \beta = 6$). For comparison, also the cluster coefficients of equivalent random graphs are shown. Again, a data point represents a time-average over cluster-coefficients of different networks, where the above annotations apply in the same manner.

Obviously, the observed networks exhibit large clustering-coefficients when compared to those of equivalent random graphs. This is not a big surprise, as our mechanism of linkage between the agents is characterized by links being made to next-to-nearest neighbours and thus directly favours the formation of cliques. When taking a look at the dependence of the cluster-coefficient on p_u, a minimum at $p_u = 0.6$ can be identified. Interestingly, this corresponds to the maximum of the number of cooperating agents in Figure 5 and to the value of p_u where we examined an apparent scaling-law behavior of the degree distribution. This is not a contradiction, but rather indicates, that - when compared to the other cases - there is still a potential for further clustering in the system. In almost the same manner, the high clustering-coefficient for $p_u = 1.0$ is the outcome of averaging over maximum clustering and states where cliques are still comparatively pronounced.

Plotting the cluster-coefficients C_i of individual agents against their degree k_i allows for a more sophisticated analysis of network structure. The corresponding plot is shown in Figure 12, where each point corresponds to a pair $\{k_i, C_i\}$. The points have been sampled from 100 different networks. Based on this data, we have calculated the mean cluster-coefficient in dependence of the degree of the nodes, denoted by $\langle C_i \rangle (k)$. The tail of this distribution is shown in double logarithmic scale in the insert of Figure 12. Clearly, there is a non-random relationship between cluster-coefficient and degree of the nodes. Specifically, Figure 12 shows that the underlying networks exhibit hierarchy: For small degrees the mean clustering is much higher than for large degrees. Equivalently, nodes with many links are - on average - less connected between each other than nodes with less links, which is exactly what one would expect from a hierarchic network. Examination of $\{k_i, C_i\}$ for $p_u = 1.0$ (not shown)

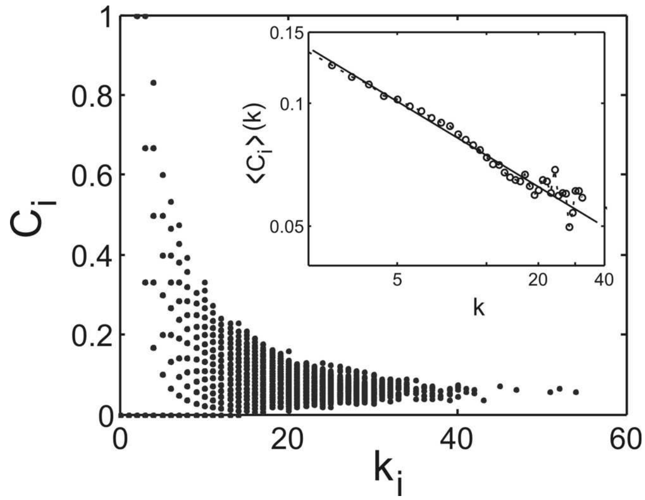

Fig. 12. Individual cluster coefficients C_i plotted against individual degree k_i for $p_u = 0.6$ and $\alpha = \beta = 6$. The insert shows the tail of the corresponding distribution in a double-logarithmic plot, where the individual C_i's have been averaged. The slope of the interpolating line is $\gamma \approx -0.4$.

confirms that the network is disintegrating for high values of p_u. The hierarchy is lost, as for degrees up to 35 the downward slope of the curve is destructed.

Finally, Figure 13 shows the average distribution of individual payoffs in the system, for $p_u = 0.1$ and $p_u = 0.6$ respectively. Although, the maximum of the distribution is at a higher payoff for $p_u = 0.1$, the average payoff is higher for $p_u = 0.6$ as the tail of the distribution is 'fatter' in this case. Further, the insert shows the tails of the distribution in a semi-logarithmic plot, indicating that the tails are almost exponential.

4 Conclusion and Discussion

In this article, we have considered the prisoner's dilemma being played on dynamic networks under full rationality and local information horizons of the

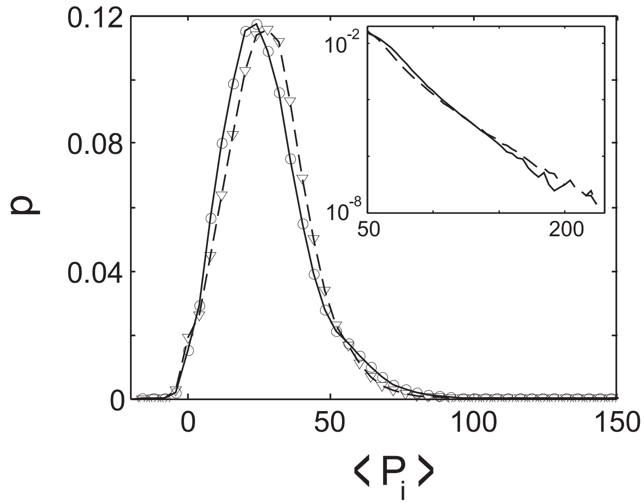

Fig. 13. Average distribution of the individual payoffs for $p_u = 0.1$ (broken line) and $p_u = 0.6$ (solid line) ($\alpha = \beta = 6$). The insert shows the tails of the distribution in a semi-logarithmic plot.

agents. Within this framework, reasonable assumptions about individual decisions lead to a model of network dynamics where defecting agents become isolated over time. Our results show, that the simulation of these dynamics leads to cooperative behavior, even within a preconstitutional setting and a framework of complete rationality. In contrast to previous work neither imitation nor memory longer than one single timestep is required. The degree of cooperation arising depends on the three introduced model parameters and on the specific choice of the payoff-matrix.

Most important, a high update-probability p_u leads to high synchronisation of the agents' decisions and significant oscillations of global parameters like the number of cooperating agents and the average linkage in the system. Much resources are wasted in collective movements. This oscillatory dynamics might provide an understanding for many cyclical synchronization patterns found in actual economic settings. In contrast, low synchronisation of the agents introduces considerable randomness in the system and seems to hinder the emergence of cooperative structures. For regimes in between high and low synchronization, we showed that the system reaches an optimum, where network characteristics resemble those of complex networks, exhibiting clearly non-random properties. The cooperative structures can be characterized by a scaling-law-behavior of the degree-distribution and a hierarchical organization of the network. In this context, a discussion of substructures and hierarchical organization in the interbank market is given by Boss et al. (2005).

In the spirit of constitutional economics and under the aspect of the observed facts, we may point to Buchanan (1975) and his conception of the social dilemma faced within a licentious society. In his work, Buchanan points out that - when generalized to social scales - 'the prisoner's dilemma will take on highly complex structural characteristics'. Eventually, we have modelled these characteristics within our model and demonstrated that the afore mentioned 'characteristics' exhibit a nature strongly reducing the fatality of the prisoner's dilemma, as its properties resemble those of complex networks. What goes beyond the presented work, but would merit closer investigation is a quantitative discussion of the resources spent by the agents to assert their decisions, combined with a comparison to the resources spent by a state-like institution leading to the same degree of cooperation.

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