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Evolutionary prisoner's dilemma in random graphs

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Abstract

We study an evolutionary version of the spatial prisoner's dilemma game (SPD), where the agents are placed in a random graph. For graphs with fixed connectivity, α, we show that for low values of α the final density of cooperating agents, ρ_c depends on the initial conditions. However, if the graphs have large connectivities ρ_c is independent of the initial conditions. We characterize the phase diagram of the system, using both, extensive numerical simulations and analytical computations. It is shown that two well defined behaviors are present: a Nash equilibrium, where the final density of cooperating agents ρ_c is constant, and a non-stationary region, where ρ_c fluctuates in time. Moreover, we study the SPD in Poisson random graphs and find that the phase diagram previously developed looses its meaning. In fact, only one regime may be defined. This regime is characterized by a non stationary final state where the density of cooperating agents varies in time.

Keywords: Game theory; Cellular automata

1. Introduction

The search for models able to account for the complex behavior in many biological, economical and social systems has lead to an intense research activity in the last years [1]. In particular, a very debated issue is the emergence of cooperation between competitive individuals [2] a problem that was studied by Axelrod [3] in the context of the Game theory [4]. Game theory was originally developed to find the optimal strategy for a given game between two intelligent players. However, its straightforward development involved the generalization toward the iterated games of N players. In this context many theories have been proposed to explain the emergence and sustainability of cooperation, kin selection [5], reciprocal altruism [6], group selection [7] and others.

The prisoner's dilemma (PD) is the archetype model of reciprocal altruism [8]. In the game, each

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player has two options: to defect, or to cooperate. The defector will always have the highest reward T (temptation to defect) when playing against the cooperator which will receive the lowest payoff S (sucker value). If both cooperate they will receive a payoff R (reward for cooperation), and if both defect they will receive a payoff P (punishment). Moreover, these four payoffs satisfy the following inequalities:

$$\begin{aligned} &T > R > P > S \\ &T + S < 2R \end{aligned} \tag{1}$$

It is not too hard to recognize that for rational players, in a two players-one round game the choice of defection will assure the largest payoff for each player independently of the other decision (Nash equilibrium) [9]. This situation, however, creates a dilemma for intelligent players, they know that mutual cooperation results in a higher income for both of them. The question then, is to find under which conditions the cooperation emerges in this game.

Nowak and May [10] have shown how cooperation can emerge between players with memoryless strategies in the presence of spatial structure (spatial prisoners dilemma, SPD). They considered a deterministic cellular automaton where agents are placed in a square lattice with self, nearest and next-nearest interaction. At each round of the game, the payoff of the player is the sum of the payoffs it got in its encounters with its neighbors. The state of the next generation is defined occupying the site of the graph with the players having the highest score among the previous owner and the immediate neighbors. It was remarkable that within these simple rules, for a certain range of values of the pay-off matrix, very complex spatial patterns show -up with cooperators and defectors coexisting.

Since then, the game has been largely extended or modified to study more complex situations. For example, Szabo et al. studied the influence of the tit-for-tat strategy [11] and the effects of the external constraint [12] in the game. Vainstein and Arenzon [13] approached the problem considering site-diluted graphs to mimic the presence of disorder in the environment and proved that, depending on the amount of disorder, cooperation can be enhanced. Moreover, Abramson and Kuperman [14] and Kim et al. collaborators [15] studied the consequences of different topologies and proved that defectors are enhanced in small-world networks. Furthermore, Ebel and Bornholdt [16] studied the response of the system upon perturbations finding different regimes for avalanche dynamics.

In this work, we try to extend previous results of other authors [10,17] and to put them in a more general framework. For a square graph [10,17], using periodic boundary conditions and starting with half of the agents as cooperators and the other half as defectors ($p = 0.5$) it was noticed that as a function of the temptation, the spatial game has three qualitative different final states: The first one, for low temptations ($1 < T < 4/3$), is characterized by a stationary or slightly periodical ρ_c that becomes a global majority ($\rho_c > 0.5$). The second, for intermediate temptations ($4/3 < T < 3/2$), is characterized by a non stationary $\rho_c(t)$ and spatiotemporal chaos [10,17], and a third one, for high temptations ($3/2 < T < 2$), is also characterized by a stationary or slightly periodical ρ_c but that is now a global minority ($\rho_c < 0.5$).

To this end we decided to study the model in a random graph [18]. The introduction of a random graph may be interpreted as a first step to the characterization of the disordered nature of the interactions in evolutionary systems.

We study two types of random graphs. The first one, usually called the Bethe lattice, has all sites with a fixed and equal number of neighbors α, while the other one is a Poisson random graph, in which the number of links per site is Poissonian distributed with a mean value α.

While, it has been recently argued that many real networks are of scale free type [19], we are convinced that the understanding of simpler graphs, like the one studied in this paper, whose structural properties are well characterized[18] will be of valuable help in the future analysis of the more complex ones. This is specially relevant in the current state of the research, where the microscopic properties of these scale free graphs are not completely understood, and that are certainly not universal at all. Moreover, despite of the recent avalanche of results showing that many real graphs show power law dependences on their degree distribution, it is not unseeing to believe that many others (even real) graphs will follow the old Erdös-Rènyi distribution.

In this paper, we show that for graphs with fixed degree the final density of cooperating agents (ρ_c) develops multiple jumps as a function of the temptation (T) of the agents, and depending of the degree these jumps may lead also to a region of the phase space where the density of cooperating agents, ρ_c, fluctuates in time. These jumps are very well characterized and we show that they can be predicted by the study of the interaction between clusters of cooperating and defecting agents. However, for Poisson random graphs the situation is more complex, the number of jumps become infinite in the thermodynamic limit, and we do not find a stationary state of cooperating agents. However, in these graphs cooperation is strongly enhanced.

The remaining of the paper is organized as follows. In the next section we present the model with all its details. Then, the numerical results for the graphs with fixed connectivity are presented, together with a comparison of the analytical predictions for the phase diagram. In the next section we present the results for Poisson random graphs and finally the conclusions are outlined.

2. Model

The model is defined by placing two kind of agents, cooperators (C) or defectors (D), in a random graph, with fixed or fluctuating connectivities (as mentioned above) and considering that the connected pairs interact through the payoff matrix (Table 1), where C stands for cooperator and D for defector and where the temptation T satisfies $1 < T < 2$ which is consistent with Eq. (1)[11].

The agents will interact simultaneously and independently from each other and the agent payoff will be the sum of the payoffs that it wins in its interaction with all its neighbors.

The evolution of the system proceeds as follows: first, each site is occupied by a cooperator (C) with a probability p, or by a defector (D) with a probability $1 - p$. Then, the agents interact following the payoff matrix 1, and in the next time step, in every site i of the graph we will place the agent with the higher payoff between the neighbors of i and i itself. The time t is then defined as the number of generations between the current one and the first. The process is repeated, erasing the later payoffs, until the system stabilizes.

Table 1
Nowak's payoff matrix for one player

	D	C
D	0	T
C	0	1

D: defector and C: cooperator.

In this way our model reproduces a synchronous deterministic interaction between memoryless agents. The only source of randomness comes from the graph structure and the initial conditions p, but, as it is shown below, this is enough to produce a complex behavior. Therefore, the only remaining relevant variables of our problem are the temptation T of the agents and the degree α of the graph.

For asynchronous interactions, taking place in biological systems where the evolution is continuous, the cooperation usually does not emerge in such simple models [20] and most be impose by an external entity [11] or by devising more complicated evolution rules. However, the synchronous evolution rules devised before are relevant for real world problems in which delays in the transmission of the information between the agents are important[20], for example, financial markets, and voting problems. It is in this context that our study intends to be relevant. We will show, in this scenario, that the random spatial structure of the graph, alone, leads to very complex patterns.

Some variables will be useful in the discussions below, so we will introduce them here. The state (D or C) of site i will be characterized by a variable θ_i that takes the value 1 if the agent is a cooperator and 0 otherwise. In this way, the state of a system with N sites at time t is fully characterized by the set of variables $(\theta_1, \theta_2, \ldots, \theta_N)$. We defined also s_i as the number of cooperative neighbors of the agent located at the site i (by definition $s_i \leq \alpha$), and $g_{\theta_i}^{s_i}$ as the total payoff of the agent placed at site i having s_i cooperating neighbors.

A simple analysis of the payoff matrix shows that the agent's payoff will be different from zero only when it plays with at least one cooperator. Thus, the agent's payoff depends on the number of its cooperative neighbors. Besides, the agent's payoff also depends on the type of agent, if it is a cooperator, it wins 1, otherwise it wins T for every interaction with a cooperative neighbor. Therefore:

$$g_{\theta_i}^{s_i} = (T - (T - 1)\theta_i)s_i \quad (2)$$

Note that for a cooperative agent $g_1^s = s$, while for a defector one $g_0^s = Ts$ as pointed out before.

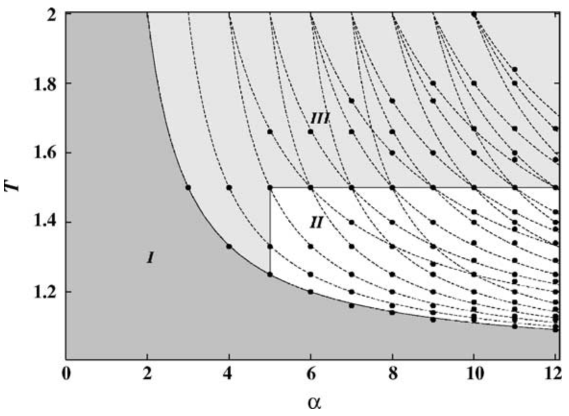

Fig. 1. Phase diagram for the PD game in a fixed degree case. The dashed lines (analytic) and the symbols (simulations) represent the points where ρ_c changes its value. The shadow characterize three different regimes: (I) high values of ρ_c, (II) non-stationary values of ρ_c and (III) low values of ρ_c.

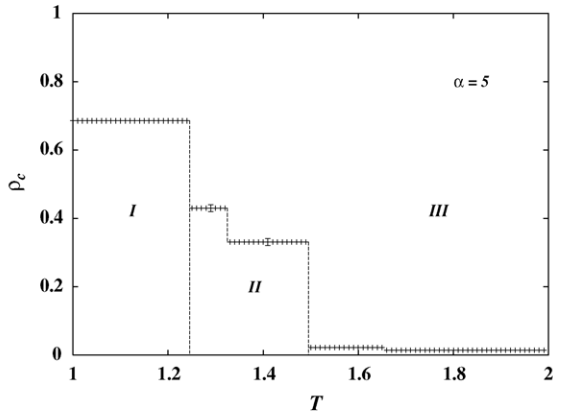

Fig. 2. ρ_c vs. T for random graphs with a fixed degree $\alpha = 5$, $N = 10,000$. (I–III) Characterize the zones of the phase diagram, see (Fig. 1). In the zone (II) ρ_c is the mean value of $\rho_c(t)$ and the bars on the lines reflects the fluctuations of ρ_c in this zone.

3. Fixed degree graphs

3.1. Phase diagram

In Fig. 1 it is shown the phase diagram of the model obtained from the simulations (black symbols) and from analytical computations (dashed lines to be discussed below) when the agents are initially distributed with probability $0 < p < 1$ in the graph. The black symbols represent critical temptations (T_c), i.e. the temptations values at which ρ_c jumps for each connectivity α. Note the perfect coincidence between the points and the predictions, and also the fact that the phase diagram does not depend on p, provided of course that it is different from 0 and 1.

In this figure the full lines define three different regimes. The first one (I) is characterized by a stationary ρ_c, the highest for these degrees α but not necessarily the global majority, as expected for the two dimensional square lattice. A second (II) is characterized by non-stationary states. These states do not necessarily emerge with probability 1. It means that for a given α and a given T in this zone, it will depend on the initial distribution of the cooperators, and on the particular graph whether this phase is observed or not. And a third (III) regime that appears for high values of T, is characterized by a stationary ρ_c that is the global minority ($\rho_c < 0.5$).

Note that for a pure PD's game ($\alpha = 1$) and for a one-dimensional chain ($\alpha = 2$) there are not jumps. For $\alpha > 2$ one can define more than one regime, and within each regime one can observe many jumps. To be illustrative about this point, the Fig. 2 shows the variation of ρ_c as a function of the temptation T for $\alpha = 5$.

From now on, and unless specified otherwise, all the simulations presented result from the average over at least 50 graphs each one initialized once.

For $\alpha = 5$ (Fig. 2) the system is characterized by three different regimes and by four jumps in ρ_c, two of them are associated to the phase transitions (I–II) at $T = 5/4$ and (II–III) at $T = 3/2$, while the other two ($T = 4/3$ and $T = 5/3$) are just jumps of ρ_c within the regimes, see again Fig. 1 to locate these points.

It is not hard to realize that all the dynamics of the model is enclosed in the competition between cooperators and defectors. Because of their small payoffs, the cooperators, to survive, must be organized in clusters therefore, we may imagine the system as a set of cooperating clusters embedded in a sea of defectors. If the boundary of the cooperator's cluster is strong enough it will grow, otherwise, the cluster keeps its size or becomes smaller.

It is important to point out that in this model only phases with cooperators and defectors coexisting may appear. In fact, the defective population may invade the whole graph ($\rho_c = 0$) if the lowest possible payoff for a boundary defector g_0^1, i.e a defector interacting with only one cooperator, is larger than the highest possible payoff of a cooperator g_1^{α}. From Eq. (2) this may happen if $T > \alpha$, but by definition $T < 2$, therefore $\rho_c > 0$ for all $\alpha \geq 2$. Moreover, a whole invasion of the graph by cooperating agents is impossible because one defector surrounded by cooperators will have the highest possible payoff g_0^{α} and is indestructible, $\rho_c < 1$.

For a graph with fixed connectivity it is interesting to see what happens when the temptation T of the agents increases. Obviously, if $T = 1$ the systems does not evolve in time, it keeps its initial distribution of cooperators and defectors. Increasing the temptation, we may ask ourselves, at which value of T, ρ_c is going to change . It is evident that either most of the clusters of cooperators become weak at their boundaries, such that they get invaded by the defectors, or they become strong enough to occupy part of the sea of defectors. Then, the condition of equilibrium that most be satisfied by all the agents in the boundary between cooperators clusters and defectors is the following:

$$g_1^{s_1} = g_0^{s_0} \tag{3}$$

where s_1 and s_0 stand for the number of cooperative neighbors of the cooperator and the defector respectively. But from Eq. (2) $g_1^{s_1} = s_1$ and $g_0^{s_0} = Ts_0$. Then,

$$T_c = \frac{s_1}{s_0} \tag{4}$$

and since we are interested in the region $T > 1$, the Eq. (4) implies that $s_1 > s_0$. Therefore, we may substitute $s_1 = \alpha - n$ and $s_0 = \alpha - n - m$ in Eq. (4) to get:

$$T_{c_{n,m}}(\alpha) = \frac{\alpha - n}{(\alpha - n) - m} \tag{5}$$

where obviously $n < \alpha$ and since $T < 2$, m must also satisfy:

$$1 \leq m \leq \text{int}\left(\frac{(\alpha - n) - 1}{2}\right) \tag{6}$$

In this way, assigning appropriate values of n and m to the Eq. (5) we characterize all the jumps of ρ_c for a given α as a function of T . This is what is represented in Fig. 1 by dashed lines. Of course, the lines are just guided to the eyes, and fixed values of α should be understood when analyzing Eq. (5).

Going deeper in this kind of analysis we may see that the strongest cooperators hold:

$$g_1^{s} > g_0^{s-1} \quad (2 \leq s \leq \alpha) \tag{7}$$

which means that the defectors can not invade the clusters of cooperators, or vice versa a cooperator surrounded by s cooperators will invade all the defectors neighbors with less than s cooperators around them. In other words, while Eq. (7) holds the clusters of cooperators grow inside the sea of defectors (except for $\alpha = 2$, when the winner cooperator does not have any defector to invade). At some point the defectors are isolated and become indestructible, stopping the propagation of cooperators.

Then, following (2), the set of inequalities (7) are satisfied for the temptations:

$$T < \frac{\alpha}{\alpha - 1} \tag{8}$$

that correspond to the first jump at the lowest $T_c(\alpha)$, from Eq. (5). Below this line, appears what we call regime (I), there, ρ_c evolves toward a stationary state where it reaches, depending on the initial conditions, its highest possible value.

On the other hand, for the temptation range:

$$\frac{3}{2} < T < 2 \tag{9}$$

the opposite condition is satisfied, i.e:

$$g_0^1 < g_1^s < g_0^{s-1} \quad (3 \leq s < \alpha) \tag{10}$$

independently on the initial conditions and the degree of the graph. Now, the spreading of cooperators over defective sites is strongly reduced and defectors dominate the system. The condition (10) implies again a stationary final state. But, due to the greatest domination of defectors, ρ_c reaches its lowest possible value for the given initial conditions (regime (III)).

In the intermediate range appears what we call regime (II). In this regime the stability conditions (7) and (10) with an absolute winner do not hold anymore and dynamics instabilities appear in the interior of the graph. Depending on the temptation, T, boundary sites are intermittently occupied by defectors or cooperators, or alternatively lines of defectors travel across the cluster of cooperators.

Moreover, note that in the phase diagram the regime (III) is not only limited to values of T larger than 3/2.

For $\alpha = 4$ the regime (II) is not present, a surprising result considering that it is present in the square graph. Unfortunately we were not able to analytically justify this, but, we are tempted to conjecture that this is a consequence of the absence of spatial correlations in the Bethe graph.

3.2. Degree and initial conditions dependence

In the previous subsection we presented numerical simulation and arguments that justify the independence of the phase diagram from the initial conditions. Here, we go a step further trying to understand how ρ_c changes with the initial conditions and the connectivity of the graph in the different regimes of the phase diagram.

The Figs. 3 and 4 show the final value of ρ_c as a function of the graph connectivity for different initial probabilities p in regimes (I) and (III). In general, a large connectivity enhances the cooperation in the first regime and reduces it in the third one. An important point is that in both curves for α greater than 5, ρ_c is independent of the initial condition (Fig. 3). A situation that holds also in the non-stationary regime-(II). This result avoids the necessity to include perturbations or other external factors to ensure the independence of the initial conditions of the system and remarks the importance of large connectivities to the emergence and sustainability of the collaboration. A similar result

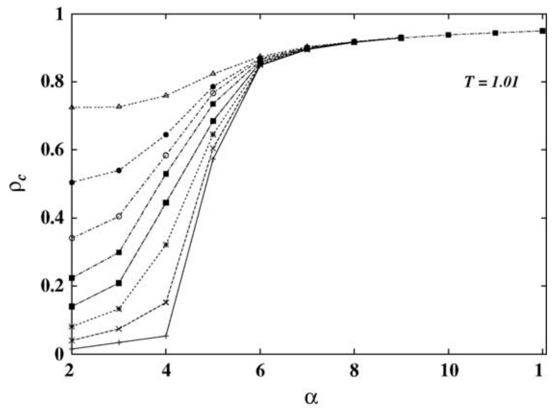

Fig. 3. Initial conditions dependence of ρ_c in regime (I). From bottom to top p varies from 0.2 to 0.9 in steps of 0.1, $N = 10,000$. Note the convergence of ρ_c when $\alpha > 5$ and the minimum reached by the one dimensional chain.

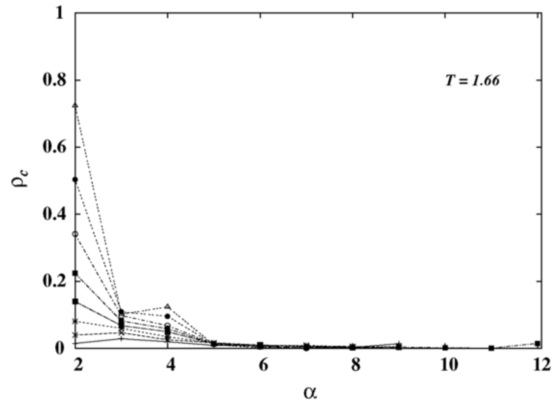

Fig. 4. Initial conditions dependence of ρ_c at $T = 5/3$, a characteristic T_c in the third phase. Note again the convergence of ρ_c when $\alpha \geq 5$.

was already remarked in a different context by Challet and Du [21] comparing the performance of closed and open source software projects.

To better understand the effects of the connectivity on the game dynamics, let us define $P_\theta^s(\alpha)$ as the probability of an agent doing θ to has, at $t = 0$, s cooperative neighbors in a graph with degree α. As before $\theta = 1$ reflects cooperation and $\theta = 0$ reflects defection.

Based on combinatorial arguments we find that:

$$P_\theta^s(\alpha) = \binom{\alpha}{s} p^s (1-p)^{\alpha-s} (p\theta + (1-p)(1-\theta)) \tag{11}$$

and the following relations hold:

$$P_\theta^s(\alpha) > P_\theta^s(\alpha - 1) \quad \text{if } s > \alpha p \tag{12}$$

$$\left.\begin{array}{l} P_\theta^{s+1}(\alpha) > P_\theta^s(\alpha) \\ P_\theta^{s+1}(\alpha) > P_\theta^s(\alpha-1) \end{array}\right\} \quad \text{if } s < \alpha p - 1 \tag{13}$$

Note that, independently of p, the probability to have s or more cooperative neighbors increases when the connectivity grows.

Therefore, from Eqs. (12) and (13), a larger connectivity implies an increase in the number of cooperators linked to both kind of agents. This leads to a local payoff increment that amplifies the domination of the winners agents in each phase. This explains the behavior shown in Figs. 3 and 4. In regime (I), as the cooperator's clusters can only grow, they are strongly enhanced when the connectivity of the graph increases. This is clearly what is shown in Fig. 3, where for $\alpha > 2\rho_c$ increases continuously. On the contrary, in the regime (III) cooperators can hardly invade the defectors, who win practically all boundary interactions (see Eq. (10)). Then, an increase in connectivity virtually exterminates all the cooperators in the system. Regime (II) is more complex, when the degree of the graph increases both kind of agents are enhanced and also the fluctuations in ρ_c are larger.

4. Poisson random graphs

To study graphs with fluctuating connectivity, we assign to each vertex of the graph a number of links determined by a Poissonian distribution with mean α:

$$P(\alpha_i) = \frac{\alpha^{\alpha_i}}{\alpha_i!} \exp(-\alpha) \tag{14}$$

For these graphs the local equilibrium conditions satisfied by neighboring and opposite agents remains:

$$g_1^{s_1} = g_0^{s_0}$$

with the only difference that now the degrees α_0 and α_1 of both sites may be different, a situation that must be taken in consideration during the analysis of the phase diagram.

Following the analysis done for the fixed degree case it is easy to realize that the following temptations characterize the equilibrium conditions for the boundary between cooperating and defecting agents.

$$T_{c_{n,m}}(\alpha_i) = \frac{\alpha_i - n}{(\alpha_i - n) - m} \tag{15}$$

where $\alpha_i = \max\{\alpha_0, \alpha_1\}$, and $T_{c_{n,m}}(\alpha_i)$ is the critical temptation for all sites with degree α_i.

The main difference here, comes from the large number of possible degrees that can be found in this kind of graphs. In fact, Eq. (15) is more general than (4). Moreover, the larger the graph size, the larger the values of α_i's that may be found in the graph. Therefore, the number of jumps defined by (15) increases with the graph size. In the thermodynamic limit $N \to \infty$, an infinite number of jumps must be expected. This is shown in Fig. 5, where we plot ρ_c versus T for different values of N and $\alpha = 6$.

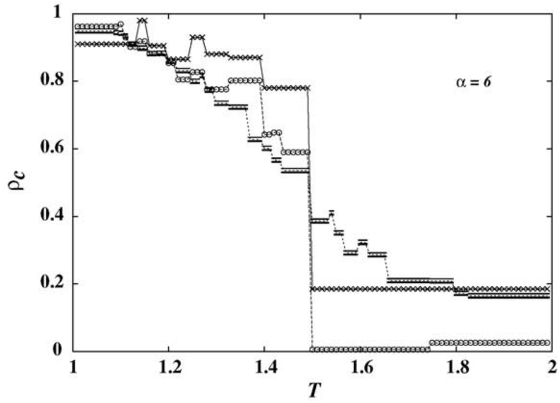

Fig. 5. ρ_c vs. T for different graph's sizes: $N = 100$ (crosses), 1000 (white circles) and 10,000 (black symbols).

The most interesting feature in this kind of graphs is that ρ_c is strongly enhanced with respect to the fixed degree graphs (see Fig. 6). This in good agreement with the results of [13] for square graphs with quenched disorder. However, here it does not reflect the topological accidents of the graph, but its random structure. In fact, from (12) and (13), we may conjecture that in Poissonian Random graphs we will find highly connected sites, that being already cooperators, at $t = 0$, will be in the long time limit, the core of a cooperator resistance for large values of the temptation.

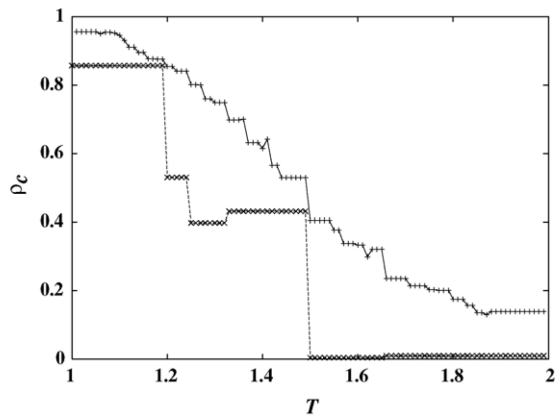

Fig. 6. Comparison between ρ_c vs. T for fixed and fluctuating degree $\alpha = 6$. Note that for the latest ρ_c is always greater, $N = 20,000$.

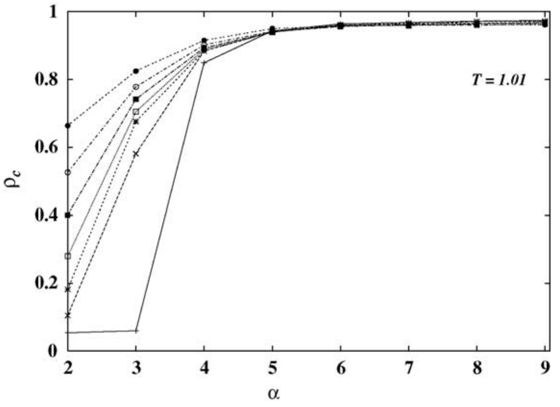

Fig. 7. Initial condition dependence for fluctuating degree graph at $T = 1.01$. Here α is the mean degree and p change from 0.2 to 0.8.

Similar arguments explain why the system will stay in a non-stationary state similar to regime (II) independently on the parameters of the game. Regime (I) is not present, because there is always a critical temptation lower than $T_c(\alpha_i)$ for any $\alpha > 2$ and therefore cooperators will never be absolute winners. Moreover, cooperative agents placed at sites with the largest degrees in their neighborhood may resist any growth of the temptation and become seeds for the spreading of cooperation avoiding the appearance of the regime (III).

Finally, Fig. 7 shows the initial condition and degree dependence of ρ_c. It is interesting that, despite of the absence of regime (I), the curve for low temptation is very similar to that for the fixed degree graphs, again ρ_c becomes independent of p for $\alpha \geq 5$ for small values of T. Again, the explanation of the degree dependence of ρ_c is similar to the one discussed for the fixed degree graph and follows directly from Eq. (11). The increase of the connectivity enhances the winner agents, therefore, for low temptations, the number of cooperating agents increases, on the other hand, for larger values of T the defectors dominate the game and prevent the spreading of cooperators.

5. Conclusions

We present a study of the characteristics of the spatial prisoner's dilemma in random graphs with synchronous evolution rules. For graphs with fixed degrees we were able to fully characterize the phase diagram of the game showing the existence of three different regimes depending on the temptation of the agents and the connectivity of the graph but independent on the initial conditions of the system. We also give analytical arguments to explain the appearance of these regimes. Furthermore, for these kind of graphs we show that for connectivities larger than $\alpha = 5$, also the density of cooperating agents, ρ_c, is independent of the initial conditions revealing the importance of large connected networks as a requirement for the emergence of stable cooperation. We give arguments that demonstrate that in the thermodynamic limit, for Poisson random graphs the density of cooperating agents changes continuously with the temptations of the agents. We also show that only a non-stationary regime exists in these graphs, independently of the temptation, the degree of the graph and the initial conditions. Finally we also showed that in these graphs the cooperation is strongly enhanced in comparison with the fixed degree graph. These results support the importance of the randomness and the connectivity in the appearance and sustainability of cooperation.

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