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The geometrical patterns of cooperation evolution in the spatial prisoner's dilemma: An intra-group model

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Abstract

The prisoner's dilemma (PD) deals with the behavior conflict between two agents, who can either cooperate (cooperators) or defect. If both agents cooperate (defect), they have a unitary (null) payoff. Otherwise the payoff is T for the defector and null for the cooperator. The temptation T to defect is the only free parameter in the model. Here the agents are represented by the cells of a $L \times L$ lattice. The agent behaviors are initially randomly distributed according to an initial proportion of cooperators $\rho_c(0)$. Each agent has no memory of previous behaviors and plays the PD with his/her eight nearest neighbors. At each generation, the considered agent copies the behavior of those who have secured the highest payoff. Once the PD conflict has been established ($1 < T < 2$), this system shows that cooperation among agents may emerge even for reasonably high T values giving rise to the well-known strategy: join to conquer, fight to share. Contrary to previous studies, in which the lattice cells are viewed as groups and are allowed to self-interact (inter-group situation), here the cells are viewed as individuals and are not allowed to self-interact (intra-group situation). Although the short time and asymptotic behavior of ρ_c are similar in both cases, the intermediate behavior is different. Oscillations in the intra-group $\rho_c(t)$ forbids data collapse. The cooperators clusters geometrical configurations are distinct between inter and intra-group models, which explains the $\rho_c(t)$ differences.

Keywords: *N*-agent game; Spatial prisoner's dilemma; Cooperation/defection; Intra-group model; Biophysics; Sociophysics; Econophysics

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1. Introduction

Although spontaneous cooperation has always been observed in natural systems, it does not have a foothold within the Darwinian Evolutionary theory, which frequently focuses on direct competition. No evolutionary modification over species can be selected by the natural selection if the new trait is exclusively beneficial to other species. Indeed, there can be a selective production of directly harmful structures to other animals, such as the viper hooks [1]. In one hand, it may also be suggested that Dawkins' emphasis on selfish

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genes [2] gives a new perspective on the role of competition since it even envisages the possibility of competition inside the genome. On the other hand, in the cellular scale, cooperation may be favored through in-group selection. In this case, although the detriment to the fitness of individuals (expressing a given gene), it may be advantageous to the group (deme) where the individuals are members [3].

The conflict between cooperation and competition, especially in the context of the prisoner's dilemma (PD) problem is well defined. Consider the rule used by Nowak and May [4]: two players have a payoff of either $R = 1$ (reward) or $P = 0$ (punishment), if both cooperate or both defect, respectively. If one of them defects, the defector has a payoff of T (temptation) while the cooperator has a payoff of $S = 0$ (sucker). In the range $T > R > P > S$ and $2R > S + T$, cooperation is the best global strategy while defection is the best individual strategy, characterizing the conflict [5]. Here, the condition $P > S$ has been relaxed without any harm and the conflict range is: $1 < T < 2$. Temptation is the only free parameter in this model. The PD applications range from gene polymorphism in yeast [6], intra-host competition in RNA virus [7,8] to predator inspection in fishes [9–11].

A major advance in the cooperation-competition conflict comprehension has been made with the memory inclusion for the agents, who play the PD repeatedly (Iterated PD). In this system, with one time step memory, cooperation has emerged as an optimum strategy component, either with tit-for-tat strategy in a deterministic environment [5,12] or with the win-stay lose-shift (Pavlov) strategy in a stochastic environment [13].

The interaction among several agents who play PD repetitively, i.e., an N-agent game has also being considered. The PD analysis, which includes the spatial component, has been investigated in details and, when defection dominates ($T \geqslant 2$), this problem is known as "the tragedy of the commons" [14].

Nowak and May [4,15] have considered the spatial prisoner's dilemma (SPD) in the conflict region ($1 < T < 2$) where the cells of a square lattice represent the agents. They have supported the argument that cooperation could emerge as a stable strategy when the dilemma is played dynamically among several agents. As pointed out by Schweitzer et al. [16] and Durán and Mulet [17], instead of viewing agents switching states as a function of time, it is interesting to view the system as two populations invading each other. The SPD has also been played on more realistic structures, which mimic social relations, such as random graphs [17], small world networks [18] and disordered lattices [19], increasing the scope of application of the single PD approach.

The SPD model [4] is particularly interesting when each cell is viewed as a conglomerate (group). This conglomerate can be either in a cooperative or in a defective state and can play the PD with itself, establishing an interacting group environment—an inter-group model. We stress that self-interaction, in this case, is well justified for successive time steps in demes and that cooperation states may emerge from group selection models as suggested before [20,21].

Nevertheless, if one considers an intra-group model, where the lattice represents the whole group and the cells the individuals, one can hardly explain this self-interaction as a viable social and biological behavior. The intra-group model increases the application scope of the SPD to sociophysics [22,23] and to agent-based models in econophysics [24,25] in which the strategy "join to conquer, fight to share" is suitable.

In this paper, we compare the intra and the inter-group models and show that self-interaction produces a bias to cooperative states leading to more stable cooperation clusters. We start describing the model in Section 2, where the general boundary effect analysis is applied to local cooperation clusters to explain their stability. In Section 3, we present and interpret the results obtained by numerical simulations. Final comments are presented in Section 4.

2. Model description

Consider one agent i who plays the PD with an agent j. The agent states are $\theta_i = 0$ for defection and $\theta_i = 1$ for cooperation [16]. The i-agent, in state θ_i, given that agent j is in the state θ_j, has a payoff:

$$g_{\theta_i} = \theta_i\theta_j + T(1 - \theta_i\theta_j)\theta_j. \tag{1}$$

In the SPD, the agents are represented by the cells of a $L \times L$ lattice and we are interested in the proportion of cooperators ρ_c as a function of time, model and T. Initial cooperators agents are randomly (uniformly) distributed according to the proportion $\rho_c(0)$. This is the only stochasticity introduced in the model. The system evolution is entirely deterministic. In each generation (time interval), each agent plays the

PD with his/her first and second neighbors. This agent then compares his/her own payoff with those of the given neighbors. Finally, the agent copies the state of the agent who had the highest payoff. This is the agent state to be used along the following generation. The agent's scores are not cumulative and all the payoffs are reset to zero after each PD round ("one-shot game").

2.1. Phase transitions

Defection is not the only evolutionary stable strategy [26]; three main regimes are observed as a function of T for this system. For $T\sim 1$ ($T\sim 2$), the asymptotic values for the proportion of cooperators ρ_c is stationary and makes up a majority (minority). For intermediate values of T, ρ_c is non-stationary, even rendering chaos (strongly dependent on the initial configurations) [4]. However, each main regime may be divided into regions where $\rho_{c,T}$ changes values according to the number of interacting neighbors α and also to the number of interacting cooperators $0\leqslant s\leqslant\alpha$. From Eq. (1) one sees that if agent i interacts with α agents having s_i cooperative agents around, his/her payoff is [17]:

$$g_{\theta_i}^{(s_i)} = [T-(T-1)\theta_i]s_i. \tag{2}$$

This leads to the following properties: $g_1^{(s)}=s$ and $g_0^{(s)}=Ts$, since $T>1$, $g_0^{(s)}>g_1^{(s)}$ and $g_\theta^{(s)}\geqslant g_\theta^{(s-1)}$. In the SPD model, the transitions on the proportion of cooperators occur when temptation on the surface of the cooperation cluster overpass a threshold value. In the conflict range $1<T<2$, these transition occur at [17]:

$$T_{n,m} = \frac{\alpha-n}{\alpha-n-m}, \tag{3}$$

where $0\leqslant n<\alpha$ and $1\leqslant m\leqslant \mathrm{int}[(\alpha-n-1)/2]$ are integers. These temptation values are shown in Table 1.

If the agents self-interact ($\alpha=9$, $n=0$ and $m=4$, see Table 1), the cooperation/defection coexistence region is in the interval $\frac{9}{5}<T<2$. Contrary to inter-group model ($\alpha=9$), where the defection region starts at $T=2$, if the agents do not self-interact ($\alpha=8$, $n=1$ and $m=3$ in Table 1), this region is: $\frac{8}{5}<T<\frac{5}{3}$, i.e., close to the middle of the conflict region ($1<T<2$) (see Fig. 1). One sees that neglecting self-interaction does not drastically modify either the stationary and the non-stationary regimes, nor the pattern found by Nowak and May [4]. Neglecting self-interaction only shifts the ρ_c curve to lower temptation values. The case where agents self-interact and interact with only their nearest neighbors ($\alpha=5$) is treated by Schweitzer et al. [16]. In this case the transitions values are given by $T_{4,1}=\frac{5}{4}$ and $T_{4,2}=\frac{5}{3}$ (see Table 1). According to Table 1, if the agents do not self-interact in the nearest neighbors model ($\alpha=4$) only one non-trivial transition is expected at $T_{5,1}=\frac{4}{3}$.

Table 1

Temptation values where the transitions on the proportion of cooperators occur. These transitions dependent on the number of considered neighbors α and are given by Eq. (3)

m	α						
	9 →	8 →	7 →	6 →	5 →	4 →	3
	n						
	0	1	2	3	4	5	6
1	$\frac{9}{8}$	$\frac{8}{7}$	$\frac{7}{6}$	$\frac{6}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$
2	$\frac{9}{7}$	$\frac{8}{6}$	$\frac{7}{5}$	$\frac{6}{4}$	$\frac{5}{3}$	2	
3	$\frac{9}{6}$	$\frac{8}{5}$	$\frac{7}{4}$	2			
4	$\frac{9}{5}$	2					

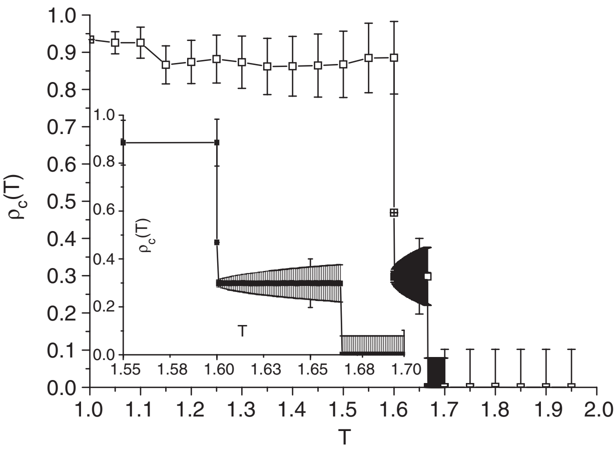

Fig. 1. The asymptotic values (1000 generations) obtained from an initial proportion of cooperators $\rho_c(0) = 0.60$ of non-self-interacting agents in 1000 clone groups in square lattice ($L = 200$) as a function of temptation values using periodic boundary condition. This pattern is consistent with the transition values considering $\alpha = 8$ in Table 1. The error bars refer to the standard error of the mean. Inset: Amplification of the cooperation/defection coexistence region.

2.2. Boundary effect

To investigate the models dynamics and to quantify the payoffs, let us classify the agents according to their position in a $L \times L$ lattice. There are: $(L-2)^2$ inner (bulk) agents who have 4 first neighbors and 4 second neighbors ($\alpha_b = 8$); $4(L-2)$ surface agents who have 3 first neighbors and 2 second neighbors ($\alpha_s = 5$), and finally 4 edge agents have 2 first neighbors and 1 second neighbor ($\alpha_e = 3$). Observe that, as the order of adjacency is decreased, the first-neighbor number decreases arithmetically while the second-neighbor number is halved (geometrical decrease).

Consider the number of times the PD is played along the x- and y-axis direction and diagonally among the cells, which are given by $I_x = I_y = L(L-1)$ and $I_{xy} = 2(L-1)^2$, respectively. Adding these terms, one obtains the number of times the PD is played in each generation for fixed boundary condition (FBC) $N_f = I_x + I_y + I_{xy} = 2[2L^2 - 3L + 1]$. For periodic boundary conditions (PBC), one has $I_x = I_y = L^2$ and $I_{xy} = 2L^2$ and the number of times the PD is played is: $N_p = 4L^2$ and $N_f = N_p - 6L + 2$. This quantifies the effect of the PBC. For $L \gg 1$, $N_p \simeq N_f \simeq 4L^2$, where the remaining difference between PBC and FBC is of order L.

The fixed boundary effects can be applied to the cells of a cooperative cluster and provides some insight into the dynamics of the system. The agents cooperation adjacency idea allows the following agent classification: the bulk agents, who have a payoff of 8 units (due to 8 cooperative surrounding agents), the surface agents with payoff 5, and the convex edge agents with payoff 3, while the surrounding defector agents payoffs are T.

In the cooperation clusters, the deterministic dynamics forbids bulk agents to switch states since all nearest and next-nearest neighbors have the same state. Here, the edge agents can be either convex, when the given agent has 2 nearest and 1 next-nearest cooperators, or concave, when the agent has 4 nearest and 3 next-nearest cooperators. The convex edge agents are the least stable ones. The cooperative cluster invasion starts at the convex edge cells. These cells have the smallest payoff and switch state, producing new edge cells, which again, are more susceptible to switch states. As illustrated in Fig. 2, cooperation clusters evolve differently when self-interaction is whether considered or not. If self-interaction is considered, the cooperative clusters are essentially squares, having 4 convex edges. When self-interaction is neglected, the cooperation clusters are essentially spherical, with more convex edges, which diminish their stability. We notice in Fig. 2 that the same cooperation cluster density on both populations only rises for smaller T when self-interaction is neglected.

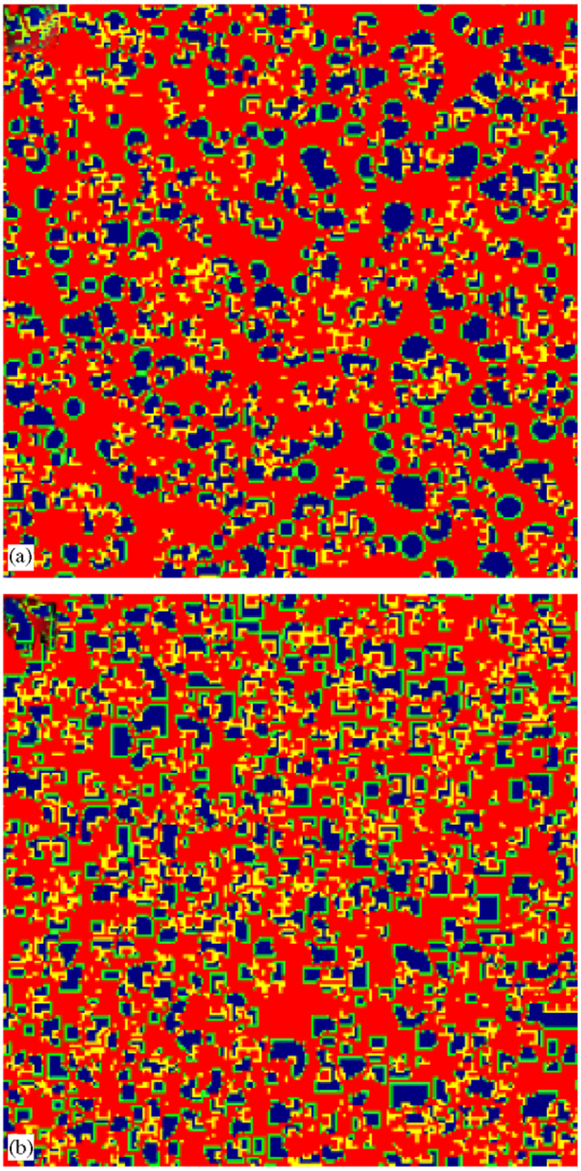

Fig. 2. One realization of cooperators and defectors with $T_{nsi} = 1.65$ and $T_{si} = 1.95$ as $t \to \infty$, with periodic boundary condition, when self-interaction is neglected (a) and considered (b). The initial values are $\rho_c(0) = 0.60$ for a square lattice of size $L = 200$. The following color coding has been used: blue (dark grey), the agent is a cooperator; green (light grey), the agent is a former defector; red (grey), the agent is a defector; yellow (very light grey), the agent is a former cooperator. Without self-interaction (top), cooperative clusters grow as round-shaped forms, while self-interaction (bottom) causes these clusters to grow in square shapes.

3. Simulation results and analysis

The comparison between the proportion of cooperators $\rho_c(t)$ as a function of time (generations) t, with and without self-interaction, is shown in Fig. 3 for fixed and PBC. In each case, we have considered up to $t_{\max} = 1000$ generations and $M = 1000$ realizations on a $L = 200$ square lattice. Around 12 h have been spent on a Pentium Celeron 1.8 GHz computer with 256 Mb of Random Access Memory (RAM) running Linux operation system to generate the data used in Fig. 3.

Starting with $\rho_c(0) = 0.60$, both systems react in the first generation by lowering $\rho_c(t)$ and then increasing it due to the formation of cooperation clusters. The seed needed to form a cooperation cluster is to have four

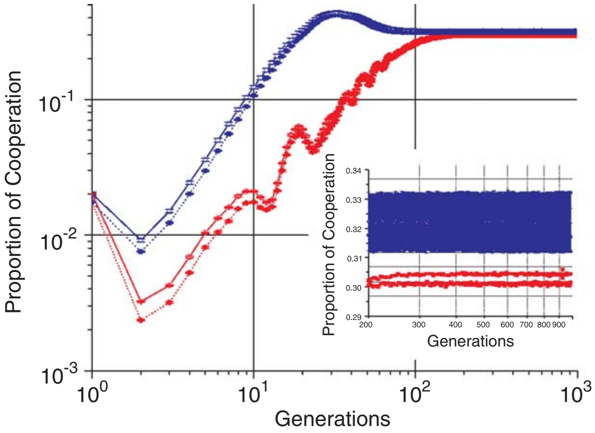

Fig. 3. Proportion of cooperators ρ_c as a function of time t for $T_{si} = 1.95$ and $T_{nsi} = 1.65$ for inter (below) and intra-group (above) models with fixed (solid line) and periodic (dotted line) boundary conditions. The initial values are $\rho_c(0) = 0.60$ for a $L = 200$ lattice and $M = 1000$ realizations. Inset: Same but with $200 < t < 1000$. The fluctuations are much smaller in the inter-group model.

nearest neighbor cells in a cooperative state. The $\rho_c(t)$ increase occurs up to ten generations for both models. We notice that $\rho_c^{(si)}(t) \sim 5\rho_c^{(nsi)}(t)$ for $1 \leqslant t \leqslant 10$. The intermediate regime is the most interesting one, as the differences between the inter- and intra-group models are striking: $\rho_c^{(si)}(t)$ is a smooth function, while $\rho_c^{(nsi)}(t)$ oscillates. Finally, after about 100 generations, both systems stabilize and the cooperation proportion is $\rho_c^{(si)}(\infty) = 0.318$ and $\rho_c^{(nsi)}(\infty) = 0.299$. These values are very close to each other but they are statistically significant. They persist indefinitely as stationary values (see inset of Fig. 3). The error bars represent the standard error of the mean. They typically have the same values but for the inter-group model with FBC. In this case, they are one order of magnitude greater due to the fragmentation of the cooperative clusters on the lattice boundaries. The initial proportion of cooperators does not alter the intermediate and stationary values, limiting its effects to the first 10 generations (initial regime).

In the intermediate regime, from 10 up to 100 generations, $\rho_c^{(si)}(t)$ grows rapidly, presenting a significant bump ($t \sim 25$) before stabilization. On the other hand, $\rho_c^{(nsi)}(t)$ grows less rapidly and presents smaller, but more frequent, bumps (oscillatory behavior). The reason behind these differing behaviors can be understood through the cooperation clusters geometrical structures consideration, which implies different cooperating cluster stabilities. As we have seen, these clusters are square-shaped when self-interaction is considered and circle-shaped when self-interaction is neglected. The former is more stable than the latter due the presence of fewer convex edges. Let us focus on the oscillations and on the $\rho_c(t)$ fluctuations. When agents self-interact, the initial cooperative clusters grow, increasing $\rho_c^{(si)}(t)$ until they intersect one another (and/or touch the boundaries in the FCB case) producing numerous, less stable cooperation clusters. This interaction generates the large bump (Fig. 3) and may produce larger fluctuation when compared to the intra-group model (see Fig. 3). When the agents do not self-interact, the clusters growing behavior is different, because they are rounded, having more convex edge cells. The clusters grow until they reach a critical size and then break into seeds, which form new cooperation clusters. These clusters have a notable ability to self-disperse into more homogeneous, smaller growing islands. This mechanism produces the transient oscillations and smaller fluctuations in the inter-group model observed (Fig. 3). This means that the "join to conquer" strategy leads to the growing of cooperation groups until interests are disturbed by other cooperation group (groups collide) if self-interaction is considered. If self-interaction is neglected, "join to conquer" strategy leads to the growing of cooperation groups up to a critical size and internal interests are enough to produce the group splitting (fight to share).

4. Conclusion

We have considered (inter-group) and neglected (intra-group) the agent's self-interaction in the spatial DP game. For short and long running times, both models present roughly the same average behavior for the cooperation proportion $\rho_c(t)$. Nevertheless, this quantity fluctuation for the inter-group case with fixed boundary condition is greater (by one order of magnitude) than in the other cases. In the intermediate regime the inter- and intra-group $\rho_c(t)$ differ drastically due to the geometrical shape of the cooperation clusters, which are typically square for inter group models and round-shaped for intra-group models. The self-interaction overall effect is to favor cooperation in all steps of evolution and increase fluctuation due to the higher stability of the cooperation clusters.

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