Evolutionary prisoner's dilemma game with dynamic preferential selection

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(Dated: May 23, 2006)

A modified prisoner's dilemma game is numerically investigated on disordered square lattices characterized by a ϕ portion of random rewired links with four fixed number of neighbors of each site. The players interacting with their neighbors can either cooperate or defect and update their states by choosing one of the neighboring and adopting its strategy with a probability depending on the payoff difference. The selection of the neighbor obeys a dynamic preferential rule: the more frequency a neighbor's strategy was adopted in the previous rounds, the larger probability it was picked. It is found that this simple rule can promote greatly the cooperation of the whole population with disordered spatial distribution. Dynamic preferential selection are necessary to describe evolution of a society whose actions may be affected by the results of former actions of the individuals in the society. Thus introducing such selection rule helps to model dynamic aspects of societies.

PACS numbers: 02.50.Le, 05.50.+q, 87.23Cc, 89.65.-s

I. INTRODUCTION

Game theory [1, 2, 3, 4] were introduced to study complex behaviors qualitatively of biological, ecological, social and economic systems. Of particular famous is the evolutionary prisoner's dilemma game (PDG), which was introduced by Axelrod [3] to study the emergence of cooperation among selish individuals, have attracted most attention in theoretical and experimental studies. Recently, more and more attentions have been focused on the applications of the PDG in the area of behavior sciences, biology and economics, etc [5, 6, 7]. In the original PDG the players could make two choices: either to cooperate with theirs co-players or to defect. They are offered some payoffs depended on their choices, which can be expressed by 2×2 payoff matrices in agreement with the four possibilities. The players get rewards $R(P)$ if both choose to cooperate (defect). If one player cooperates while the other defects, then the cooperator (C) gets the lowest payoff S (sucker's payoff), while the defector (D) gains the highest payoff T (temptation to defect). Thus the elements of the payoff matrix satisfy the conditions: $T > R > P > S$ and $2R > T + S$, so that lead to a so-called dilemma situation where mutual cooperation is beneficial in a long perspective but egoism can produce big short-term profit.

In the studying of the PDG, one of the most interesting items is to study under what conditions will the mutual cooperation emerge and sustain stably or how to facilitate the cooperation of the whole population [2, 3, 4]. In the PDG, the state where all players are defectors has proved to be evolutionary stable state [8], which has inspired numerous investigations of suitable extensions that enable cooperative behavior to persist. Nowak and May [8] have introduced a spatial evolutionary PDG model in which individuals located on a lattice play with their neighbors and with themselves. The dynamics of the game is govern by a deterministic rule: individuals adopt the strategy that has received the highest payoff among its neighbors including themselves. It had been shown that the spatial effects promote the survival of cooperators [8, 9, 10, 11]. Szabó and Tőke extended the deterministic dynamics of the model to a stochastic evolutionary one: rather than following the most successful neighbor's strategy straightly, the adoption of one of the neighboring strategies is allowed with a probability dependent on the payoff difference [12]. This revised version took account into the irrational choices of the players and observed that below certain critical values b_c (noise-dependent) a stable absorbing state of all C emerged. Recently the spatial PDG have been studied on different social networks models, it was found that cooperation can be maintained on these networks in a wide range of network parameters [13, 14, 15, 16, 17, 18]. In addition, dynamic network model [19] and dynamic payoff matrices [20] were also introduced to sustain high concentration of cooperators in the evolution of PDG.

In the present work, we study the PDG using Szabó-Tőke version [12] on disordered lattices with slightly different dynamics. Rather than randomly selecting a neighbor and adopting its strategy with a probability between two rounds [12, 17, 18], the players select one of their neighbors to update their states according to a dynamic preferential rule: the neighbor whose strategies were adopted more frequent by them in the previous rounds will be selected with lager probability. Our main aim is to investigate how the underlying structure of interaction and the preferential rule affect the evolution the game. Using systematic Monte Carlo (MC) simulations, we calculate the density of cooperators as a function of the temptation to defect b for different disorder levels ϕ of the lattice and impact factor α of the "successful "strategy (see the definition of the model). It is found that both the structural parameter ϕ and the preferential selection rule have an influence on the evolutionary results of the game. In the case of regular square lattice $\phi = 0$, the preferential selection rule benefits slightly the spreading of defectors, while for mixed population $\phi > 0$ cooperative behavior can be greatly enhanced by forming clusters of cooperators in a wide range of parameter b. In addition, disordered structure is also proved to be favor for the persistence of cooperation. These results are distinct from previous researches [8, 9, 10, 11] which believe that the spatial structure may promote the survival of cooperators.

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II. MODEL AND SIMULATION

We consider an evolutionary PDG with players located on disordered square lattices with a ϕ portion of random rewired links and fixed number of neighbors of each site. The players are pure strategists and can follow only two simple strategies: C (always cooperate) and D (always defect). Each player plays a PDG with itself and with its neighbors and collects payoff depended on the payoff-matrix elements. The total payoff of a certain player is the sum over all interactions. Following previous studies [8, 12, 13, 15], the elements of payoff matrix can be rescaled, i.e., we can choose $R = 1$, $P = S = 0$, and $T = b(> 1)$ without any loss of generality in the evolutionary PDG.

In society, some special persons may influence others munch stronger than the average individual, still these influential persons are coupled back to their social surroundings [15]. In other words, different neighbors would have different impact on one's behavior. In general, one can expect that the influence between two people is asymmetric and would evolve with time. To model this situation we define a quantity $A_{ij}(t)$, which describes the impact weight of jth player to ith player at time t and possesses asymmetric property, i.e., independent of the corresponding quantity $A_{ji}(t)$. In this way, we hope to catch some general effects that dynamic asymmetric influence among the players might have on the dynamical behavior of the game.

The randomly chosen player i revises its strategy by selecting one of its neighbors j with a probability γ according to a preferential selection rule:

$$\gamma_{ij} = \frac{A_{ij}(t)}{\sum_{k\in\Omega_i} A_{ik}(t)}, \tag{1}$$

where Ω_i is the community composing of the nearest neighbors of i. Eq. (1) means that the larger the impact weight of a neighbor, the more probability it is selected to compare with. If and only if their strategies are different, the ith player's state as well as the neighbor's impact weight will be updated, otherwise nothing happens (no strategy transformation and weight updating). Accepting the idea suggested by Szabó [12, 17, 18], given the total payoffs (E_i and E_j) from the previous round, player i adopts the neighbor's strategy with the probability

$$W = \frac{1}{1 + \exp\left[-(E_j - E_i)/K\right]}, \tag{2}$$

where E_j is the neighbor's payoff and K characterizes the noise introduced to permit irrational choices. Note that the decision is only affected by their payoff difference. Since the work by Szabó-Tőke [12] the parameter K is usually fixed to 0.1, therefore in the present study we use the same value. Generate a random number r uniformly distributed between zero and one, if $r < W$, the neighbor's strategy is imitated and a revising on $A_{ij}(t)$ is performed according to the following rule

$$A_{ij}(t+1) = A_{ij}(t)(1 \pm \alpha), \tag{3}$$

where the minus corresponds to the case of $r > W$ with no strategy updating for ith player. For the initial condition, all $A_{ij}(0)$ are assigned as 1.0. The parameter α in Eq. (3) can be depicted as impact factor which characterizes qualitatively the relative change of the impact weight of once comparison. Since in most realistic cases the influence of the successors would be greater than those losers on one's behavior, this rule could be termed as: 'win-strengthen, lose-weaken'.

Two groups of systems will be considered subsequently. In the first case $\alpha = 0.0$ with $\phi = 0.0, 0.1$, and 1.0 are studied. This means that the impact weight is independent of time, namely the neighbors of each player will be selected with equal probability to compare with during the whole process of the evolution. This allows us to understand how the underlying lattice structure would affect the evolution of the PDG. In the second case $\alpha = 0.01$ with corresponding ϕ values are investigated, i.e., the dynamic preferential rule is taken into account to study what influence of this rule will have on the evolutionary PDG. Starting from a random initial state, the rules of the model are iterated with parallel updating by varying the value of b for fixed ϕ and α values. We have found that a small amount of external noise is efficient to avoid the slowing-down phenomenon towards the stable state of the system. To do this, after a round of play, we chose one player at random and flip its strategy. This is enough to speed the system to attain dynamic equilibrium. The total sampling times are 6×10^4 MC steps and all the results shown below are averages over the last 5000 steps.

III. RESULTS

In the following we show the results of simulations performed in systems with 300×300 players. Our key quantity is the cooperator density ρ_C, the average fraction of players adopting the strategy C of the equilibrium state. The main features of the steady-state phase diagram are similar to the results obtained in Ref. [12], i.e., there exist two different absorbing states ($\rho_C = 1$ and $\rho_C = 0$) whose stability regions are separated by the active phase. We have found numerically that $\rho_C \simeq 1$ in all cases we are interested if $b < 5/4$, which can be regarded as a homogeneous cooperation state. Since our main aim goes beyond this trivial steady-state, we will only concentrate on the region of $b > 5/4$, where many new features may emerge.

Our main results, i.e., the b dependence of the average density ρ_C of cooperators in the equilibrium state for different values of ϕ and α, are shown in Fig. 1. First we consider the model without preferential selection ($\alpha = 0$). In the case of $\phi = 0.0$, which corresponds to square lattice structure, we recover the result of the stochastic model [12]. ρ_C decreases monotonically with increasing b until a certain threshold b_c, where the cooperators vanish. With more long range links emerging on the lattice ($\phi = 0.1$), the level of cooperation is promoted unexpectedly, which is contrary to the previous researches whose results support that the local interaction may promote the cooperation of the whole population (see Refs. [8, 9, 10, 11] and the references therein). Particularly, in the

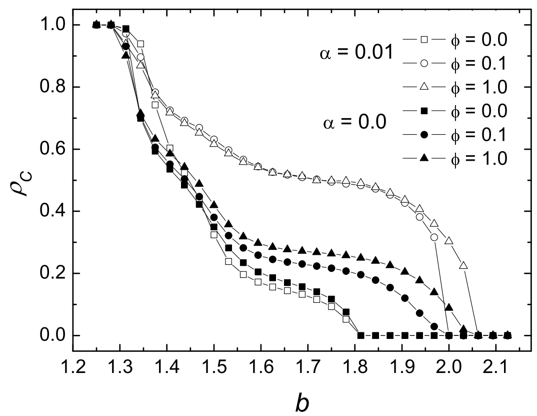

FIG. 1: Average density of cooperator ρ_c as a function of the temptation to defect b in the equilibrium state. Open and closed symbols correspond to the case of random selection and preferential selection of one neighbor to compare with respectively.

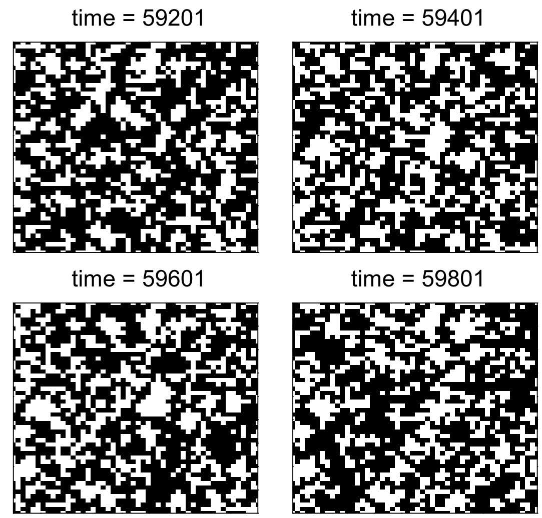

FIG. 2: Snapshots of equilibrium configurations of cooperators (white) and defectors (black) in the evolutionary PDG on a disordered lattice ($\phi = 0.1$) for $b = 1.906$. A 50×50 portion of the full 300×300 players is illustrated.

case of $\phi = 1.0$ corresponding to a random mixed population, where there is no spatial structural advantage, the cooperation is enhanced extensively. Even in the case $b > 2.0$, minor fractional cooperators can be found in a sea of defectors. Recently, Huaert and Doegeli have studied another famous evolutionary game, snowdrift game, on different types of lattice [21]. They have found that spatial structure eliminates cooperation for intermediate and high cost-to-benefit ratio of cooperation because benefits of costly cooperative acts accrue not only to others but also to the cooperator itself [21, 22]. Review of the present PDG model we studied, each player C plays with itself besides its nearest neighbors, which indicates that it will gain at least R payoffs even in the worst case (surrounding by defectors). In a different interpretation, besides their neighbors, the cooperators' investment will benefit themselves too. In addition, the high cost-to-benefit ratio of cooperation in snowdrift game corresponds to large values of the temptation to defect b in PDG. Then there is not surprising that the disordered structure would promote the cooperation in the present model.

We now consider the influence of the dynamic preferential selection on the evolution of the game. The results obtained for $\alpha = 0.01$ are summarized in Fig. 1 using open symbols. Though the qualitative behavior referred by the calculations is similar to those of the previous version, there are some remarkable differences. For well-structured populations, dynamic preferential selection promotes cooperation for small b; however, for large b, the fraction of cooperators is lower than in random selection case, i.e., the defectors are favored. While for mixed populations ($\phi = 0.1$ and 1.0), cooperative behavior can be greatly promoted and maintained in a wide range of the parameter b. Even in the extreme defection circumstance ($b > 2.0$), cooperators can survive and persist with a minor level as illustrated by open triangles in Fig. 1. We expect that the dynamic preferential selection induces the emergence of

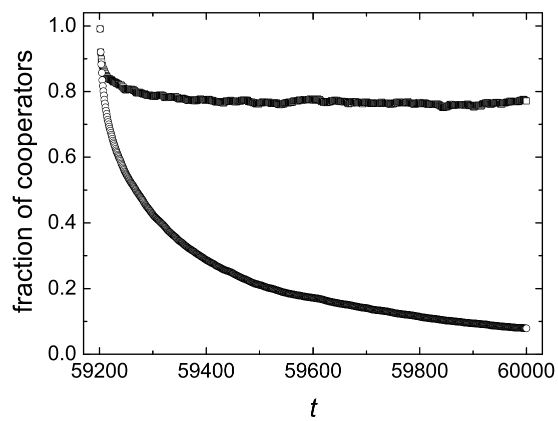

FIG. 3: The average fraction of those who cooperate at a special time $t = 59201$ in the steady-state, whom again adopts strategy C in the subsequent evolutionary process (squares); and the fraction of those who always cooperate after time $t = 59201$ (circles).

influential players; and if some of them are cooperators, then compact communities consisting of their neighbors and themselves could be formed and survive stably in the background of defectors, which would contribute to the persistence of the cooperation.

In order to check this statement and also get an intuitive understanding of the evolution, four typical snapshots of the steady-state distribution of cooperators and defectors are illustrated in Fig. 2. These snapshot are a 50×50 portion of the full 300×300 players. From these configurations, one can observe how the communities of cooperators persist. Though their center, size, and shape change continuously and two communities may unite or single community may divide into more parts or disappear, their space distribution is rough persevered in a long time scale (even after six hundreds MC time steps). In a distinct view, the average fraction of those who cooperate at a special time $t = 59201$ in the steady-state, whom again adopts strategy C in the subsequent evolutionary process, is reported in Fig. 3, also illustrated the fraction of those who always cooperate after that time. A detailed numerical analysis results in approximate eighty percent and ten percent of them respectively after eight hundreds MC steps. These results suggest that, instead of the random walk and annihilation of the clusters of cooperators [12], they fluctuate stably in the background of defectors.

IV. CONCLUSIONS

To sum up, we have explored the general question of cooperation formation and sustainment from the perspective of coevolution between the dynamics of the players' state and their interactions. Both factors of the underlying structure and the dynamics of the game were considered. On the one hand, disordered lattices are introduced to study the effect of the topological structure of interaction. Our investigations support the results obtained in Refs. [21, 22], i.e., spatial extension generally fails to promote cooperative behavior in a system where every individual contributes to a common good and benefits from its own investment. In the present model, this condition is realized by simply letting the players play with themselves besides their neighbors.

On the other hand, considering the asymmetric influence in many nature populations, we defined the individuals' impact weight, which describes the strength of the influence of the players to their neighbors. Based on this quantity, a dynamic preferential selection rule was introduced to the dynamics of the game. The state updating of the players is performed by selecting one of their neighbor to compare with and determine whether adopt it's strategy or not dependent on their payoff difference. The larger impact weight of a neighbor, the more probability it was selected. The simulation results have indicated that this selection rule have a remarkable influence on the evolutionary results of the PDG. In the case of well-structured populations $\phi = 0$, the preferential selection rule benefits slightly the spreading of defectors for large b; while for mixed populations $\phi > 0$, cooperative behavior can be greatly promoted and maintained by forming communities consisting of influential cooperators and their neighbors in a wide range of parameter b. Dynamic preferential selection are necessary to describe evolution of a society whose actions may be affected by the results of former actions of the individuals in the society. Thus introducing such selection rule helps to model dynamic aspects of societies.

V. ACKNOWLEDGEMENT

This work was supported by the Doctoral Research Foundation awarded by Lanzhou University.

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