Modelling spatial practices and social representations of space using multi-agent systems

Jean-Luc Bonnefoy* - Christophe Le Page- Juliette Rouchier** - François Bousquet****

** UFR de géographieUniversité de Provence*
29, rue R. Schuman
13 628 Aix-en-Provence

Jean-Luc.Bonnefoy@aixup.univ-mrs.fr

*** Cirad Tera Ere*
Campus de Baillarguet
BP 5035, 34032 Montpellier Cedex

Lepage@cirad.fr
Rouchier@cirad.fr
Bousquet@cirad.fr

ABSTRACT: *This paper demonstrates that multi-agent systems have the capacity to model a region in all its complexity. An example is developed to show that these tools are not only capable of spatializing and distributing the behaviour of individuals, but above all, that they allow individuals to integrate different perceptions of space as well as the constraints imposed on them by a community. A dialectic is established between individuals, spaces and society, which is used to simulate a region using clearly defined social representations and spatial practices, which are suitable for testing our geographical theories and hypotheses.*

RÉSUMÉ : *Il s'agit de mettre en évidence la capacité des systèmes multi-agents à modéliser un territoire en sa complexité. Un exemple est développé pour démontrer que ces outils sont non seulement susceptibles de spatialiser et de distribuer le comportement des individus mais qu'ils autorisent surtout l'intégration de perceptions différenciées de l'espace par les individus et de contraintes qui leurs sont exercées par une collectivité. Une dialectique s'instaure entre individus, espaces et société, qui contribue à la simulation d'un territoire par le biais de pratiques spatiales et de représentations sociales clairement définies, propres à tester nos théories et hypothèses géographiques.*

KEY WORDS: *modelling, multi-agent systems, geographical space, social representations, sustainable development*

MOTS CLÉS: *modélisation, systèmes multi-agents, espace géographique, représentation sociale, développement soutenable.*

1. Background:

This paper relates to research that has been undertaken by the authors specifically on the modelling of spatial or social dynamics. It presents the methodological approach that is common to their research. F. Bousquet and C. Le Page are interested in the modelling of interactions between natural and social dynamics using multi-agent systems (MAS) in the context of research on renewable resource management[1]. J. Rouchier specialises in the relationships of exchanges between individuals in a society and, particularly, on the role of trust in the context of renewable resource management. J.L. Bonnefoy is interested in networks of individuals' spatial practices which constitute a region or are influenced by a region. The authors hope that this paper will encourage a fresh approach to the concept of region because the development of research on the subject tends to follow a social theory that sometimes neglects spatial constraints and sources and vice versa.

Multi-agent systems are used because they have "the potential to model individuals, their behaviour and interactions directly and offer radically new solutions to modelling" [Ferber, 1995]. We consider that these properties could be beneficial to geography. In fact, "multi-agent modelling is based on the capacity of current software programmes to give individuals a degree of autonomy". The individuals or agents can represent people, animals, trees, etc., or, more broadly, a town, village, herd or forest. An agent is an "entity capable of acting on itself or its environment, which reacts to its changes and has a partial representation of its environment" . By evolving in a modelled space—in the form of a regular grid in which resources are spread out, indeed a more complex reconstruction of an observed reality—each agent builds up its own representation of space and by acting, the agent transforms the space for others. Interactions are central to this type of modelling.

This kind of approach is in itself a theoretical and methodological response—ie a theory is really played out—to dealing with complex phenomena and it has an important contribution to make to environmental issues. For the geographer, it is another way of putting a behavioural approach in a spatial context at the level of individuals. This is achieved by defining the agents and the rules that govern their interaction, and not by applying heavily parametrized formulas that represent dynamic systems and which take more account of inflows and outflows than the behavioural aspects of interactions . Lastly, it is an effective way for researchers to construct experiments, in other words to play out their theories, spatial models or hypotheses and to simulate and compare what happens in a multi-agent universe with an observed "reality".

2. Multi-agent systems and geographical space

A novel geographical approach using MAS involved modelling the dynamics of the evolution of a system of towns [Bura, 1993], particularly the hierarchies in terms of the urban functions and the population. The towns observed expansion, which fitted with the theories on urban hierarchy and activities became more specialized because of supply and demand mechanisms. However, the agents' (towns') intrinsic immobility meant that the model did not use the multi-agents' capacities to the full in terms of spatial interaction.

Some research has been conducted on the application of MAS to problems of spatialized resource management. For example, Schmitz [Schmitz, 1997] studied different ways of organizing agents for managing a resource distributed in space. In addition to research in the field of ecology and ethology, where scientists seek to understand the mechanisms for finding food [Folse, 1989 ; Roese, 1991; Drogoul, 1993; Krebs, 1996; Pepper, 1999], studies have also been conducted on societies of social agents that manage common resources [Epstein and Axtell, 1996; Kohler and E., 1996].

These models do not incorporate the different levels on which space can be considered. When resources are put into a spatial context, this is usually a question of simple geographical coordinates in a continuous space or elementary cells in a defined space. The representation of natural spatialized processes or agents' representations of space

[1] The application used here benefited from advances made with the software Common Pool Resources for Multi-Agent Systems (CORMAS) developed at CIRAD.

presupposes that spatial entities are modelled on several different levels which can be manipulated by the agents.

F. Bousquet and D. Gautier [Bousquet, 1999] have demonstrated two ways of approaching the integration of space in MAS using the example of a process of agricultural expansion: farmers cultivate land around their village, whose population is growing; animals roam freely which degrades the forest savannah at varying rates. The first approach is a classic-type integration where space is the support for the resources used by herds (forest) and farmers (fertile areas). The agents have rules that govern their spatial behaviour. They modify their environment and perceive all the modifications that occur within their field of perception objectively. In the second model, the spatial entities such as the forest, fields and savannah are established *a priori*. In this way, the authors present the spatial structure of the multi-agent universe. These spatial units have the characteristics of agents but the herds and farmers are only apparent in the rules that govern how the entities function. It is as if the forest makes way for the savannah, the fertile areas may way for fields, etc. The authors consider that these two approaches mark the beginning of research on the integration of space in MAS.

When the two models are considered from a geographical viewpoint, there are several points to note. In the first model, space is central to the interactions between agents and the model takes account of the individual movements of the farmer and herd agents, their interactions as well as the spatial occurrences that are linked to the relative position of resources and their transformation. However, some aspects are lacking when it comes to converting the model of this support space into a model of a region. Firstly, the level of interaction between space and society is zero because, in the machine, space is merely the support for the forest, savannah, etc., and individuals only act on these elements if they happen to meet. Secondly, over time the agents' individual action produces a space where spatial entities like the forest, fields, savanna and the village community are simply the result of a visual construction assembled by the observer (the simulator). There are no models of other types of contingencies, namely, the relative importance of social issues on behaviour and of collective and individual representations of space. For example, the collective representation of the forest which forms during the simulation cannot be reintroduced in the simulation. Only the observer-simulator's perception can be introduced empirically when the results of the simulation are interpreted, which is a different point of view again.

In the second model, which is the opposite of the above approach, the agents are spatial entities which presupposes their existence *a priori*. The observer-simulator models a geographical construction which requires a thorough understanding of how to balance the degree of spatial generalization and the semantic level of these entities and their rules of exchange. Although modelling exchanges between spatial entities may seem unnatural, it is possible that the behaviour of individuals could give rise to spatial structures that can be identified by the agents during or after the simulation, which is not the case here. In particular, exchanges are totally objective, everything occurs as if an unexpected power was regulating the relationships between entities, overriding individual action. In addition, there is no longer any real spatialization apart from the position of objects in the multi-agent space and the dynamics can only be managed quantitatively. Lastly, in this particular example, it is highly likely that the results will only confirm the original spatial hypotheses [Bonnefoy, 1998]. Nonetheless, this approach can produce results of interest for resource management or the development of an area. To achieve this, the scale of the study must be appropriate to the definition set by the construction of the spatial entities. In this sense, it is advisable to work on a smaller scale and to involve several village units and differentiated topographic configurations, in other words to provide the complexity required for the chosen spatial scale.

One of the benefits of MAS is their capacity to reveal how different stakeholders use space and their perception of it and, if the case should arise, the characterization of types of space, their differentiation and organization, which is every geographer's preoccupation. We envisage a model that associates individual spatial practices and the group's appreciation of space. Here, the region is no longer a spatialized sub-unit of the

MAS, on the contrary, the MAS becomes the modelled region. This method is not just a question of integrating space into multi-agent systems, it is the arrangement of theoretical concepts of geographical space. It involves the construction of a dialectic between a space produced by the society and a space that restricts individuals, using individual and collective representations. This calls for what Distributed Artificial Intelligence experts refer to more generally as learning. In addition, the existing interaction between space and society must be integrated, ie the dynamic constraint that the space—which is produced by society—imposes on the society. Our example uses spatial representations to model the interactions between the path taken by shepherds who graze their flocks in the undergrowth and a forestry resource, with the aid of spatial representations. The grazing is actively controlled, to a greater or lesser extent, by the individual or collective representations which are built up during the simulation, integrating the successive states of the forestry resource between savannah deterioration and re-growth. In the model of interactions, the representations act as mediator between the agents and the common resource [Bousquet, Barreteau et al., 1999]. Here, however, the representations are individual or collective constructions that come from spatial practices during the simulation instead of being common references established *a priori* as in the case of religious practices, for example [Lansing, 1994]. Thus, in the model, we consider three elements that interact dynamically: individuals' spatial practices, individual representations of space and collective spatial representation. These three elements interact as time goes by in the machine and the individual practices and representations simultaneously ensue from the collective representation and influence it.

3. Description of the model of the forest's social representations:

The aim of the model is to determine whether multi-agent systems have a valuable contribution to make to the understanding of a geographical space and, more generally, to investigate the dynamics between individual and collective representations and how they are manifested. Above all, the model is useful as an illustration. We can draw up a work hypothesis and ask whether the combination of individual and collective spatial representations in a multi-agent "forest" system is sufficient for simulating the management of the forest resource. In other words, how much emphasis should there be in the model on the shepherd agent's "awareness" of the need to manage his environment and on the restrictions that the community imposes on individual spatial practices if a system for using the forestry resource—ranging from simple predation to sustainable management—is to be established?

The model is as follows: each shepherd agent takes his flock to graze in a forest that is divided into groves. This leads to a degradation of the savannah forest. At the same time, the forest regrows naturally according to a probability that gives priority to regeneration at the edges of the forest. Through their spatial practices, the shepherd agents memorize the areas where they have been and form their representations: the state of the whole forest is then judged on the basis of these partial perceptions. The shepherd agents proceed by extrapolation to form their global representation of the forest. The more forest spaces they come across, the more their representation is of an abundant forest and vice versa. Periodically, these different individual representations (referred to as individual thresholds in the model) "meet" in the village and a collective representation is put together. Here, this construction is symbolized by an average evaluation which sets a collective threshold limiting the future grazing. In theory, the groves that are smaller than the collective threshold will not be grazed. However, several strategies are open to the shepherd agent. His individual practices can conform to the group's orders, in which case his strategy is called "collective". He can ignore the collective rules and graze his flock systematically, a strategy which is called "personal". Lastly, he can go against the collective rules by modifying his practices, and so strike a middle course between his habitual practices and the collective threshold. This strategy is called "arrangement". In the model, the shepherds' strategies are established at the start of the simulation and are permanent. The elements: village, shepherd, forest and land are identified by variables and processes which give them their autonomy in the programme (Figure 1)[2].

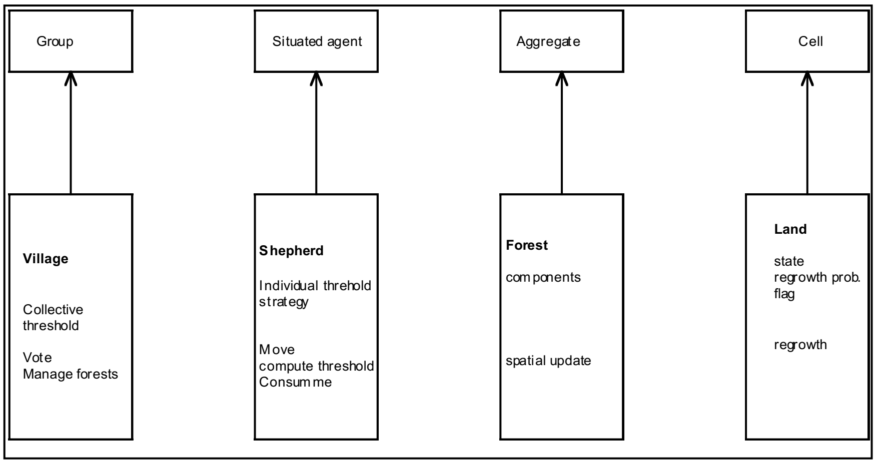

Figure 1: the model's components. Inheritance from Cormas classes

In a multi-agent system, space can be represented by a grid made up of cells—that represent a particular use or resource—onto which the agents move as a function of a time step that regulates the artificial world. In our case, the shepherd agents and their flocks move at random but, as far as possible, they stay in the forest once they have reached it. The forest is a set of cells that may or may not be adjacent (Figure 2).

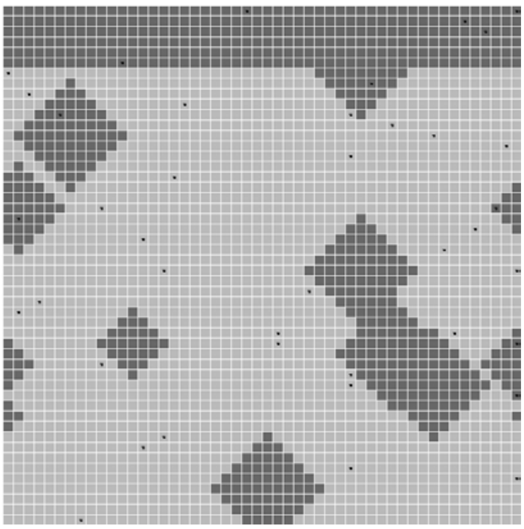

Figure 2: initial state. The forest is in dark grey (686 cells are arranged into 11 groves), the savannah is pale grey. The shepherds and their flocks are represented by a dot. This multi-agent universe is a closed grid of 50 x 50 squares, each one has eight adjoining squares.

Each grove (group of forest cells, or one isolated forest cell) is identified cyclically by the community and, depending on its size (the number of squares it contains) and the set

[2] Figures 1, 3 and 4 have been set up using Unified Modelling Language which makes it is possible to overcome the constraints relating to the multi-agent systems' programming environment.

collective threshold, an indicator (flag) indicates the village agent's decision concerning its use. The shepherd agent who then wants to go into the grove may or may not graze his animals. The following diagram shows the agents' general behaviour and perception which depends on their surroundings (in or outside the forest) and their strategy (Figure 3). It will be detailed during the course of the simulations.

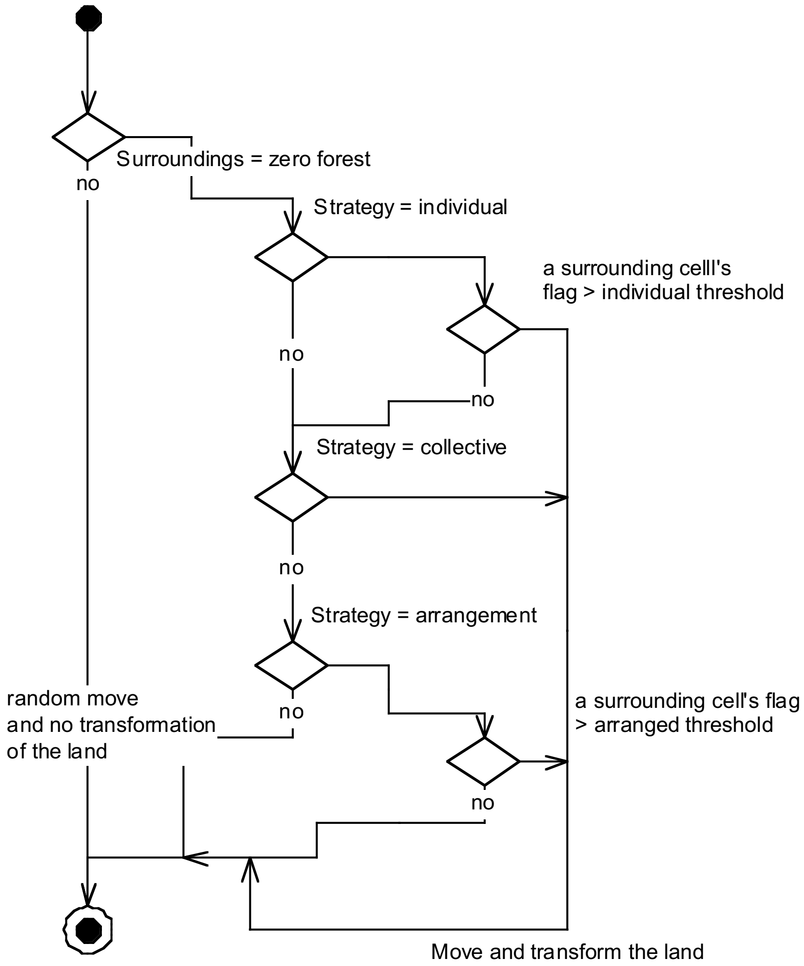

Figure 3: behaviour and perception of the shepherd agent. The section in the dotted box is a variation of the "personal" strategy.

The model's rhythm is set according to the rationale described above which is illustrated in Figure 4. Here, we can see the series of different sequences mentioned above: the shepherd agents' behaviour and perception; forest regrowth which depends on a random draw; the calculation of the size of new groves. The last stages have a longer periodicity: the calculation of the individual threshold based on the routes undertaken; the calculation of the collective threshold and the differences between the collective and individual thresholds; updating the flags that ban grazing which the shepherd agents will cross depending on their strategy. Then, the cycle begins again.

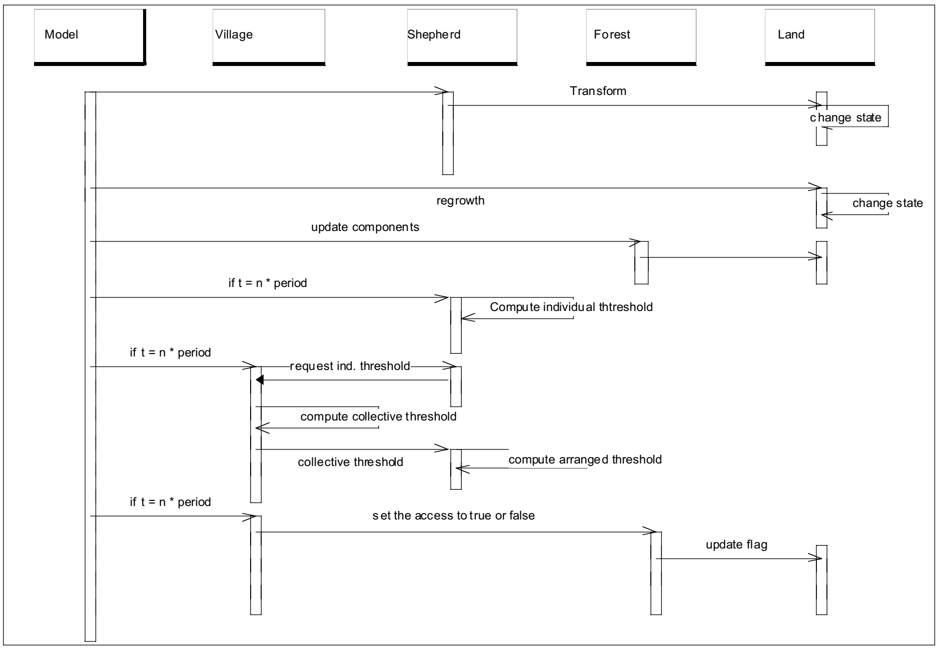

Figure 4: diagram of sequencing for one time step.

4. Several simulations

We plan successive simulations where the play of individual and collective representations will increase. In each simulation, 40 shepherd agents move through the space at random. Two simulations involve the "personal" strategy. This "method of modelling interactions is similar to what economists refer to as externalities" (Bousquet *et al*., to be published). In fact, the shepherd agents' practice influences the practice of others even when there is no direct contact between them. In the first scenario, the representations are nonexistent and the shepherd grazes his flock as soon as he comes across the resource. In each simulation, the forest disappears after 120 to 250 time steps. This scenario illustrates the tragedy of the commons where collective goods are exploited to the point of exhaustion because profits are individualized and costs are shared [Hardin, 1968]. Later research that integrates awareness of the resource and social interactions, in particular (for example, [Bousquet, Duthoit et al., 1996]), has demonstrated the shortcomings of this hypothesis. In addition, the major criticism of this theory concerns the fact that common resources are not necessarily freely available. Societies organize rules to regulate access, and this is one of the objects of our simulations.

A second scenario includes individual representations of the forest that come within the "personal" strategy and take no account of the group. It is the shepherd's own past perception of the forest—his learning—that conditions where he grazes his flock. If the forest is degraded, the shepherd does not memorize many wooded spaces on his route and his new individual threshold goes up[3] which means he will not be able to graze small groves (Figure 3). In this scenario, we could consider that the shepherd agent becomes "aware" of the finite nature of the resource being modelled. The shepherd agent anticipates in accordance with his perception of the immediate surroundings. The individual threshold is comparable to an indicator that is inverse to the flock size. In "reality", the limitation that the shepherd imposes on himself can be interpreted as him giving up a number of animals to adjust flock size to the forest's new carrying capacity.

[3] In fact, the "route memory" cumulates the number of squares of forest perceived in a period of 10 time steps in order to make up the individual threshold that results from the balance period – route memory (with a period equal to 10). In this way, the individual maximum threshold is 10.

The results of the simulations show that a model with the capacity to integrate the individual representation and individual management of a resource-flock combination, provides an alternative to the tragedy of the commons (table 1).

Of the initial forest, 25% is maintained in 20 groves (Figure 5 and Table 1). The average number of squares covered by the shepherd agent is low (26 squares). Given the considerable standard deviation, these figures show the huge disparity between individual spatial (coming across a large grove, etc.) or economic opportunities (absence of competition with other shepherd agents). The average individual threshold is very high which suggests that flocks are small.

Strategy	***Forested cells***	***Number of groves***	***Average grazing***	***Mean threshold***
Individual	***150***	***20***	***26***	***8***
Collective	***230***	***61***	***32***	***5***
Mixed	***233***	***63***	***32***	***6***

Table 1: Results of simulations with the different strategies

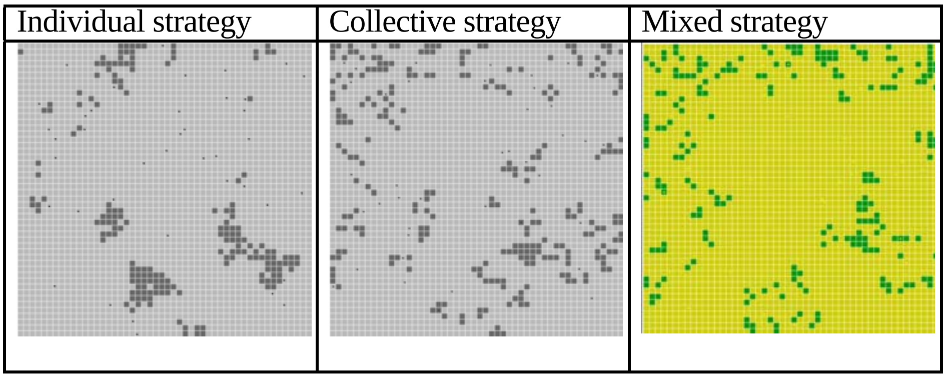

Figure 5: state of the forest after 300 time steps

Our next simulations introduce the restrictions imposed by the group. The first strategy is "collective". An average of the individual thresholds (in other words the individual perceptions of the state of the forest) is calculated every 10 time steps once they have been updated, and groves that are smaller than this threshold are excluded from grazing which is indicated by "hanging flags" to denote a ban. The result of the simulations shows that much more of the forest is maintained than in the previous example (35% of initial forest) but it is fragmented into small groves (Figure 5 and Table 1).

This is only the spatial translation of the method used by the group (and by the simulator) to impose a restriction on the shepherd agents because the ban relates to a minimum size. If a restriction of this type imposed by the group bears little relation to a real situation (protected forest spaces are often large and there are few of them), it seems to be socially effective here: the flocks graze more and the deviations are smaller. There is an apparent normalization carried out by the group which reduces the inequalities between the shepherd agents. In addition, the average threshold indicates that flock size is greater. It could be that the fragmentation of the forest into small groves means that there are more spatial opportunities available to each shepherd agent for reaching forest areas. The inadequacy of this spatial explanation becomes clear later on.

The dynamics and the regulations are activated by the agents' or the group's incomplete knowledge of the forest environment. The collective threshold can be compared to the events in this multi-agent universe in order to determine the difference because the "average" individual "learning" is a quantitative (ie nonspatial) interpretation of the size of the forest. When the value of the collective threshold is five at the end of the simulation, this presupposes that the average shepherd agent estimates that half of the space is wooded. Yet, only 10% of the space is forested. The same applies to the previous model in which the shepherd agents perceived that 20% of land was wooded when in fact the figure was only 7%. It is true that in our model, one forest square can be counted several times in the reference period. But this very biased representation is obviously part of the regulation. Deviation is caused by numerous factors. The first, which is very important, is the use of an average to represent the behaviour of a group! Another factor is that the orders for management apply to the next 10 time steps while grazing continues, the forest grows back and the orders are also out of sync (in time) because they reflect the memory of the routes undertaken during the preceding 10 time steps. The natural increase in grove size can invalidate the grazing ban. Groves that are only just bigger than those subject to a ban can be totally deforested before the next collective decision is taken because it is the village agent and not the shepherd agent that has the power to impose a ban. This raises an interesting question about whether or not an individual should be given responsibility in the context of sustainable management. This option was simulated in the model so that its impact on forest cover could be assessed: the shepherd agent can exercise self control by comparing the collective threshold with the size of the grove that he wants his animals to graze. The effect is immediate because the inertia due to the decision-making intervals disappears. However, the results from a limited number of simulations were not that different from the above strategy. In fact, it does not answer the frightening question as to what becomes of collective responsibility when there is a "transfer" to individual reason.

The "arrangement" strategy, based on the "collective" strategy, appears like a dispensation adopted by the shepherd agent. By meeting halfway, he allows his flock to graze the groves that are of the size set collectively plus half the difference between that and his own threshold of perception (cf. Figure 3). This violation of the group's orders could be considered as a necessary delay because it gives the agent time to reduce the size of his flock (individual threshold) which would occur anyway since the forest in his immediate environment will continue to diminish. Simulations of this type produce results very similar to the "collective" strategy

We observed that the number of forested squares fluctuated considerably from one simulation to another. The figures explicitly translate the oscillations between "personal" and "collective" strategies. It is also interesting to note that when an individual adaptation of the collective rule is modelled, there is more deviation between agents in relation to the accumulation of grazed spaces. This adaptation leads to slightly more forest fragmentation and slightly smaller flocks (collective threshold is 6 instead of 5). These results have a small contribution to make to the hypothesis of relationships proposed above concerning the fragmentation of the forest into small groves, the grazing opportunity and the difference in grazing between agents with a "collective" strategy. In the light of the last results, it appears that respecting the collective rules reduces the initial deviation in time, i.e. the deviation linked to the relative position—advantageous or otherwise—of each one (in relation to the forest and the other shepherd agents).

5. Conclusion

This model, which is a very simple construction, can be used to simulate a wide range of situations and a great deal of interaction because of the dynamic established between the space and the individual and collective representations. In fact, the individual representations only reflect the learning process that each shepherd agent goes through to find the forest spaces. It does not reflect the "objective reality" of this multi-agent universe because the agent has only evolved in a very small part of the available space and his personal and past experience are no indication of the actual state of the forest. From these individual perceptions, a collective representation emerges which provides a common rule and means that each shepherd has access to the other shepherds' representations. This incomplete knowledge means that there is a disparity between the reality and the perception of the multi-agent universe, which helps enrich the dynamics and regulate the resource. Thus, if the "personal" strategy refers to the concept of externality, we could say that the "collective" strategy refers to the theory of conventions, the "collectivization" of representations within the agents' society which acts here as a stimulus to the triptych of individual, space and society. In terms of modelling, there is a mediator and a catalyst between the three poles which are driven by the disparity between the events that occur in the multi-agent universe. Modelling the play of spatial representations developed by the agents during the course of their action is interesting in the framework of MAS and as an approach to geographical space and even sustainable resource management. Multi-agent modelling can include experts' representations as well as their decisions, which means it is possible to understand their implications for a resource and how they are linked in a social context. The research presented here illustrates the theoretical issues being discussed in the field of MAS on accounting for social constraints and individual autonomy [Gilbert, 1995] in a dynamic environment. Collectively, the agents decide on the restrictions that they impose on themselves for using an environment, they adapt individually to these social restrictions and, thus, transform their common environment, then strengthen or change the social rules depending on their degree of satisfaction.

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