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Efficient Nash equilibria in a federal economy with migration costs

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Abstract

We consider a federation of two regions populated by identical individuals, in which interregional migration is costly. We define a federation as an economy in which migration may not be restricted by governments. We compare and contrast first-best efficiency (with migration controls) and federal efficiency (without migration controls). We show that first-best efficiency requires maximising total product net of migration cost, while federal efficiency does not. We also show that migration costs may lead to a discontinuous federal utility–possibility frontier and to discontinuous regional reaction functions. We establish that decentralised equilibrium allocations may not be first-best efficient but are federally efficient. We conclude by tying together well-understood results from the limiting cases of free mobility and immobility with our results for the intermediate case.

Keywords: Fiscal externality; Interregional transfers; Migration costs; Efficiency

JEL classification: H7; R5

1. Introduction

Consider a federation of regions. This is a two-level hierarchical public system with a federal government at the upper level and with regional governments

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(looking after the economic interests of their own constituency) at the lower level. The population can migrate between regions within the federation. Under these institutions, what is the rôle of a federal government in dealing with potential problems associated with interregional population movements? There is a growing literature about this problem. In the simplest case, the federal economy involves costless interregional migration and identical individuals who supply labour in their region of residence. These conditions imply equal utility across the land in equilibrium. An additional feature, such as regional provision of public goods or regional rent sharing, creates a fundamental interdependence among regional public economies which is realised through individual migration decisions. In this context, Flatters et al. (1974) and Boadway and Flatters (1982) have shown that interregional transfers are necessary for efficiency. The reason provided for the inefficiency associated with private behaviour was that individuals do not take into account the implications of their move on the tax price of the public good, or on per capita rents, in both regions of origin and destination. The presence of such a fiscal externality prompted the initial belief that a federal government is necessary for efficiency. Subsequently, however, Myers (1990) and Krelove (1992) have shown that the necessary efficient transfers will be made by the regional governments, simply because maximising a regional utility subject to equal utility across regions is equivalent to maximising the national equilibrium utility level. The presence of such incentive equivalence among governments renders a federal government unnecessary for efficiency. Furthermore, since there is only one possibility for distribution in equilibrium, namely, equal utility across the land, there is also no distributive role for a federal government.

What happens when interregional migration becomes costly? Hercowitz and Pines (1991) have shown in a dynamic and single-good model that the equilibrium of decentralised regional governments choosing regionally-uniform head taxes (or subsidies) and interregional transfers is not characterised by production efficiency, in other words, the equilibrium is not first-best efficient. That a decentralised equilibrium is not first-best efficient does not necessarily imply a rôle for a federal government. The question is whether or not a federal government, constrained to operate with the same set of policy instruments used by the regional governments, could do better from an efficiency perspective. Thus a first objective of this paper will be to build a model with migration costs and to characterise and contrast first-best efficiency with constrained efficiency. In what follows, the concept of constrained efficiency, henceforth federal efficiency, will be motivated by showing that the associated instrument restrictions are necessary for consistency with fundamental federal institutions.

Non-pecuniary costs of migration have been introduced by Mansoorian and Myers (1993), who build a model where agents derive utility from consumption of a single good and directly from their region of residence. An individual has a different degree of attachment to different regions, the region of strongest attachment being defined as 'home'. The non-pecuniary migration costs arises when an agent migrates from home. It is assumed that the cost of migrating ranges from zero to one and that there is at least one individual associated with each level of cost. As a result, when both regions are populated, there is an individual who is indifferent between the two regions (a marginal agent) and policy change induces migration. Mansoorian and Myers do not characterise first-best efficiency, but do show in their model that a federal government using the same instruments as regional governments cannot do better than the decentralised equilibrium from an efficiency perspective, that is, the equilibrium is constrained efficient. We differ from Mansoorian and Myers in various ways. First, we assume the usual pecuniary migration costs. A more important distinction stems from our assumption that all residents of a region face the same migration costs. As a result, there is a non-trivial set of policy choices associated with no migration. Furthermore, as will be explained below, we find that imposing the same migration costs for everyone may imply discontinuous utility–possibility sets and regional reaction functions. This structure for migration costs, because it is rather common, allows for closer ties to Hercowitz and Pines (1991) and Wildasin (1991) (section III), as well as for a diverse set of models in other literature, for example, Steiger and Tabellini (1987) and Boadway and Wildasin (1990).

We structure our paper as follows. We begin with a description of technology and preferences of a single-good and homogeneous-population model in Section 2. We then study first-best efficiency in Section 3. We find that a necessary condition for first-best efficiency is a population partitioning between regions that maximises national product net of migration costs (the 'pie'). We also find that, given an initial population partition, there is a unique first-best partition.

As usual, in characterising first-best efficiency, the central planner can assign individuals both to a residence and to a level of consumption. We remove these assumptions in Section 4. An important, if not defining, characteristic of a federation is that every individual in it can choose residence without legal impediment. For example, the Treaty of Rome, a founding document of the European Union, allows citizens of a member state the right to seek employment and to obtain social benefits without legal prejudice in any other member state. As a result governments in the federalism literature typically cannot impose immigration quota, cannot force emigration, and cannot discriminate by taxing or subsidizing individuals according to their place of origin.[2] We follow that literature by not allowing migration controls and by restricting governments to a regionally-uniform taxation on their homogeneous residents.

In order to characterise federal efficiency and the decentralised equilibria we must know what are feasible allocations under the available instruments. Section 5 determines feasibility. There we show how different migration regimes (e.g.

[2]The ability of a government to tax-discriminate on the basis of origin is equivalent to the imposition of an immigration toll—a powerful instrument for controlling immigration (see Myers and Papageorgiou, 1994b).

migration versus no migration) impose different constraints on the problem. In particular, when the initial partition does not maximise the pie there is a production advantage of inducing migration, but staying at the initial partition has the distributive advantage of making feasible a larger set of consumption distributions.

Federal efficiency is analyzed in Section 6. Two points are drawn from our analysis in this section. (1) With our instruments and structure for migration costs, the utility–possibility frontier (UPF) in a federal economy is discontinuous. Since different migration regimes impose different constraints on the central planner, the UPF cannot be characterised by solving a single constrained optimisation problem. A different problem must be solved for each migration regime yielding solutions conditional on the regime which can then be compared in deriving federal efficiency. Under these conditions discontinuities will often arise. (2) A central planner in a federal economy may not maximise national product net of migration costs. This happens because, contrary to the case of a first-best economy, federal institutions restrict the policy instruments available to the central planner so that distributive issues and maximising resources available for consumption can become interdependent. We show that a central planner may stay at an initial partition which does not maximise the pie in exploiting its less restrictive constraints on distribution.

Once federal efficiency has been characterised, we are ready to address the question of economic reasons for the existence of a federal government under costly migration. Two points are drawn from our analysis in Section 7. (1) We graph regional governments' reaction functions for all cases and show that our structure for migration costs lead to discontinuities in the those functions. This result is consistent with results emphasised in Mintz and Tulkens (1986), who study tax competition under cross-border shopping with transportation costs. Unlike that model, however, the discontinuities in our model do not lead to problems of existence. As with the UPF, different migration regimes impose different constraints on the problems of regional governments. Hence a government's behaviour cannot be characterised by a single constrained optimisation problem. The associated discontinuities in reaction functions arise because a marginal change in one region's policy can lead the other to desire a complete switch in migration regimes, an associated switch in constraints, and thereby a large change in the policy which represents its optimal response.[3] To the best of our knowledge, the existence of these discontinuities has not been recognised in

[3] This logic is very analogous to Mintz and Tulkens, the difference being that our migration regimes translate into their trade regimes (e.g. trade versus autarchy). Mansoorian and Myers (1993) is not characterised by discontinuities in either the UPF or reaction functions. The assumption that there is at least one individual associated with each level of migration cost trivializes the no migration regime so that there is no change in constraints in moving between regimes. The possibility of discontinuities does not require that all agents have the same migration costs, as in our paper, only that there are gaps in the distribution of individuals over migration costs.

the literature on costly migration. (2) We establish that the equilibrium allocation of decentralised regional governments is federally efficient but not first-best efficient. That is, there is no efficiency rôle for a federal government with the same set of instruments in our model. Finally, in Section 8, we conclude by tying together the well-understood limiting cases of free migration (no migration costs) and no migration (prohibitive migration costs) with our intermediate case. We also discuss various extensions and limitations.

2. General features of the economy

There is a nation represented by a closed system of two regions, 1 and 2, each one endowed with a geographically fixed resource T_i for $i=1, 2$. A large population, N, is partitioned between the two regions. A population partitioning between the two regions, henceforth 'partition', is given as (N_1, N_2) with $N_1 + N_2 = N$ and $N_i > 0$. Since one of the two regional populations is sufficient to determine the partition, we specify it from now on by using the population of region 1. The partition varies because individuals can migrate from region i to region j at a positive cost m_{ij}. We define the initial partition as N_1^0.

Individuals are identical with respect to both ability and tastes. They supply one unit of homogeneous labour in the region where they live and they derive utility from the consumption of a numéraire good given by a concave function U. Local labour and resource are combined to produce the numéraire under a linear homogeneous and concave technology $X_i(N_i, T_i)$. Marginal products are denoted y_i and r_i for labour and resource respectively, and the total amount of production is denoted $X \equiv X_1 + X_2$.

3. First-best efficiency

We characterise first-best efficiency by choosing a population partition and an allocation of consumption to individuals so that no individual's utility can be increased without decreasing the utility of at least one other individual. A necessary condition for first-best efficiency is that the resources available for consumption are maximised. These resources are the economy's total product net of migration costs, the 'pie'. We show that the first-best efficient population partitions are given by,

$$N_1^* = \begin{cases} N_{1,\mathrm{L}}^* & \text{if } N_1^0 < N_{1,\mathrm{L}}^* \\ N_1^0 & \text{if } N_{1,\mathrm{L}}^* \le N_1^0 \le N_{1,\mathrm{H}}^* \\ N_{1,\mathrm{H}}^* & \text{if } N_{1,\mathrm{H}}^* < N_1^0, \end{cases} \tag{1}$$

where $N^*_{1,L}$ is the unique N_1 such that $y_1 - y_2 = m_{21}$ and $N^*_{1,H}$ is the unique N_1 such that $y_2 - y_1 = m_{12}$.[4] Notice, if there are no migration costs, we have the standard result that N^*_1 corresponds to equal marginal products.

Fig. 1 can be used to illustrate this result. Marginal products in that figure are shown by lines A. The increase in the pie due to the move of one individual from region 2 to region 1, which is given by $(y_1 - m_{21}) - y_2$, appears as the vertical difference between lines B_1 and A_2. This is positive for a partition less than $N^*_{1,L}$, zero for a partition equal to $N^*_{1,L}$ and negative otherwise. On the other hand, the increase in the pie due to the move of one individual from region 1 to region 2, which is given by $(y_2 - m_{12}) - y_1$, is shown as the vertical difference between lines B_2 and A_1, and is positive for any partition larger than $N^*_{1,H}$, zero at $N^*_{1,H}$ and negative otherwise. If the initial partition is smaller than $N^*_{1,L}$, moving individuals from region 2 to region 1 increases the pie until the efficient partition $N^*_{1,L}$ is reached. On the other hand, if the initial partition is larger than $N^*_{1,H}$, moving individuals from region 1 to region 2 increases the pie until the efficient partition $N^*_{1,H}$ is reached. In-between $N^*_{1,L}$ and $N^*_{1,H}$, any initial partition is first-best efficient because moving individuals between regions would decrease, rather than increase, the pie. Therefore $[N^*_{1,L}, N^*_{1,H}]$ represents all first-best efficient partitions and, given N^0_1, there is a unique N^*_1. Once the pie is maximised consumption is divided among individuals.

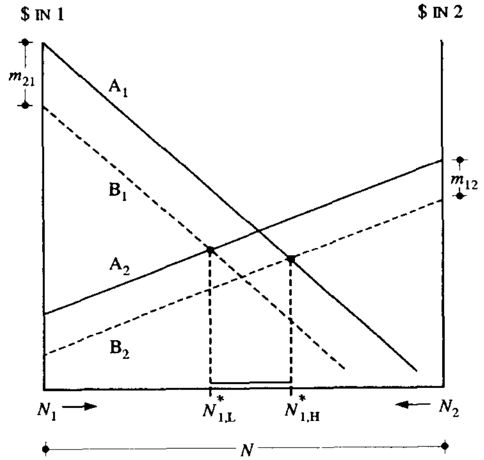

Fig. 1. First-Best Efficient Partitions.

[4]Throughout the paper the subscript L (H) will denote the lowest (highest) value of the variable. A formal proof of the claims in this section is available in the longer version of this paper Myers and Papageorgiou (1994a).

4. Federal economy

Under first-best efficiency, the central planner has the ability to assign a residence and a level of consumption to an individual. Governmental actions, however, are institutionally restricted, it is often assumed that governments can impose taxes and provide goods and services but they cannot interfere otherwise in private choice and markets. Furthermore, as we pointed out in the introduction, a defining characteristic of a federal economy is that interregional migration cannot be restricted by governments. Following these, we assume that regional governments may only choose uniform head taxes on their homogeneous residents (see, for example, Wildasin (1986) section two).[5] A central goal of our paper is to work out how such standard instrument restrictions affect feasibility and efficiency under costly migration.

We next decentralise the economy of Section 2. Competitive firms produce the good, pay labour and the resource their marginal products and earn zero profits. Regional governments have two instruments: a uniform head tax τ_i and a per unit tax t_i on the fixed resource in their jurisdiction. Since at most they can tax away all resource rents, we have $t_i \leq r_i$ (free disposal). The budget constraint of regional government i is given by

$$\tau_i N_i + t_i T_i = 0. \tag{2}$$

Finally, for homogeneity, we assume that all individuals own equally the resource available in the nation. Thus their budget constraint is written as

$$x_i = y_i - \tau_i + (r_i - t_i)\frac{T_i}{N} + (r_j - t_j)\frac{T_j}{N} \tag{3}$$

for $i \neq j$, where x_i is the after-tax income for an individual in region i expressed in terms of the numéraire. When there is migration into a jurisdiction, the original residents consume their entire after-tax income while migrants must first deduct their migration costs.

Combining (1) and (2) to eliminate τ_i, and invoking the zero-profit condition for firms, yields regional feasibility

$$N_i x_i = X_i - S_{ij} + S_{ji} \qquad \text{with } S_{ij} \equiv N_j(r_i - t_i)\frac{T_i}{N} \tag{4}$$

where S_{ij} is the total transfer of after-tax rents from region i to region j, and $S_{ij} \geq 0$

[5] Since individuals differ only by their place of origin and since tax discrimination is not allowed on this basis, uniform taxation is required for consistency with federal institutions. The absence of discrimination by place of origin eliminates entrance prices. We also do not allow exit subsidies. We will discuss this in our concluding section.

by free disposal.[6] In what follows we treat S_{ij}, rather than t_i, as a decision variable. By (4), choosing one of those two determines the other. Using (4), the after-tax income of individuals in terms of interregional transfers is given by

$$x_i = \bar{x}_i - \frac{S_{ij} - S_{ji}}{N_i} \tag{5}$$

where $\bar{x}_i$ is the average product of labour in region i. For simplicity we define the net interregional transfer relative to region 1:

$$S_1 \equiv S_{12} - S_{21}. \tag{6}$$

5. The migration equilibrium and interregional transfers

Since no migration restrictions are allowed in a federal economy, governments cannot choose their regional population size directly. They can, however, choose interregional transfers which affect regional per capita income by (5)—hence the partition. More specifically, a change in income caused by a change in transfers can create migration because individuals will move from region j to region i if they can increase their consumption by doing so. Since the individual benefit of moving from region j to region i equals $(x_i - m_{ji}) - x_j$, individuals migrate if $x_i - m_{ji} > x_j$.

We imagine that government decisions about interregional transfers and individual decisions about location occur in two stages. We begin with an initial partition N_1^o. During the first stage, regional governments choose their interregional transfer taking any transfer by the other regional government as given. We discuss the issue of commitment in the concluding Section 8. During the second stage, individuals take transfers and the location of other individuals as given and choose their residence, production then takes place, government policies are implemented, and consumption occurs. Governments are fully cognizant of the consequences of their first-stage choices on second-stage behaviour. We then solve for the subgame-perfect equilibrium. The migration equilibrium in the second stage will be established when the migration requirement $x_i - m_{ji} > x_j$ fails in both regions: the equilibrium partition is such that

$$x_i^e - x_j^e \leq m_{ji} \tag{7}$$

[6] An alternative rent-sharing framework, where regional governments own the resource would also result in (4). In that model, $x_i = y_i - \tau_i$. Allowing for non-negative regional government transfers (regional governments cannot tax the other jurisdiction), the regional budget constraints become $\tau_i N_i + r_i T_i - S_{ij} + S_{ji} = 0$. Regional feasibility (4) is obtained once again by following the same procedure as before.

holds for both regions, where an e superscript indicates variables in migration equilibrium.

In Appendix A we specify how transfer policies affect migration decisions at the second stage—hence the equilibrium allocation. Knowing what is feasible with transfers will allow us later on to characterise both efficiency and equilibrium.

An understanding of feasibility is facilitated by Fig. 2. Begin by considering the case of no migration costs in Fig. 2(1). In this case N_i^e is determined from $x_1^e = x_2^e$ by (5). With $S_1 = 0$, N_1^e is given by the unique population partition which equalises average products. We can then imagine S_1 being larger during the first stage (in which case N_1^e falls), or S_1 smaller (in which case N_1^e grows).[7] Fig. 2(2), on the other hand, illustrates the case with migration costs. Again start with $S_1 = 0$. With the figure as drawn (i.e. $\bar{x}_i^o - \bar{x}_j^o < m_{ji}$ for all $i \neq j$), we have $N_1^e = N_1^o$. By increasing S_1, x_1^e decreases and x_2^e increases until $S_{1,H}$ is reached where $x_2^e - m_{12} = x_1^e$ at N_1^o. For any larger transfer, migration from region 1 to 2 proceeds until $x_2^e - m_{12} = x_1^e$. A similar explanation applies to the part of the figure with $S_1 < 0$. In summary, the transfer $S_{1,L}$ is the lowest net transfer from region 1 to 2 which stops migration to region 1, and $S_{1,H}$ is the highest net transfer from region 1 to 2 which stops migration to region 2. Further, changing the net transfer under the

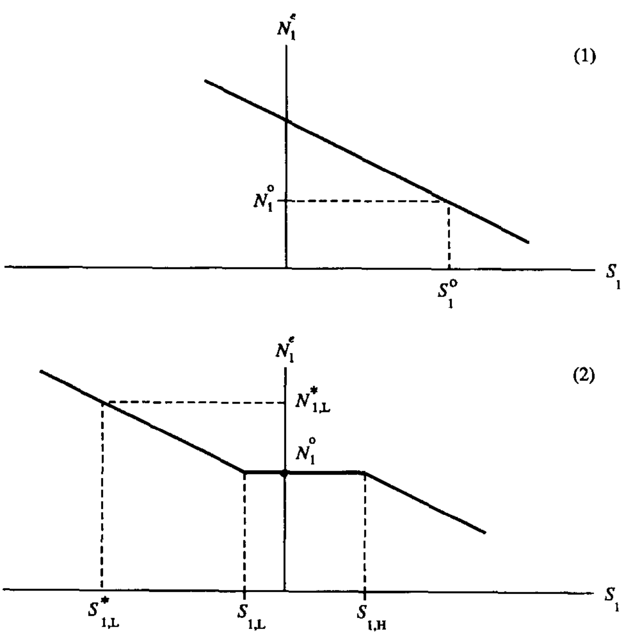

Fig. 2. Equilibrium partitions and net transfers.

[7]In the figure, S_1^o denotes the transfer necessary to support N_1^o as an equal utility partition.

downward-sloping portion of the graphs changes the partition and leaves the distribution unaffected, while changing the net transfer under the flat portion of the graphs leaves the partition unaffected and changes the distribution in favour of the country receiving the larger transfer.[8]

There are four implications which stem from our analysis about what are feasible allocations of population among regions and consumption among individuals given the instruments available to governments.

I. If $N_i^{\mathrm{o}} = N_i^{\mathrm{e}}$ then the relationship between x_i^{e} and x_j^{e} is restricted to $x_i^{\mathrm{e}} - x_j^{\mathrm{e}} \leq m_{ji}$ for all $i \neq j$ by (7).

II. If $N_i^{\mathrm{o}} < N_i^{\mathrm{e}}$ then $x_i^{\mathrm{e}} - m_{ji} = x_j^{\mathrm{e}}$.

With $N_i^{\mathrm{o}} < N_i^{\mathrm{e}}$ and $x_i^{\mathrm{e}} - m_{ji} < x_j^{\mathrm{e}}$, some individuals originating in region j have migrated to region i against their own interest. With $x_i^{\mathrm{e}} - m_{ji} > x_j^{\mathrm{e}}$, (7) is violated.

III. No matter where individuals finally reside, the N_i^{o} individuals who originated in region i each consume x_i^{e} for $i = 1, 2$.

This is obvious for all non-migrants. Migrants to region i consume $x_i^{\mathrm{e}} - m_{ji} = x_j^{\mathrm{e}}$ by implication II.

IV. Any feasible and stable population partition can be a migration equilibrium by the appropriate choice of net interregional transfer.

A stable population partition is one in which $x_i^{\mathrm{e}} - x_j^{\mathrm{e}}$ decreases as N_i^{e} increases. Taking the case of migration into region 1, using (5) and (6) on $x_1^{\mathrm{e}} - m_{21} = x_2^{\mathrm{e}}$ by implication II, we obtain upon total differentiation

$$\frac{\mathrm{d}N_1^{\mathrm{e}}}{\mathrm{d}S_1} = \left(\frac{1}{N_1^{\mathrm{e}}} + \frac{1}{N - N_1^{\mathrm{e}}}\right) \Big/ \left(\frac{\partial x_1^{\mathrm{e}}}{\partial N_1^{\mathrm{e}}} + \frac{\partial x_2^{\mathrm{e}}}{\partial N_2^{\mathrm{e}}}\right). \tag{8}$$

Since the stability of a population partition requires that the denominator be negative, the RHS is negative. This implies that the graph of N_1^{e} against S_1 is downward-sloping for $N_1^{\mathrm{e}} \neq N_1^{\mathrm{o}}$ as in Fig. 2. A feasible population partition is one with $-X_2 < S_1 < X_1$. This condition is necessary for the transfer which corresponds to a population partition being feasible, but it also ensures that consumptions are

[8] With $N_1^{\mathrm{o}} < N_{1,\mathrm{L}}^*$, $S_{1,\mathrm{L}}^*$ represents the transfer necessary to support $N_{1,\mathrm{L}}^*$. We will use this aspect of Fig. 2 in what follows.

positive. To see this, in the case of migration into region 1, use (5) and (6) on $x_1^e - m_{21} = x_2^e$ and observe that $-X_2 < S_1$ ensures positive consumptions. Thus Fig. 2 represents the graph of N_1^e against S_1 on the assumption that all feasible population partitions are stable, and then any feasible and stable partition can be achieved by the right choice of transfer during the first stage.

6. Federal efficiency

In Appendix B, in order to characterise federal efficiency, we use the conceptual device of a central planner who chooses all available policy instruments consistent with federal institutions. There, we constrain the planner's problem by the four implications derived above and consider the case $N_1^o < N_{1,L}^*$. In Fig. 3, the axes represent the utility of the N_i^o individuals originating in region i for $i = 1, 2$ (using implication III). We show that the utility–possibility frontier (henceforth UPF) is given by $[BD) \cup G$.

The intuition behind this finding follows. With the population partition at N_1^o, the pie consists of the total product X^o. If migration could be controlled then the utility possibilities for this case would be given by AF in Fig. 3. Once we restrict differences in consumption to be consistent with the migration equilibrium conditions (implication I), the utility possibilities are restricted to [BE].[9] When

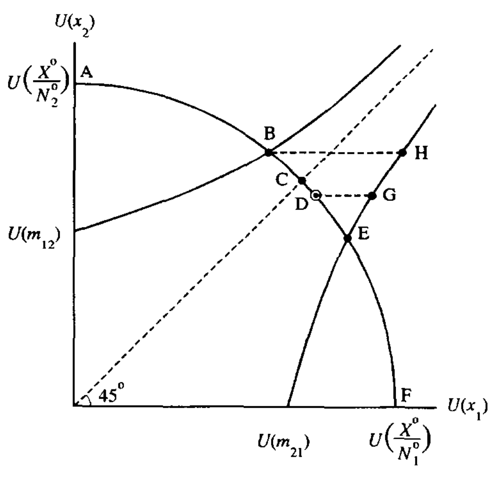

Fig. 3. A UPF of a Federal economy.

[9]The curves with intercepts at $U(m_{12})$ and $U(m_{21})$ respectively specify utility combinations which satisfy $x_2 - m_{12} = x_1$ and $x_1 - m_{21} = x_2$ respectively. These curves approach the 45° line as m_{ij} become small relative to x_i.

there is migration, implications II and III are constraints which provide a rule for dividing any given pie among all individuals. With $N_1^o < N_1^e$, for example, the central planner is restricted to distribute the pie along the curve which starts at $U(m_{21})$ in Fig. 3. Thus, with migration, the problem of the central planner is simply to maximise the pie. With $N_1^o < N_{1,L}^*$, the pie is maximised at $N_1^e = N_{1,L}^*$ which we represent by point G in Fig. 3. Clearly, the allocation represented by point E is not efficient and this is also true for the set of allocations represented by points on the curve [DE]. However, with an initial allocation along [BD), inducing migration into region 1 to increase the pie worsens the well-being of those originating in region 2 because exploiting the productivity benefit requires $x_1^e - x_2^e = m_{21}$ in order to induce individuals to move out of region 2. Thus [BD) (all N_1^o allocations) are federally efficient. Now consider different initial population partitions. For N_1^o close to $N_{1,L}^*$, G and D are close to E. For smaller N_1^o, the distance between E and G increases because the loss in the pie associated with staying at N_1^o is larger. Only when N_1^o is sufficiently small, so that G is north-east of point H, would all efficient allocations correspond to a first-best population partition $N_{1,L}^*$. In Appendix B we derive the lower and upper bounds for potentially efficient partitions. These lead to our first result:

Proposition 1. *A central planner in a federal economy may not maximise total product net of migration costs when either* $N_1^o < N_{1,L}^*$ *or* $N_1^o > N_{1,H}^*$. *In particular,* $N_1^{e*} = N_1^o < N_{1,L}^*$, *or* $N_{1,H}^* < N_1^o = N_1^{e*}$ *are possible efficient population partitions once federal institutions are imposed.*

This is in contrast with the first-best case of Section 3. There, maximising the pie was socially desirable in every case because the policy instruments available to the planner were sufficient to keep the issue of maximising the pie independent from the issue of dividing it among individuals.[10] However, in a federal economy where federal institutions restrict the policy instruments available to the planner, those two issues become interdependent. For example, a central planner maximising a Rawlsian social welfare function under the conditions described by Fig. 3 would find moving an individual from region 2 to region 1 socially undesirable. On the other hand, for welfare functions with less aversion to inequality, such a move could be desirable. This is in contrast to the case of first-best efficiency where each N_1^o implies a unique N_1^*.

The approach we used to characterise the UPF is worth discussing because it has implications for a broader literature. With our instrument restrictions and migration costs, it is clear from implications I and II that the existence of migration and the direction of that migration, if it exists, impose different

[10]Because the distribution of consumption among individuals is unrestricted in the first-best case, graphing the first-best UPF requires one dimension for each individual. If we restrict the first best distributions to equal consumption for every agent originating in a particular region, however, we can graph this slice of the first best UPF in Fig. three. It has intercepts $U(X^*/N_1^o)$ and $U(X^*/N_2^o)$, passes through point G and lies everywhere outside of AF.

constraints on the choices of a central planner. Therefore the UPF cannot necessarily be characterised by a single constrained optimisation problem. The approach we use in Appendix B is to solve three different constrained optimisation problems—corresponding to no migration, migration into region 1 and migration into region 2. We then compare the three different conditional solutions to determine the federally efficient solution.[11] Within such an environment discontinuities may arise. This does not happen only in the case of a federal economy under interregional migration costs but, more generally, in cases where constraints equivalent to those generated by federal institutions apply and where population movement between categories is costly. For example, both Steiger and Tabellini (1987) and Boadway and Wildasin (1990) involve costly intersectoral labour shifts, the former between the import and export sectors of a small open economy, the latter between two industrial sectors. Since in some part of both papers (a) intrasectoral tax discrimination on identical individuals is not allowed, and (b) intersectoral labour migration is costly and cannot be restricted, both economies require a two-stage approach in characterising their discontinuous UPF.

Finally it is useful to map allocations represented by points in Fig. 3 into the corresponding transfers of Fig. 2(2). A first-stage $S_1 < S^*_{1,L}$ involves a second-stage allocation with $N^e_1 > N^*_{1,L}$ and $x^e_1 - m_{21} = x^e_2$, and thus an allocation on the curve starting at $U(m_{21})$ but south-west of G. As we increase S_1, we move north-east along the curve because the pie is increasing until $S_1 = S^*_{1,L}$ (point G). Increasing S_1 further, we move south-west from G until $S_1 = S_{1,L}$ (point E) where $N^e_1 = N^o_1$ and the distribution is $x^e_1 - m_{21} = x^e_2$. Increasing S_1 still further, we move north-west along BE until $S_1 = S_{1,H}$ (point B) where $N^e_1 = N^o_1$ and the distribution is $x^e_2 - m_{12} = x^e_1$. For $S_1 > S_{1,H}$, there is migration from region 1 to 2. As we increase S_1, we move south-west along the curve starting at $U(m_{12})$ as the pie is decreasing ($N^e_1 < N^o_1 < N^*_{1L}$). If we define the transfer which corresponds to point D as $S_{1,D}$, we have $S_{1,L} > S_{1,D} > S_{1,H}$. Notice that most efficient allocations require a non-zero transfer. If, for example, $S_{1,D} > 0$, a non-zero transfer is necessary for efficiency. However, also notice that with $S_{1,D} < 0$ a zero transfer is efficient.

7. Subgame-perfect equilibria

At the first stage, regional governments simultaneously choose an interregional transfer $S_{ij} \geq 0$ in order to maximise the utility $U(x_i^{\,e})$ of their original representative resident. These choices take fully into account their implications for behaviour at the second stage, in particular, Section 5.

Since federal efficiency may require interregional transfers, the question is whether or not the two regional governments in equilibrium will make the

[11] In explaining Fig. 3, we first characterised efficiency conditional on no migration, then efficiency conditional on migration into region 1, and finally compared the two solutions.

transfers necessary to support federal efficiency. In particular, the existence of externalities (e.g. the fiscal externality) may be expected to cause inefficiency. However, in Appendix C, we prove the following result:

Proposition 2. *All subgame-perfect equilibrium allocations are federally efficient allocations.*

Furthermore those equilibria need not be first-best efficient. For example, all subgame-perfect equilibria allocations of case I in Appendix C are first-best efficient, but the unique subgame-perfect equilibrium allocation of case II.3 is not because the equilibrium partition is $N_1^o < N_1^*$. The same conclusion, that equilibria are not necessarily first-best, has been reached by Hercowitz and Pines (1991). Nevertheless, we point out that a failure to achieve first-best efficiency here cannot be traced to a fiscal externality since all equilibria are efficient. That is, a federal government which internalizes externalities by construction and operating under federal institutions could not do better from an efficiency perspective. This is consistent with the conclusions of Mansoorian and Myers (1993).

Finally, in the appendix, we show that positive migration costs may lead to discontinuities in the governments' reaction functions. We have already pointed out in Section 1 that this is consistent with results emphasised by Mintz and Tulkens (1986). Unlike their model, however, we show that the discontinuities here do not create problems of existence. There is, in fact, a unique allocation corresponding to all subgame-perfect equilibria for each set of parameters. As with the UPF, different migration regimes impose different constraints on the problems of regional governments. Consequently, a government's behaviour cannot be characterised by the first-order conditions of a single constrained optimisation problem. The associated discontinuities in reaction functions arise because a marginal change in one region's policy can lead the other to desire a complete switch in migration regimes, an associated switch in constraints, and thereby a large change in the policy which is its optimal response.[12] Consider the case represented by Fig. 2(2) and Fig. 3 where we assume $S_{1,D} > 0$. In Appendix C (case II.2), we prove that the reaction function for region 2 is given by $S_{21} = S_{12} - S_{1,L}^* \geq 0$ for $0 \leq S_{12} \leq S_{1,D}$, yielding a net transfer which, from Fig. 2(2), leads to the migration of $(N_{1,L}^* - N_1^o)$ individuals to region 1 and to an allocation given by point G in Fig. 3. This allocation is preferred by region 2 because, with migration, implication II holds while, without migration, the best transfer conditional on the no migration case, $S_{21} = 0$, would lead to a net transfer $S_1 = S_{12} \leq S_{1,D}$ or the allocation corresponding to a point on [DE] in Fig. 3. However, as soon as we marginally increase S_{12} from $S_{12} = S_{1,D}$ region 2's best response jumps to $S_{21} = 0$, yielding a net transfer which involves a change in migration regime to no

[12] Such discontinuities did not exist in Mansoorian and Myers, 1993. In Mansoorian and Myers migration costs range from 0 to 1 and there is one individual associated with each level of cost. As a result there would be no horizontal segment in the graph of population against the transfer instrument (see Fig. 2) and there would be the same equal utility constraint for the marginal agent under either migration regime. Thereby there is no change in constraints across migration regimes.

migration and leads to an allocation just above point D on [BD), an allocation which region 2 prefers over that at G. The reaction functions for this case are graphed in Fig. 6(1.2) of Appendix C[13] To the best of our knowledge the potential for these discontinuities and the associated complications for characterising efficiency and equilibria have not been recognised in the literature on costly population migration.

8. Concluding remarks

Imagine that migration costs get smaller. In Fig. 3, this would imply that as the two curves which begin at $U(m_{12})$ and $U(m_{21})$ approach the 45° line, the UPF narrows down. At the limit, where migration costs become zero, the two curves collapse upon the 45° line. The comparison of the migration and no migration cases becomes trivial: since [B, E] has diminished to the single point C, and since G is to the northeast of C, the UPF is represented by a single point G on the 45° line of Fig. 3. That is, costless migration of individuals in the system forces interregional equality at the first-best efficient allocation. In a modified Fig. 1, where lines A and B are coincident, allocation G corresponds to the first-best partition $N^*_{1,L}=N^*_{1,H}\equiv N^*_1$ at the intersection of lines A_1 and A_2. There, marginal products of labour are equalised. Since both regional governments prefer the unique efficient allocation over all feasible allocations, both desire the same first-best efficient net transfer S^*_1. Consequently, no federal government is required to enforce the efficient net transfer (Myers, 1990). Moreover, since interregional utility inequalities cannot be sustained in a federal economy when migration is costless, no federal government is required to resolve interregional distribution problems. We conclude that, under costless migration, there is no need for a federal government in our model.

Imagine next that migration costs get larger. When they become large enough, the two allocations given by $U(m_{12})$ and $U(m_{21})$ in Fig. 3 are outside A and F respectively, and the UPF is then represented by the entire [A, F]. Since the initial partition determines a pie which cannot be further enlarged because prohibitively high migration costs preclude any movement from the initial partition, no government is required to ensure efficiency. With respect to distribution, note that since any split on the fixed [A, F] is first-best efficient, any feasible interregional

[13]We can offer some intuition as to why the discontinuities do not lead to existence problems in our model. First ignore non-negativity in focusing on the desired net transfers by the regions. Unsurprisingly region 1 (2) never wants a larger outgoing net transfer, S_1 ($-S_1$) then region 2 (1) wants to receive. Each either wants the same net transfer (reaction functions are coincident) or a smaller outgoing net transfer (e.g. the reaction function for region 2 is to the left of the reaction function for region 1 in (S_{21}, S_{12}) space). Given this and the fact that discontinuity is a jump in the desired outgoing net-transfer to a smaller level (left) the discontinuity never passes through the other region's reaction function which could lead to existence problems.

transfer is also first-best efficient. Such transfers will only be made by a federal government because making a transfer is not in the interest of a regional government in the absence of migration. We conclude that, when migration costs are high enough to trap people geographically, a federal government is needed in order to resolve interregional distribution problems.

Let us now return to the intermediate situation described in our paper. As in the two extreme cases above, a federal government is not required to ensure efficiency because all subgame-perfect equilibria allocations are federally efficient. Moreover, as in the case of prohibitively high migration costs, interregional distribution may still require the existence of a federal government. Finally, the discontinuities disappear both when mobility is free and when migration costs become large enough. In the first case, there are no discontinuities because the constraints across migration regimes are identical (see implications I and II at $m_{ij}=0$) and in the second case because there is only one migration regime.

We close our paper by considering extensions and limitations of the model. The model could be extended to allow for public goods and population heterogeneity. Because the potential for discontinuities will still exist these extensions would require, as in this paper, solving for both the UPF and reaction functions in two steps; solve for the solution conditional on the migration regime and then compare across the regimes in determining the unconditional solutions.[14] In considering the robustness of the efficiency result for all degrees of migration in this paper, we would note this is undoubtedly not robust. When there is costless migration of a homogeneous population an environment with externalities between governments does not lead to inefficiency because the incentive equivalence induced by migration internalizes externalities. But once the population is heterogeneous or migration becomes costly the incentive equivalence is broken and externalities (strategic interaction) among regional governments may lead to inefficiency. So any environment which involves externalities among governments (e.g. spillouts of public goods, capital tax competition, transboundary pollution etc.) may yield inefficient equilibria.[15]

Also notice that our results were predicated on the assumption that governments could commit to a policy. It is possible to show that, because of costly migration, regional governments may have the incentive to renege on announced transfers if such an action is feasible. In particular, consider case I of Appendix C under Fig. 2(2), where region 2 announces $S_{21}{}^{n}=-S^{*}_{1,L}>0$. In response, $N^{*}_{1,L}-N^{\circ}_{1}$ migrants move from region 2 to region 1. Once migration is complete, will the announced transfer represent the best policy of region 2? Observe that, once migration is

[14]For a model with population heterogeneity and local public goods but with $m_{ij}=0$ see Burbidge and Myers, 1994.

[15]We would also speculate that adding regions may lead to sufficient complications in the nature of the strategic interaction among regional governments that equilibria may be inefficient. This is the subject of current research.

complete, the government of region 2 faces a new economy determined by the new partition $N^*_{1,L}$. If m_{12} is high enough to trap the residents of region 1, region 2 can renege on the transfer without causing further migration. Since part of the transfer goes to the original residents of region 1, reneging is preferred if the government of region 2 aims, for example, to maximise the total utility of its original residents provided that utility is not too concave in the good. These incentives are a problem if commitment is not possible in our model because individuals will not migrate in response to a non-credible policy announcement. Under these circumstances, a federal government is needed in order to guarantee regional government announcements to transfer (tax) policies. The need for such guarantees disappears both when mobility is perfect and when migration costs become large enough. In the first case, regional governments will never renege because no individual can be trapped. In the second case, transfers become irrelevant for efficiency because no individual can move.[16]

This paper has been built on the fact that federal institutions preclude immigration controls such as immigration tolls or quota and forced emigration. Some federations do not, however, preclude emigration (exit) subsidies. Canadian provinces (e.g. Alberta and Ontario) pay local welfare recipients their migration costs for moves to other provinces. Our model has not allowed for this instrument. Modelling this possibility complicates the analysis (e.g. one must allow for the possibility of cross-hauling of population—individuals migrating in both directions) and would lead to different results both in terms of efficiency and equilibrium. But it also would allow for many interesting possibilities. Cross-hauling may in fact be going on in Canada with its obvious associated inefficiencies. Further there is the potential for a reintroduction of incentive equivalence if the migrant sending country can commit to covering the migration costs of its residents leading to the possible promotion of efficiency and horizontal equity. Finally there is the impact of the federal government paying migration costs on the decentralized equilibria (in some countries the federal government subsidizes moving costs through the income tax system). Building a model with emigration subsidies is left for future research.

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[16]This issue also does not arise in Mansoorian and Myers, 1993 as individuals can not be trapped because migration costs are not pecuniary.

in Marseille, France, and at the annual meeting of the North American Regional Science Association in Niagara Falls, Canada.

Appendix A

Interregional transfers

Since what is feasible with the specified instruments depends on the existence and the direction of population movement, we proceed to discuss three separate cases (A, B, C) specified according to whether the initial partition is smaller than, equal to, or larger than the corresponding equilibrium partition. For each case we determine the conditional net transfer, denoted as $S_{1|k}$ for $k=\mathrm{A}$, B, C which is necessary to support a second-stage equilibrium population partition. We also determine the associated feasible consumption levels for each region. Toward this end, it is useful to define

$$S_{1,\mathrm{L}} \equiv \frac{N_1^{\mathrm{o}}(N - N_1^{\mathrm{o}})}{N}(\bar{x}_1^{\mathrm{o}} - \bar{x}_2^{\mathrm{o}} - m_{21}) \tag{9}$$

$$S_{1,\mathrm{H}} \equiv \frac{N_1^{\mathrm{o}}(N - N_1^{\mathrm{o}})}{N}(\bar{x}_1^{\mathrm{o}} - \bar{x}_2^{\mathrm{o}} + m_{12}). \tag{10}$$

By $m_{ij} \geq 0$, $S_{1,\mathrm{L}} \leq S_{1,\mathrm{H}}$. When $m_{ij}=0$, $S_{1,\mathrm{L}}=S_{1,\mathrm{H}}=S_1^{\mathrm{o}}$. See Fig. 2.

Case A^{e} ($N_1^{\mathrm{o}}<N_1^{\mathrm{e}}$): This case requires that individuals from region 2 migrate to region 1. Therefore, after-tax income in region 1 net of migration costs at the initial partition, $x_{1|\mathrm{A}}^{\mathrm{o}}-m_{21}$, is larger than the corresponding after-tax income in region 2, $x_{2|\mathrm{A}}^{\mathrm{o}}$. Using (5) and (6), we have

$$S_{1|\mathrm{A}} < S_{1,\mathrm{L}}. \tag{11}$$

Given $S_{1|\mathrm{A}}$, individuals will migrate from region 2 to region 1 for as long as $x_{1|\mathrm{A}}-m_{21}>x_{2|\mathrm{A}}$. Assuming migration stability $x_{1|\mathrm{A}}-m_{21}-x_{2|\mathrm{A}}$ decreases as N_1 increases. Therefore, in equilibrium,

$$x_{1|\mathrm{A}}^{\mathrm{e}} - x_{2|\mathrm{A}}^{\mathrm{e}} = m_{21}. \tag{12}$$

Using (5) and (6) on (12) yields $N_{1|\mathrm{A}}^{\mathrm{e}}$ as an implicit function of $S_{1|\mathrm{A}}$:

$$S_{1|\mathrm{A}} = \frac{N_{1|\mathrm{A}}^{\mathrm{e}}(N - N_{1|\mathrm{A}}^{\mathrm{e}})}{N}(\bar{x}_{1|\mathrm{A}}^{\mathrm{e}} - \bar{x}_{2|\mathrm{A}}^{\mathrm{e}} - m_{21}). \tag{13}$$

Given $S_{1|\mathrm{A}}$ and $N_{1|\mathrm{A}}^{\mathrm{e}}$ consumptions are then determined by (5) and (6).

For our work on equilibrium in Appendix C, it is useful to specify the migration equilibrium partition when there is no transfer. For $S_{1|\mathrm{A}}=0$, (13) requires

$$\bar{x}^{e}_{1|A} - \bar{x}^{e}_{2|A} = m_{21} \tag{14}$$

which, in turn, implies a unique partition $N^{e}_{1,L}$. When there are no transfers, individuals receive an after-tax income equal to their average product of labour by (5), so that the individual benefit of moving from region 2 to region 1 is $(\bar{x}_1 - m_{21}) - \bar{x}_2$. Migrants will be attracted to region 1 for as long as this benefit remains positive.

Case B^{e} ($N^{o}_{1} = N^{e}_{1}$): Since no migration occurs in this case, conditions (7) apply. In consequence,

$$-m_{12} \leq x^{e}_{1|B} - x^{e}_{2|B} \leq m_{21}. \tag{15}$$

Then, using (5) and (6) on (15), we have

$$S_{1,L} \leq S_{1|B} \leq S_{1,H}. \tag{16}$$

Given $S_{1|B}$ and N^{o}_{1} consumptions are determined by (5) and (6).[17]

Case C^{e} ($N^{o}_{1} > N^{e}_{1}$): We apply an argument exactly analogous to that of case A^{e} to obtain

$$S_{1|C} > S_{1,H} \tag{17}$$

$$x^{e}_{2|C} - x^{e}_{1|C} = m_{12}.$$

Using (5) and (6) on (18) yields $N^{e}_{1|C}$ as an implicit function of $S_{1|C}$:

$$S_{1|C} = \frac{N^{e}_{1|C}(N - N^{e}_{1|C})}{N}(\bar{x}^{e}_{1|C} - \bar{x}^{e}_{2|C} + m_{12}). \tag{19}$$

Given $S_{1|C}$ and $N^{e}_{1|C}$ consumptions are determined by (5) and (6). For $S_{1|C} = 0$, (19) requires

$$\bar{x}^{e}_{2|C} - \bar{x}^{e}_{1|C} = m_{12} \tag{20}$$

which, in turn, implies a unique equilibrium partition $N^{e}_{1,H}$ when there are no transfers. To compare $N^{e}_{1,L}$ with $N^{e}_{1,H}$, recall that average products of labour decrease with increasing regional population. Therefore, since $\bar{x}^{e}_{1} > \bar{x}^{e}_{2}$ at $N^{e}_{1,L}$ by (14) and $\bar{x}^{e}_{1} < \bar{x}^{e}_{2}$ at $N^{e}_{1,H}$ by (20), we have $N^{e}_{1,L} < N^{e}_{1,H}$.

Fig. 2(2) was drawn such that $S_{1,L} < 0 < S_{1,H}$, therefore there are two other variants of Fig. 2 which may be appropriate: $S_{1,L} < S_{1,H} < 0$ in which case a zero transfer corresponds to $N^{e}_{1,H}$ (see Fig. 4(1)) and $0 < S_{1,L} < S_{1,H}$, in which case a zero transfer corresponds to a partition of $N^{e}_{1,L}$ (see Fig. 4(2)). The graphs of Fig. 2 and Fig. 4 are composites of the three cases, A, B, C. For $-X_2 < S_1 < S_{1,L}$ case A^{e}

[17] A perfectly immobile population corresponds to the special case in which $S_{1,L} - X_2$ and $S_{1,H} \geq X_1$. These conditions must hold if migration costs are large enough. At the other extreme, where $m_{ij} = 0$, case B^{e} requires the unique transfer $S_{1|B} = S_{1,L} = S_{1,H} = S^{o}_{1}$.

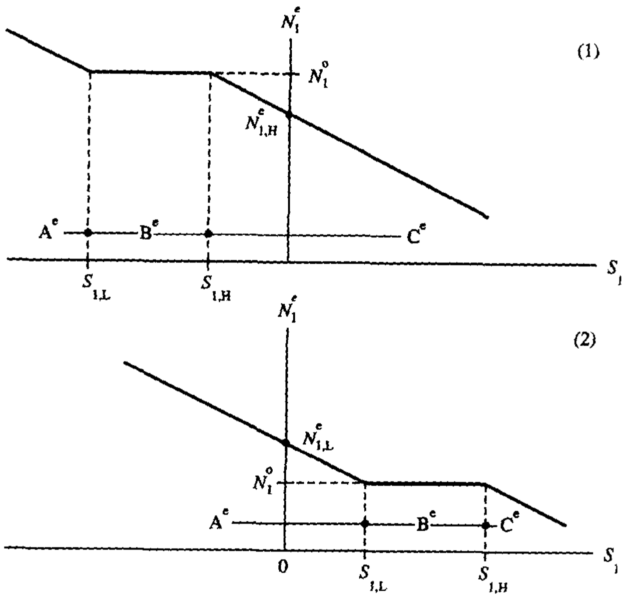

Fig. 4. Equilibrium partitions and net transfers (cont.).

applies, for $S_{1,L} \leq S_1 \leq S_{1,H}$ case B^e applies, and for $S_{1,H} < S_1 < X_1$ case C^e applies. Implication I follows from (15), implication II from (12) and (18), implication III from implication II and implication IV was explained in the text.

Appendix B

Federal Efficiency

We characterise federal efficiency by choosing a feasible allocation of the good to individuals and a net transfer so that it is not possible to increase the utility of one individual without decreasing the utility of at least one other individual. It is convenient to solve this problem by using an equivalent problem in which the choice of a net transfer is replaced by the choice of a partition. This is possible by implication IV. The choice of an efficient net transfer S_1^{e*} is then made implicit in the choice of a corresponding partition N_1^{e*} (see the last paragraph in Section 6). Toward this end, we eliminate transfers by combining (4) and derive national feasibility or

$$N_1^e x_1^e + (N - N_1^e) x_2^e \underset{(4)}{\leq} X^e \tag{21}$$

By implications I–IV, it is necessary to follow the three cases of Appendix A in characterising federal efficiency.

Case A^{e*} ($N_1^o < N_1^{e*}$): The problem is to choose N_1^e, x_1^e, and x_2^e subject to (21) and the additional constraint imposed by implication II,

$$x_1^e - x_2^e \underset{(12)}{=} m_{21}. \tag{22}$$

Using (22) solved for x_2^e in (21), and solving for x_1^e, makes it clear that *the only issue in case* A^{e*} *is to choose the* N_1^e *that maximises aggregate consumption*, $X^e - (N_1^e - N_1^o)\, m_{21}$. In consequence,

$$y_{1|A}^{e*} - y_{2|A}^{e*} = m_{21} \tag{23}$$

and (21) binds. Comparing this result with results in Section 3, we conclude that the conditionally efficient partition in a federal economy for this case coincides with the first-best efficient partition, $N_{1|A}^{e*} = N_{1,L}^{*}$. Finally, we can solve (21) and (22) at the efficient allocation to obtain

$$x_{1|A}^{e*} = \frac{1}{N}(X_L^* + (N - N_{1,L}^*) m_{21}) \qquad x_{2|A}^{e*} = \frac{1}{N}(X_L^* - N_{1,L}^*) m_{21} \tag{24}$$

where X_L^* is the economy's total output at $N_{1,L}^*$.

Case B^{e*} ($N_1^o = N_{1|B}^{e*}$: Here, by definition, the partition is not a choice variable. Thus the problem is to choose x_1^e and x_2^e subject to,

$$N_1^o x_1^e + (N - N_1^o) x_2^e \underset{(21)}{\leq} X^o \qquad x_1^e - x_2^e \underset{(15)}{\leq} m_{21} \qquad x_2^e - x_1^e \underset{(15)}{\leq} m_{12}. \tag{25}$$

Since the pie is fixed, *the only issue in case* B^{e*} *is distribution*. Characterising efficiency here is particularly simple—the only efficiency condition is not to waste or, equivalently, the first constraint in (25) binds. The second and third constraints, when binding, yield positively-sloped graphs in utility space. Thus, given diminishing marginal utility and these constraints, the feasible utility combinations will be a convex set with the efficient allocations on the outer envelope as shown in Fig. 3. A utility combination can then be chosen to maximise a social welfare function. By defining a potentially efficient difference in consumptions as

$$\Delta x_{1|B}^{e*} \equiv x_{2|B}^{e*} - x_{1|B}^{e*}, \tag{26}$$

we can later specify the full set of potentially efficient regional consumptions. Using (26) and the first constraint in (25), we obtain

$$x_{1|B}^{e*} = \frac{1}{N}(X^o - (N - N_1^o)\Delta x_{1|B}^{e*}), \quad x_{2|B}^{e*} = \frac{1}{N}(X^o + N_1^o \Delta x_{1|B}^{e*}). \tag{27}$$

Case C^{e*} ($N_1^o > N_1^{e*}$): By using an argument exactly analogous to that of case A^{e*}, we obtain

$$y_{2|A}^{e*} - y_{1|A}^{e*} = m_{12} \tag{28}$$

and (21) binding which, upon comparison with results in Section 3, implies once again that the conditionally efficient partition in a federal economy for this case coincides with the corresponding efficient partition of the first-best case, $N_{1|C}^{e*} = N_{1,H}^*$. As before we solve the constraints at the efficient allocation to obtain

$$x_{1|C}^{e*} = \frac{1}{N}(X_H^* - (N - N_{1,H}^*)m_{12}), \quad x_{2|C}^{e*} = \frac{1}{N}(X_H^* + N_{1,H}^* m_{12}). \tag{29}$$

We must now compare the conditionally efficient allocations of the three cases to determine the federally efficient allocations. Since case A^{e*} corresponds to $N_1^o < N_{1,L}^*$ and case C^{e*} to $N_{1,H}^* < N_1^o$, they solve the problem for distinct initial partitions. In contrast, initial partitions may not be distinct between case B^{e*} and one of cases A^{e*} or C^{e*}. For cases A^{e*} and B^{e*} initial partitions are not distinct when $N_1^o < N_{1,L}^*$ while, for cases B^{e*} and C^{e*}, the same is true when $N_{1,H}^* < N_1^o$. For $N_1^o < N_{1,L}^*$, the relevant comparison between cases A^{e*} and B^{e*} is made in Fig. 3 and explained in the main text.

We now provide the lower and upper bounds for federally efficient partitions. The lower bound for potentially efficient partitions corresponds to the initial partition which yields the allocation given by point H in Fig. 3. We denote this bound by $N_{1,L}^{e*}$ and we determine it by observing that, at H, $x_{2|A}^{e*} = x_{2|B}^{e*}$ for $\Delta x_{1|B}^{e*} = m_{12}$. Using (24) and (27), $N_{1,L}^{e*}$ must satisfy

$$X_L^* - X_L^{e*} = N_{1,L}^* m_{21} + N_{1,L}^{e*} m_{12} \tag{30}$$

where X_L^{e*} corresponds to output at $N_{1,L}^{e*}$. Notice that because the RHS must be positive $N_{1,L}^{e*} < N_{1,L}^*$ so that the LHS is positive

If we compare cases B^{e*} and C^{e*}, we arrive at an upper bound $N_{1,H}^{e*}$ that satisfies

$$X_H^* - X_H^{e*} = (N - N_{1,H}^*)m_{12} + (N - N_{1,H}^{e*})m_{21} \tag{31}$$

By a similar argument as above $N_{1,H}^{e*} > N_{1,H}^*$. Consequently, the full range of potentially efficient partitions is given by $[N_{1,L}^{e*}, N_{1,H}^{e*}]$. Proposition 1 follows directly.[18]

[18] In the longer version of this paper Myers and Papageorgiou (1994a) (Fig. 4) we show how to derive $N_{1,L}^{e*}$ and $N_{1,H}^{e*}$ graphically.

Appendix C

Subgame-Perfect Equilibrium

By (5) and (6), we can write x_i as a function of N_1 and S_1. Furthermore, by (13), (16) and (19), we can write N_1 as a function of S_1. Taking once again (6) into account, we conclude that utility levels can be expressed as $U_i(S_{12}-S_{21})$. We define a Nash equilibrium at the first stage as a pair $(S_{12}{}^n, S_{21}{}^n)$ such that $S^n_{12}=\arg\max U_1(S_{12}-S_{21}{}^n)$ and $S_{21}{}^n=\arg\max U_2(S_{12}{}^n-S_{21})$ for $S_{12}{}^n\geq 0$ and $S_{21}{}^n\geq 0$. A subgame-perfect equilibrium is therefore characterised by a $(S_{12}{}^n\ S_{21}{}^n)$ and the second-stage location equilibrium determined by Appendix A under $S_1{}^n=S_{12}{}^n-S_{21}{}^n$. In what follows, we examine the efficiency characteristics of the subgame-perfect equilibria with the help of Figs. 2–4. Since $0<N^{e*}_{1,L}<N^*_{1,L}<N^*_{1,H}<N^{e*}_{1,H}<N$, we distinguish five possible cases according to the location of N^o_1 in $[0, N]$. We label the division of the pie $(x^e_i-x^e_j)$ as the 'split'.

Case I ($N^o_1\leq N^{e*}_{1,L}$): Determining the efficient partition for a federal economy here involves comparing the conditionally efficient solutions of cases A^{e*} and B^{e*} because $N^o_1<N^*_{1,L}$. Therefore Fig. 3 applies. Since, in addition, $N^o_1<N^{e*}_{1,L}$, point G in that figure must be found to the northeast of H. It follows that the UPF is represented by the unique efficient allocation of case A^{e*}, and that the efficient partition is $N^{e*}_{1|A}=N^*_{1,L}$. Also, since by (24) $\Delta x^{e*}_1=\Delta x^{e*}_{1|A}=-m_{21}$, the efficient split is most favourable for those in region 1. We now turn to the efficient net transfer $S^*_{1,L}$ necessary to support $N^{e*}_{1,L}$. According to Figs. 2 and 4, this can be either positive or negative. A positive efficient net transfer corresponds to Fig. 4(2) and requires $N^*_{1,L}<N^e_{1,L}$. A negative one corresponds to Fig. 2(2) or 4(1). In any case, since both regional governments prefer the unique efficient allocation over all feasible allocations, both desire the same net transfer $S^*_{1,L}$. Taking into account (6), the best-response function [RF] of region 1 to any S_{21} is given by $S_{12}=S^*_{1,L}+S_{21}$ and $S_{12}\geq 0$, and the RF of region 2 by $S_{21}=S_{12}-S^*_{1,L}$ and $S_{21}\geq 0$. These are graphed in Fig. 5 for $S^*_{1,L}>0$, in which the solid line represents the RF of region 1

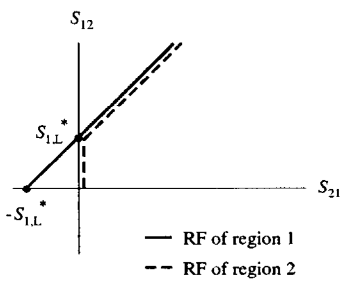

Fig. 5. Best-Response functions in Case I.

and the dashed line the RF of region 2. We conclude that there is a continuum of subgame-perfect equilibria in this case, all satisfying $S_{12}{}^{n}-S_{21}{}^{n}=S^{*}_{1,L}$ for $S_{12}{}^{n}\geq 0$ and $S_{21}{}^{n}\geq 0$. Each equilibria corresponds to the same efficient allocation represented by point G, which in this case is, northeast of H in Fig. 3.

Case II ($N^{e*}_{1,L}<N^{o}_{1}<N^{*}_{1,L}$): In this case Fig. 3 still applies because $N^{o}_{1}<N^{*}_{1,L}$. Point G, however, is now to the southwest of H, so that federal efficiency requires an allocation that belongs to [B, D)$\cup$G exactly as in that figure. In particular, we need either the allocation with $N^{*}_{1,L}$ and the split given by G, or an allocation with N^{o}_{1} and splits given by [B, D). Whether or not a transfer is necessary to support N^{o}_{1} depends on the relationship among N^{o}_{1}, $N^{e}_{1,L}$ and $N^{e}_{1,H}$ given by the three graphs, Fig. 2(2), Fig. 4(1) and Fig. 4(2). We consider each graph separately and we show that subgame-perfect equilibria are federally efficient. What complicates matters, as compared to case I, is that regions can prefer different transfers and that there exist inefficient feasible allocations which could be preferred over a particular efficient allocation by a region. In every subcase, the RF of region 1 to any S_{21} is given by $S_{12}=S^{*}_{1,L}+S_{21}$ and $S_{12}\geq 0$ because it maximises the pie and yields the most favourable split for that region. We therefore concentrate on the RF of region 2.

Case II.1 ($N^{o}_{1}<N^{e}_{1,L}$ as in Fig. 4(2)): Firstly, suppose that $N^{o}_{1}<N^{*}_{1,L}\leq N^{e}_{1,L}$, which implies $S^{*}_{1,L}\geq 0$. In order to determine the RF of region 2, we use $S_{1,D}$ as defined at the end of Section 6. Using Fig. 4(2), we have $0<S_{1,L}<S_{1,D}<S_{1,H}$. As S_{12} increases with S_{21} held at zero, the allocation approaches D. For as long as $S_{12}\leq S_{1,D}$, the government of region 2 desires the allocation given by G which dominates all allocations in [D, E]. However, when $S_{12}>S_{1,D}$, the allocation moves to the northwest of D and it cannot be further improved for those in region 2 by the government of that region. In consequence, the RF of region 2 is given by $S_{21}=S_{12}-S^{*}_{1,L}$ and $S_{21}\geq 0$ for $S_{12}\leq S_{1,D}$, and by $S_{21}=0$ for $S_{12}>S_{1,D}$ (see Fig. 6(1.1)). Secondly, suppose that $N^{o}_{1}<N^{e}_{1,L}<N^{*}_{1,L}$. The situation is exactly as before except that, now, $S^{*}_{1,L}<0$ (see Fig. 6(1.2)). In either case, there is a continuum of subgame-perfect equilibria satisfying $S_{12}{}^{n}-S_{21}{}^{n}=S^{*}_{1,L}$ for $S_{12}{}^{n}\geq 0$ and $S_{21}{}^{n}\geq 0$. Each one corresponds to the same efficient allocation specified by a point G in (E,H) of Fig. 3.

Case II.2 ($N^{e}_{1,L}\leq N^{o}_{1}\leq N^{e}_{1,H}$ as in Fig. 2(2)): Following the arguments of case II.1, the RF of region 2 is given by $S_{21}=S_{12}-S^{*}_{1,L}$ and $S_{21}\geq 0$ for $S_{12}\leq S_{1,D}$, and by $S_{21}=0$ for $S_{12}>S_{1,D}$. Since $N^{o}_{1}<N^{*}_{1}$, Fig. 2(2) implies that $S^{*}_{1,L}<0$. Consequently, for $S_{1,D}>0$, Fig. 6(1.2) applies. However, unlike case II.1, the sign of $S_{1,D}$ is now ambiguous since Fig. 2(2) only implies $S_{1,L}<S_{1,D}<S_{1,H}$. Thus we must also consider $S_{1,D}=0$ and $S_{1,D}<0$, which correspond to Fig. 6(2.1) and (2.2) respectively. When $S_{1,D}>0$, there is a continuum of subgame-perfect equilibria satisfying $S_{12}{}^{n}-S_{21}{}^{n}=S^{*}_{1,L}$ for $S_{12}{}^{n}\geq 0$ and $S_{21}{}^{n}\geq 0$, each one corresponding to the efficient split G in (E, H) of Fig. 3. When $S_{1,D}=0$, we have a unique subgame-perfect equilibrium $S_{12}{}^{n}=0$ and $S_{21}{}^{n}=-S^{*}_{1,L}$ which again corresponds to the efficient G. Finally, when $S_{1,D}<0$, the original split must be found to the

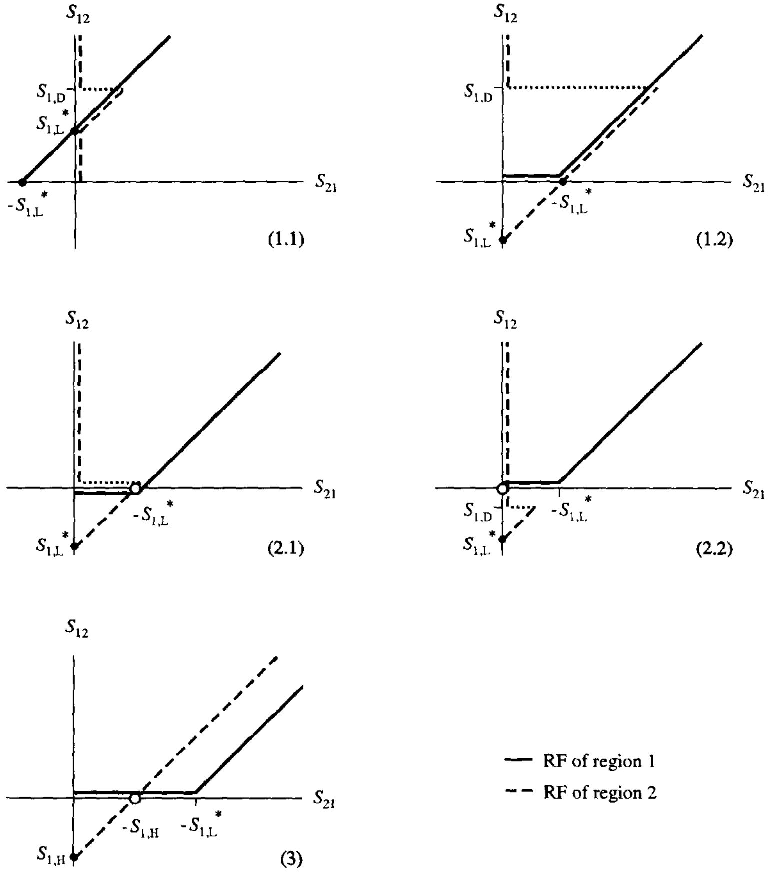

Fig. 6. Best-Response Functions in Case II.

northwest of D in Fig. 3. Under those circumstances, the original split cannot be further improved for those in region 2 by the government of that region, and a subgame-perfect equilibrium calls for zero transfers $S_{12}{}^{n} = S_{21}{}^{n} = 0$ and an efficient split in [B, D).

Case II.3 ($N^{e}_{1,H} < N^{o}_{1}$ as in Fig. 4(1)): The most favourable allocation for region 2 is represented by point B in Fig. 3, where $\Delta x^{e*}_{1|B} = m_{12}$. We know from (10) and (19) that B can be achieved by $S_{1,H}$ which also supports the initial partition. Hence the RF of region 2 to any S_{12} is given by $S_{21} = S_{12} - S_{1,H}$ and $S_{21} \geq 0$. This is shown in Fig. 6(3), in which we also take into account that $S_{1,H} > S_{1,L} > S^{*}_{1}$ since Fig. 4(1) applies and $N^{o}_{1} < N^{*}_{1,L}$. The unique subgame-perfect equilibrium satisfies $S_{12}{}^{n} = 0$ and $S_{21}{}^{n} = -S_{1,H}$, and corresponds to the efficient split B in Fig. 3.

Case III ($N^{*}_{1,L} \leq N^{o}_{1} \leq N^{*}_{1,H}$): An efficient allocation here belongs to case B^{e*} of Appendix B, and it is characterised by $N^{e*}_{1|B} = N^{o}_{1}$ and by any split consistent with

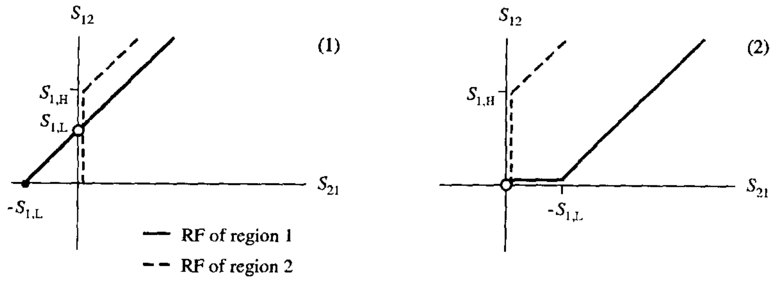

Fig. 7. Best-Response Functions in Case III.

the equilibrium condition (15). Since determining efficient allocations now does not involve a comparison of case B^{e*} with any other case, we can imagine a simplified version of Fig. 3 in which the continuous UPF is given by [B, E]. The most favourable split for region 1 is represented by point E in the simplified Fig. 3, where $\Delta x^{e*}_{1|B} = -m_{21}$. We know from (9) and (13) that E can be achieved by $S_{1,L}$ which also supports the initial partition. Hence the RF of region 1 to any S_{21} is given by $S_{12} = S_{1,L} + S_{21}$ and $S_{12} \geq 0$. On the other hand, following case II.3, the most favourable split for region 2 is B, and the RF of that region to any S_{12} is given by $S_{21} = S_{12} - S_{1,H}$ and $S_{21} \geq 0$. As with case II above, we consider subcases corresponding to each graph of Fig. 2(2) and Fig. 4 separately. For Fig. 4(2), where $0 < S_{1,L} < S_{1,H}$, the only subgame-perfect equilibrium satisfies $S_{12}{}^{n} = S_{1,L}$ and $S_{21} = 0$, and corresponds to the efficient split E in the simplified Fig. 3 (see Fig. 7(1)). For Fig. 2(2), where $S_{1,L} \leq 0 \leq S_{1,H}$, the only subgame-perfect equilibrium satisfies $S_{12}{}^{n} = 0$ and $S_{21}{}^{n} = 0$, and corresponds to any original split in (B,E) of the simplified Fig. 3 (see Fig. 7(2)). Finally, for Fig. 4(1), where $S_{1,L} < S_{1,H} < 0$, the only subgame-perfect equilibrium satisfies $S_{12}{}^{n} = 0$ and $S_{21}{}^{n} = -S_{1,H}$, and corresponds to the efficient split B in the simplified Fig. 3. This situation can be illustrated as a modified Fig. 6(3) in which $-S^{*}_{i,L}$ on the abscissa is replaced by $-S_{1,L}$.

The arguments for cases IV ($N^{*}_{1,H} < N^{o}_{1} < N^{e*}_{1,H}$) and V ($N^{e*}_{1,H} \leq N^{o}_{1}$) are antisymmetric to those for cases II and I respectively. Thus we conclude that all subgame-perfect equilibria of the two decentralised regional governments are federally efficient.

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