

A generative bottom-up approach to the understanding of the development of rural societies

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Abstract

This study analyses the complexity in determining the physical carrying capacity of a society in a specific environmental setting, and highlights the necessity for developing an agent-based modelling approach. In the context of this generative bottom-up approach, this study introduces an artificial agricultural society model developed in terms of the complex interactions among intelligent agents over space and time. Sample simulation results are presented to show the emergent macroscopic patterns of agricultural land use and of agents' travel and transport networks around a market, along the variations of society's demography and agents' trading prices at various time scales. The implications of the simulated results for policy making are also analysed.

Index words: carrying capacity, complexity and emergence, artificial society, agent-based modeling, diffusion limited aggregation, agriculture location theory; Java technology

1. Introduction

Over the course of last several decades a rapid development of socio-economy has been achieved in many parts of the world. This achievement, however, has been in company with a continual increase of human population in an exponential form. Under the effect of the two factors, a significant increase has been resulted not only in the use of many essential resources but also in the generation of many kinds of pollutants. In some areas where a large number of human populations are endowed with a limited quantity of natural resources, such as fresh water, cultivatable lands, fisheries, coals and many others, this increase has led to the occurrence of numerous socio-economic crises. Though taking place regionally in most cases, this increase has also caused concerns over the future of human beings on Earth. There have been even claims that the limits to the physical carrying capacities of the whole Earth, or societies in certain areas may have already been approached or even overshoot to a considerable degree (e.g. Meadows et al., 1972, 1992; Brown, 1994, 1995, 1997; Cohen, 1995). In addressing these concerns, the concept of sustainable socio-economic development has been widely adopted and practiced in the fields of natural resource management, environmental conservation, and socio-economic development (e.g. Harris, 1996, 1999; Giddings et al., 2002).

In the context of sustainable development, quantification of the physical carrying capacity of a society in a specific environmental setting has been the main focus. This quantification, however, has met considerable challenges. In biological science, the physical carrying capacity for a species in a specific environmental setting can be simply determined by a head-counting method (e.g. Hardin, 1991). The determination of the physical carrying capacity for human beings in a special environmental setting yet needs to reflect quantitatively the effects of many factors, including social demands, cultural difference, levels of technology application, biological and ecological conditions, physical environmental constraints, patterns of production and consumption for various resources, forms of human clustering, life styles, and trade methods and scales (e.g. Hardin, 1976; Daly, 1990, 1992; Dean et al., 2000; Yao et al., 2005). Due to the complexity, this quantification has been presented with a plethora of mathematical or computing models capable of solving a multi-objective problem. These include various supply and demand based models for resource utilization, modes of dynamic systems, and multi-objective based optimisation models (e.g. Makropoulos et al., 2003; Midgley and Reynolds, 2004; Yao et al., 2005). In contrast to the development of these analytical models, a number of researchers and practitioners have been attempting to find some quantitative criteria or what are called sustainable indexes (e.g. Daly, 1992; Lawn, 2003; Spangenberg, 2004). As has been believed, these indexes are able to signal if a socio-economy system develops into an unhealthy or an unsustainable form.

All of these analytical models and sustainable indexes, however, use both past and present information to predict the future growth trends in human population and socio-economic development. For making such predictions, many techniques have been applied, typically including regression analysis, fuzzy logic, grey theory, stochastic process modelling, and lots of their variants. The use of these techniques, however, has been largely based on an assumption that a system's development is always in equilibrium. The predictions made through using these techniques thus cannot reflect accurately the future behaviour of a population-economy-environment system. Such a system is generally well structured and with a prevailing industrial ethic of continuous growth for increasing returns due to labour increase and/or human beings' aspiration of living better lives. As a result, both socio-economic development and human population growth of the system generally have considerable momentum, and disequilibrium prevails in the whole evolutionary process. This makes the response of ecological systems to human abuse come only after long delays, and the human population act only after further delays. Such a system, therefore, is normally bounded for, unless altered by human intelligence and human self-restraint, overshoot and collapse because the resource base of the system is both limited and subject to continual erosion (e.g. Meadows et al., 1972, 1992). The human population growth in mainland China over the last several decades is a typical example of this kind. If there were no any measures being imposed to control human population growth rate from early 1980's, it is without doubt that the well-structured population-economy-environment system of mainland China would have encountered much severer and more frequent socio-economic and ecological crises. Even under over 20 years' harsh controls, the human population growth in mainland China still has considerable momentum and causes concerns over the future development of socio-economy at both regional and national scales (e.g. Yao et al., 2005).

To understand the behaviour of such a complex system with considerable feedback delays, slow physical responses, and disequilibrium nature, the newly emerged science of complexity and the recent significant advancement in both information technology and computing power have offered an effective alternative approach. This approach treats a complex system as an aggregation of multiple agents. It then starts from the bottom, simulating the complex interactions of all autonomous rule-governed agents and exploring the emergent behaviour of the whole system. This approach is rather agent-based modelling and has led to the development of a new field for research – Artificial Life (e.g. Simon, 1981; Langton, 1999; Heudin, 1999; Lake, 2000; Kohler and Gumerman, 2000). Recently, this approach has gained an application in building up an artificial society model (Epstein and Axtell, 1996). To explore how the agent-based modelling approach can be applied to the study of the physical carrying capacity of a society, this paper details the logic thinking behind this approach in a comparison with classic bottom-up and top-down approaches. It then introduces the development of an artificial agricul-

tural society model that simulates the evolutionary process of rural societies. Finally, some sample simulation results are presented and analysed.

2. Agent-based modelling for complex systems

For a long time, a large number of philosophies and scientists have believed that a real understanding of the nature of things can be gained only when one gets to the bottom of things, and somehow the things at the bottom explain everything else all the way up. This bottom-up thinking has led to a methodology in which studies of the behaviour of a large system are carried out by breaking the system into a manageable combination of sub-systems. These sub-systems are in turn broken into smaller sub-systems and so on until fundamental sub-systems are identified. This classic bottom-up approach has been commonly deployed in three means. For systems that cannot be simply described with solvable mathematical formulations, empirical analysis has been regarded as a relatively reliable tool. Through very detailed field investigations and indoor experiments, the behaviours of the fundamental sub-systems can be generalized and gradually summed up so as to form a theory that is able to characterise the general behaviour of the whole system. This means is generally known as the empirical bottom-up method.

In parallel to this empirically based reductive approach, numerical computing techniques of finite difference, finite element, boundary element and more have been widely applied. Beginning on an economically and computationally acceptable small scale, these techniques are able to provide an approximate solution to the complex differential formulation of a system as a whole. This means illustrates the computational bottom-up method. When the differential formulation of a system can be mathematically solved on the scale of fundamental subsystems, on the other hand, a pure mathematical integration analysis is without doubt the most effective tool. This means represents the mathematical bottom-up method. These three methods of the classic bottom-up approach have laid the base for classical Newtonian and analytical physics as well as contemporary science that looks to mass-energy or the murky indeterminacies of quantum mechanics. For this, the bottom-up approach has even been regarded as the creed of science. It is evident that this approach embodies the logic thinking of reduction.

While this classic bottom-up approach has gained very wide applications in understanding the behaviour of many physical and social systems, an alternative approach has also been proved successful in both explaining natural phenomena and designing artificial systems and objects. This approach begins with a theory well established on a large scale, such as Newtonian laws or Hamilton's principles. For solving a concerned problem on a very small scale, this approach dissects the theory gradually into specific forms as the scale becomes smaller and smaller. This classic top-down approach has been

applied to study the structure and function of hierarchy, hybrid, and network in various fields. It has also been regarded as providing a highly rational way for accomplishing large collective tasks, such as designing complex buildings, enterprises, aircrafts, governmental organizations, transport and electric networks, supply chains, computers, and many more. Clearly, this top-down approach manifests the logic thinking of deduction.

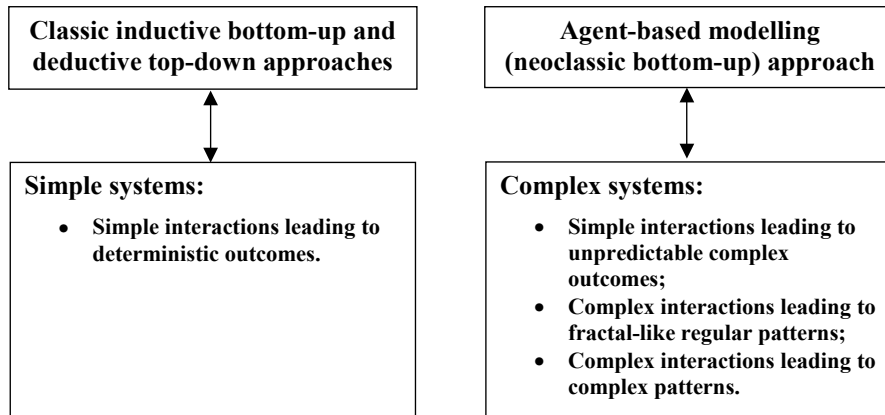


Fig. 1. Some characteristics of simple and complex systems

Both classic bottom-up and top-down approaches have been based on a belief that the behaviour of any natural and human systems is deterministic and predictable. In some “simple” physical and social systems, they two approaches have indeed led to fruitful results. In some “complex” systems, however, numerous studies have recently shown that a direct application of the two approaches cannot lead to deterministic solutions. This is because these complex systems exhibit some macroscopic properties that are generally not discernible from a pure study of the fundamental components of a system in isolation. In some cases, these emergent properties are merely surprising or counter intuitive, while in the other cases, they are in principle impossible to determine from knowledge of the parts of a system and their relationships alone. These phenomena have led to the emergence of complexity as a specialised field for research (e.g. Kauffman, 1995; Bak, 1996; Bar-Yam, 1997, 2004; Kohler and Gumerman, 2000). Fig. 1 presents a simple description of the characteristics of “simple” and “complex” systems in terms of our knowledge.

The significant improvements in both computing power and modelling sophistication over the last few decades have made agent-based modelling an effective approach for understanding the behaviour of complex systems. Rather than concerning the behaviours of fundamental sub-systems, agent-based modelling approach concerns mainly the behaviours of primary actors or agents interacting in sub-systems. It seeks rather than the mathematical solutions of the system but the emergent collective behaviour of all agents in

the whole system. Specifically, this approach works in a way of viewing a system as a large population of agents, starting from the interactions among all the agents (bottom), and constructing large aggregates of autonomous rule-governed agents that interact with one another nonlinearly.

Evidently, this agent-based modelling approach provokes an additional dimension of logical thinking on how complex physical and social systems may behave. As stated above, this logical thinking is different from that behind traditional bottom-up and top-down approaches. In philosophical terms, nevertheless, the agent-based modelling approach is still bottom-up based. Within the context of what is so called “generative social science” advocated by Epstein and Axtell (1996), this neoclassic bottom-up approach may be more accurately called generative bottom-up approach.

This agent-based modelling approach deployed in the forms of self-reproducing automata and cellular automata (CA) has led to the generation of what has been so called “Game of Life”, or simply CA life. CA life, originally discovered by John Conway at Cambridge (Gardner, 1970), has shown complex dynamics of life under the governance of a set of simple rules (Berlekamp, 1982; Poundstone, 1985; Bays, 1987 1988, 1990, 1992; McIntosh, 1990). Recently, CA life has been extended to the development of “Swarm Intelligence”, an artificial intelligence technique for studying the collective behaviour of various types of agents in decentralised, self-organised biological and socio-economic systems (e.g. Bonabeau and Théraulaz, 2000). By taking a few simple rules that govern the behaviour of human agents as the ones governing CA life, Epstein and Axtell (1996) advocated a project of what is so called “generative social science”. In the project, a multi-agent based society model named Sugarscape was developed to demonstrate how fundamental collective behaviours such as group formation, cultural transmission, combat, and trade emerge from the interactions of individual agents.

The artificial society model of Sugarscape, however, has several shortcomings, including treating human clustering as the result of resource distribution, ignoring labour activity on production and transport, and limiting trade to pairs of neighbours. In order to produce the results resembling the behaviours of real societies to a higher degree, we are developing an artificial agricultural society model that has microeconomic theories as the physical base and tackles the connected problem of agents’ decision-making for travel and transport pathway selection, production and consumption planning, and market interactions.

3. The AgriSociety model

3.1 Model structure

A society is an integrated whole of many essential parts in which human agents interact in different forms. The most essential parts of a society in a spatial setting include mainly the decision-making of human agents on settlement choice, travel and/or transport route selection, production (and land use) and consumption planning, and market interactions. In the artificial society model of Sugarscape developed by Epstein and Axtell (1996), however, there is use of labour in the gathering of resources, but no transformation of those resources, except by consumption, is considered. To tackle the connected problem of the essential parts of a society is thus the main focus in our developing an artificial society model. As can be noted in the development of Sugarscape model, the development of a successful artificial society model is not an easy task. For this, our present model concerns only the relatively simple case at this stage that of the evolutionary process of agricultural land use around a single market. Our model is thus named AgriSociety. Fig. 2 shows how our AgriSociety model tackles the connected problem of agents' decision-making for travel and transport pathway selection, production and consumption planning, and market interactions in a specific environment at a specific time interval. This time interval resembles the period of one year in reality.

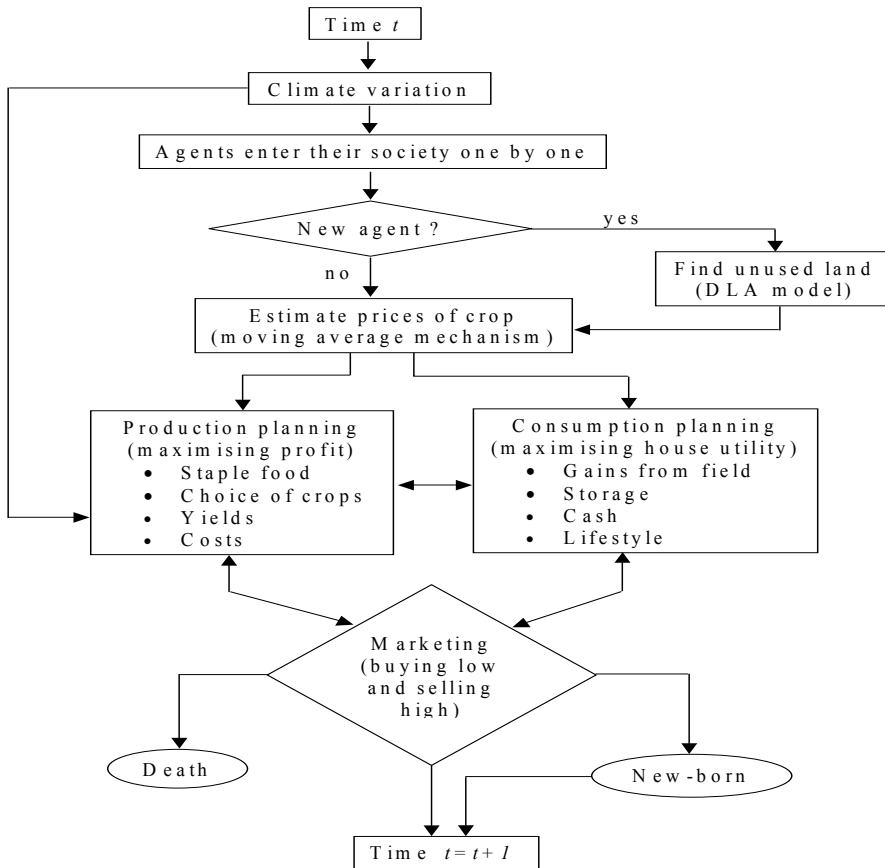


Fig. 2. Flow chart of AgriSociety model with the inclusion of major governing mechanisms and parameters for each agent's behaviour (DLA stands for Diffusion Limited Aggregation).

Our AgriSociety model is firmly based on microscopic socio-economic theories, which are reflected mainly in the following three basic ingredients of the models: agents, an environment or space, and rules.

3.2 Agents

As can be understood, agents are the “people” of artificial societies. In both CA life and Sugarscape model, a few simple rules have been applied to study the emergent macroscopic behaviours of life and society consisting of groups of agents. However, human agents normally have a culturally and genetically diverse background and behave complexly in using labour, consuming goods, bargaining in a market, and choosing their lifestyle. To reflect these, the agents in our AgriSociety model are assumed to be as intelligent and culturally and genetically diverse as humans. Specifically, they have the following attributes:

- All agents have self-interest and are capable of making their own decisions, in which attention is paid to their needs and desires in the present as well as in the future;
- Agents come from different cultural and genetic backgrounds and have a different attitude toward life style;
- Each agent has a capacity to supply a finite quantity of labour in each time period;
- Each agent consumes two types of goods – essentials and non-essentials, with staple (a basic foodstuff) as an essential good for each agent to survive;
- Leisure is an essential good to each agent;
- All agents are able to use money as a medium of trade.

3.3 Environment

Similar to CA life and Sugarscape model, the interactions of agents in our AgriSociety model occur in a physical environment that is represented with a finite, regular, rectangular grid of cells. Nevertheless, our AgriSociety model has an interface with GIS technology. So it can directly assess the spatial usability of land resources in the concerned physical environment and then inputs the information into our AgriSociety model. This makes it possible to study the complex interplay between the physical environment and societal development within the context of “generative social science” as detailed by Macmillan and Huang (2005).

Furthermore, the interface with GIS can make the physical environment be viewed either as a 3-D DTM (Digital Terrain Model) or as a 2-D computing environment that consists of a rectangular grid of cells. Detailed information

on the spatial usability of land resource is also provided with the interface on which cell is usable for growing crops. These information is shown on the left-hand side of the initial state of the simulator in Fig. 3. This simulator is developed for implementing our AgriSociety model and its function is detailed in the following section.

Although in our AgriSociety model, the 2-D computing environment is represented by a finite, regular, rectangular grid of cells, it needs to be pointed out that this does not mean that we apply cellular automata (CA) technique in building up our artificial society model. The field cells change their states as the result of the decision-making of agents in our artificial society model other than by themselves in terms of some imposed transitional rules as CA technology requires (e.g. Wolfman, 2002).

3.4 Rules

In our AgriSociety model, a large number of rules are applied to govern the behaviours of agents in various decision-making processes. These rules are based on microscopic socio-economic principles, which are mainly in four forms, with the first principle as:

Principle 1: Agents select travel pathways with a diffusion limited aggregation mechanism

In Sugarscape model developed by Epstein and Axtell (1996), human clustering has been treated as the result of resource distribution. In reality, however, most people tend to settle down in a centralized zone (village, town, or city) and to build an effective network to travel and transport their products produced in distant fields. To tackle the problem of human clustering in our development of our artificial society model, we apply a DLA (Diffusion Limited Aggregation) model to illustrate the process of each agent's decision-making for finding a suitable field to grow crops. DLA was originally introduced by Witten and Sander (1981, 1983) as a model for irreversible colloidal aggregation. Since its development, the model has gained very wide applications because it produces "fractal" objects with a dynamic growth process (Halsey, 2001). DLA has also gained wide applications in urban growth studies (e.g. Batty, 1991; Longley et al., 1991; Batty and Longley, 1994; Benenson, and Portugali, 1997; Benenson, 1998; Ward et al., 2003). These wide applications, together with the fact that real people tend to live along a road for traffic and transport convenience, lay a physical base for us to use DLA to simulate the process of travel and transport of agents.

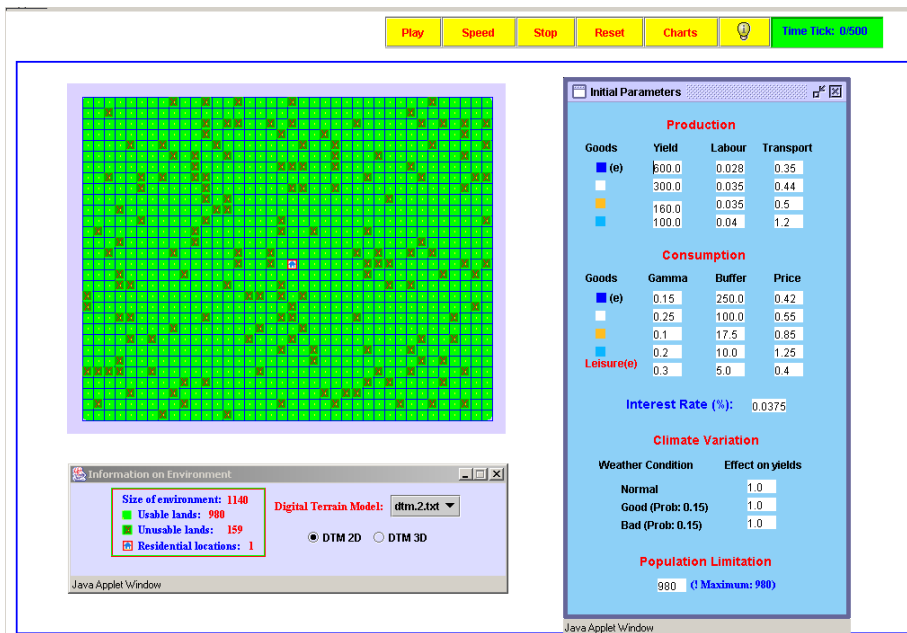


Fig. 3. Structure of the simulator and the initial state of AgriSociety mode for a specific physical environment (in “Production” section, “Yields” represent annual yields of crops, and “Labour” and “Transport” are the costs of labour and transport per unit travel length, respectively; in “Consumption” section, “Gamma” and “Buffer” represent γ and δ in the calculation of house utility using $u = (q - \delta)^\gamma$ where q is the quantity of consumption for a specific good and γ varies with the taste of agents in a small range).

The second principle governs the behaviour of agents in making production and consumption plans:

Principle 2: Agents’ production and consumption plans are made in terms of location and household utility theories.

In implementing this rule in our AgriSociety model, four types of crop are considered to be cultivable in all usable lands. Among the four types, only one type of crop is the essential good for survival, like rice or wheat, while the other types of crop are nonessential goods, which are more or less like vegetables or fruits. For agents without any storage, they grow only the crop that is essential to survival. While those agents that have the essential good sufficient for survival in their storage, they tend to grow the crop that yields a maximum profit. Each agent calculates its potential profit in terms of location theory in spatial economics, which is a function of the yield of the selected crop and the costs due to travel, transport and labour. Among the costs, the cost due to transport is regarded a linear function of travelling distance to the market/settlement. Both gains from field and goods in store determine the

budget that can be spent on a bundle of consumption goods (including leisure) that maximizes household utility.

The third principle governs the trading activities of agents:

Principle 3: Trade among agents is governed by market clearance mechanism

Trading among agents occurs in a fully free market and all agents in the residential area enter the market in a random form. Once all agents enter the market, a buyer searches for the seller with the lowest bidding price, while a seller is happy to negotiate with the buyer with the highest bidding price. As a result, they end up with compromised prices. The compromised or transaction prices for the goods are remembered by the agents and used to predict the prices of the goods in the next period with a moving-averaging mechanism. This illustrates a simple adaptive mechanism of agents.

The fourth principle determines the status of death or birth for each agent:

Principle 4: Status of agents' death and birth is determined by both staple food and life style

Under the governance of this principle, the following rules are applied in our artificial AgriSociety model:

- If, in any period, the cost for an agent to travel and transport goods from its field is greater than the potential profit that the agent can gain, it dies;
- If, in any period, an agent cannot obtain the quantity of staple food sufficient to survive, it dies;
- A new agent is born when an agent has sufficient staple food and prefers to produce a descendant in any period.

4. Sample simulations

For implementing our AgriSociety mode, a simulator is built on Java platform as shown in Fig. 3. By doing so, the detailed evolutionary process of an agricultural society can be directly visualized using an Internet Web Browser, typically Microsoft Internet Explorer 6.0 or higher. The whole program is written in object-orientated Java programming language, and modelling results are displayed in a dynamical form using Java Applet technology. The initial state of the 2-D computing environment consisting of 1140 cells in total (38 rows times 30 columns) is shown on the left-hand side of the simulator, while the initial subjective values of parameters governing the decision-making of agents in production and consumption planning, banking and weather predicting are presented on the right-hand side of the simulator (Fig. 3). The physical environment presented in a 2-D form in the left-hand side of Fig. 3 is extracted from some DEM data available to us. The detailed location

of these data is unclear and we use them for demonstrating the performance of our AgriSociety model only in this paper.

Some simulated results of the application of our AgriSociety model are shown in Fig. 4. They are obtained through following a single simulation with time. In the simulation, one cell that is a usable land and close to the centre of the whole environment is taken as the sole settlement site, at which trading also takes place. Thus, the sole settlement site is also the market place of the agricultural society. The evolutionary process of the development of an agricultural society starts from the entrance of one new agent into the environment. When the total number of agents is less than 10, one new agent is added at each time step and no trading but the exchange of different types of crop among the agents takes place. Once the total number of agents reaches 10, trade takes place and the numbers of both new agents coming into society and dead agents occurred in the society are then determined by each agent's gains from both field and market and its attitude towards life style. In this simulation, variations in weather condition are ignored with the effects of both good and bad weather conditions on yields taking a value of 1.0. Bank interest also remains unchanged.

In the simulating process, the panels for showing the initial subjective values of parameters in AgriSociety model as well as the physical environment and its relevant information are all hidden. Instead, four new panels are displayed on the screen (Fig. 4). At each time step, the spatial distribution of four types of crop chosen by agents is shown on the top left-hand corner, while the growth of road networks is displayed on the top right-hand corner. At the bottom, variations with time in society's demography and agents' trading price for a specific good are presented.

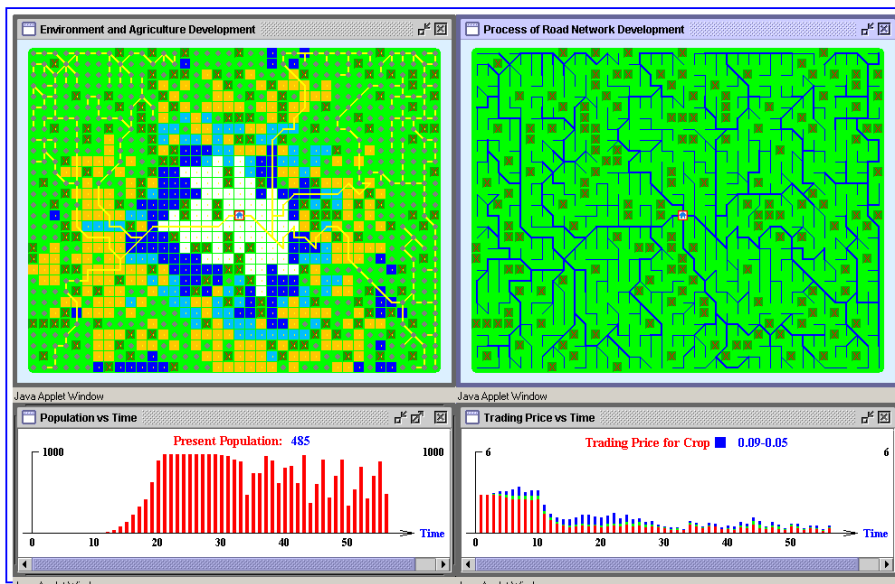


Fig. 4. Sample results of simulations using AgriSociety model (time variation is presented in the charts that show the variations of population and trading price for essential good).

It is noticeable in the four panels presented in Fig. 4 that although each agent's travelling distance to the settlement site or the market is nonlinear as the result of applying a Manhattan-distance based model of DLM (diffusion limited aggregation), the spatial distribution of the types of crop chosen by all agents are essentially in the widely recognised form of von Thünen rings. As is well known, the phenomenon of von Thünen rings has been observed in countless environments since its introduction in 1826 and has laid empirical foundation for the development of location theory in spatial economics (e.g. Wartenberg, 1966; Samuelson, 1983; Mäki, 2004). Furthermore, road network extends in a *fractal*-like pattern, which is also consistent with observations in real transport systems (e.g. Batty and Longley, 1994). At the early evolutionary states (total number of agents greater than 10), the population (labour) of the agricultural society increases at an exponential form.

When the society reaches its carrying capacity, new agents that are unable to find suitable fields to cultivate crops are presumed migrating out of the concerned physical environment. However, the trading prices of agents in the society fluctuate even when the society reaches its carrying capacity, at which the supply and demand in the whole market is well balanced. Population also fluctuates beyond the time at which the society reaches its carrying capacity. These fluctuations mean that a steady state economic growth pattern cannot be maintained for a long time. The obvious reason for this is that agents still gain from field such that the society as a whole produces more than it consumes. As the over-production accumulates, the society will eventually behave chaotically. All of these imply that when an agricultural society reaches its carrying capacity, it may still have momentum for growth and can result in the collapse of the whole society. In order to make a society maintain a sustainable growth, therefore, measures need to be taken much earlier before the society reaches its physical carrying capacity. This result, however, is still premature and a further detailed study is under way.

5. Discussion and Conclusions

The physical carrying capacity of a society has been determined previously with both classic bottom-up and top-down approaches. These approaches rely on the past and present information on socio-economic development and population growth. However, a society develops generally with an industrial ethic of continuous growth due to labour increase and/or human agents' aspiration of increasing returns and living better lives. To understand the effects of this disequilibrium mechanism and how a society tends to behave when its physical carrying capacity is reached, this paper details the necessity for adopting an agent-based modelling approach.

In the context of this generative bottom-up approach, this paper introduces the development of an artificial agricultural society model named AgriSociety. Following a single simulation through time, this study shows that the land use pattern yielded from our AgriSociety model is essentially in the same form as the widely recognised von Thüene rings. The patterns of pathways chosen by agents for travelling and transporting yields from distant fields are very similar to real transport networks. However, the simulated results also demonstrate that it is almost impossible for an agricultural society to maintain a steady state economic growth unless some labours are released. This implies that when the agricultural society reaches its carrying capacity, it still has momentum for growth and can result in the collapse of the whole society. In order to make a society maintain a sustainable growth, therefore, measures need to be taken much earlier before the society reaches its physical carrying capacity. This study, however, is still premature and further detailed studies are required.

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