# *Geometria amostral*

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&
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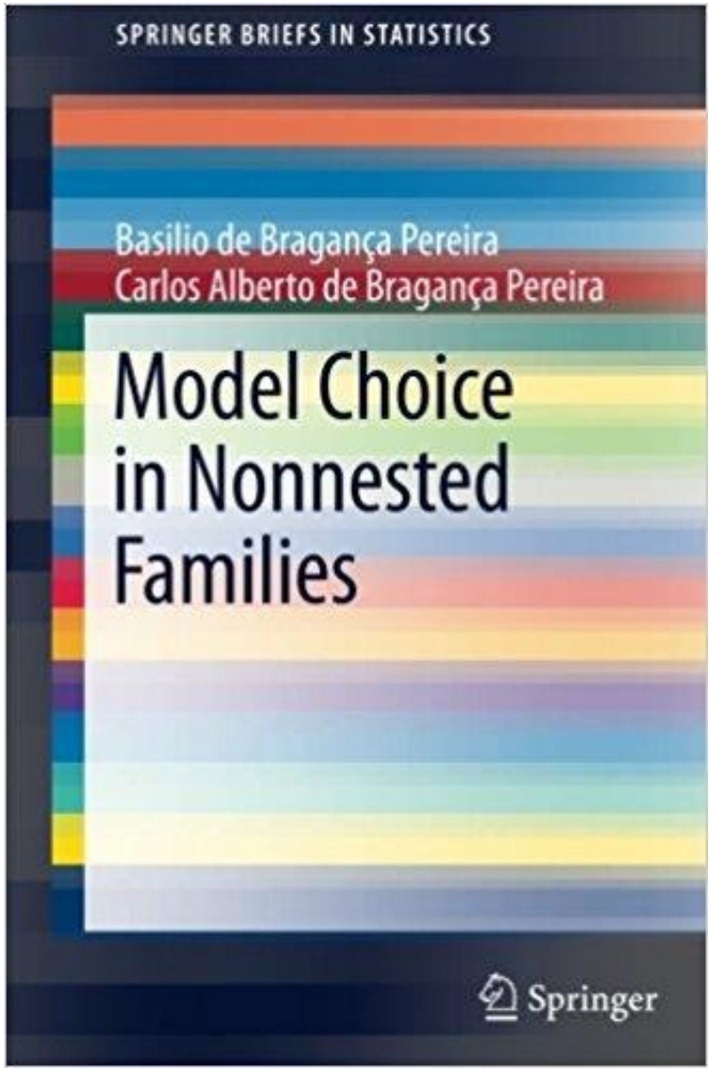
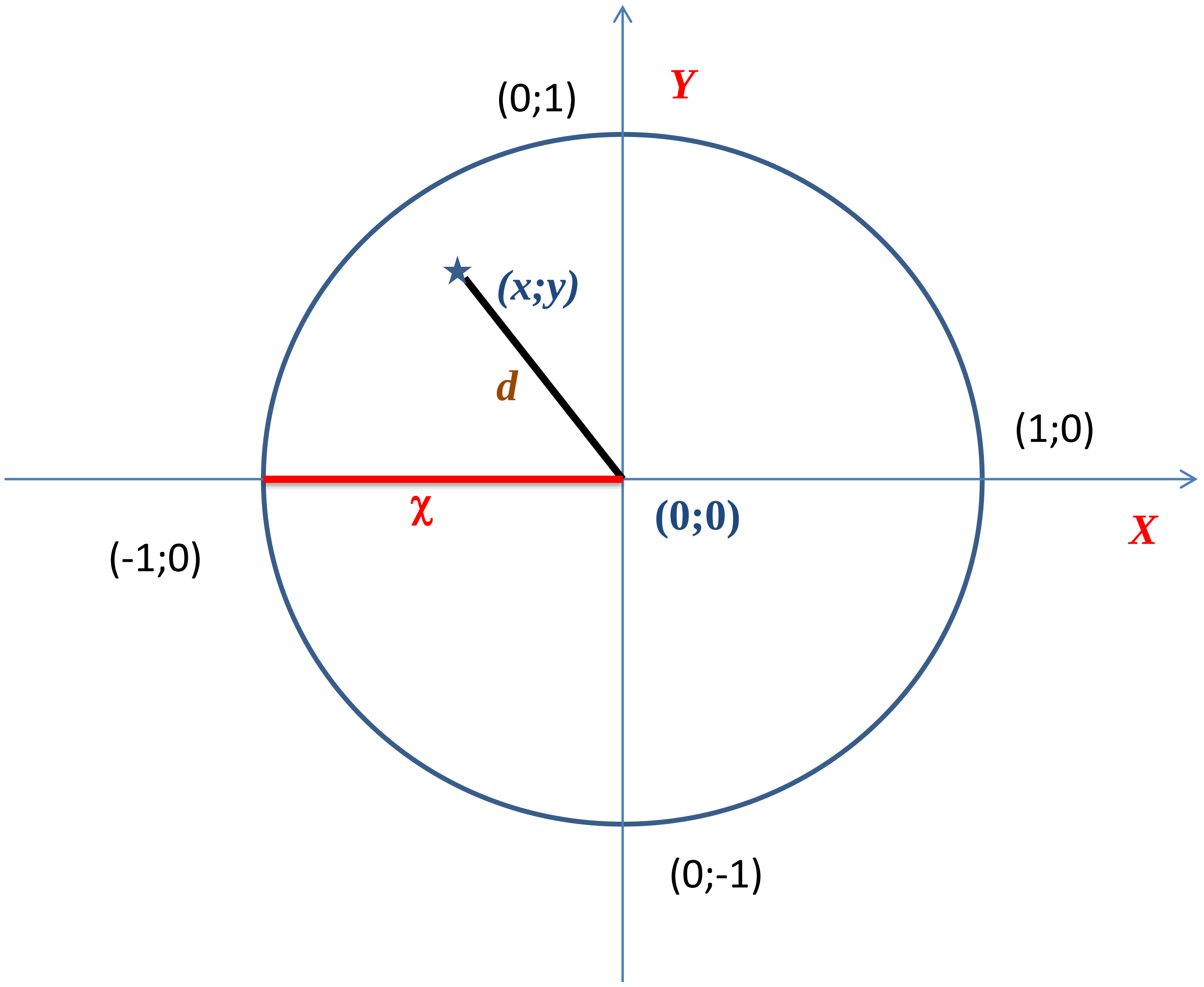


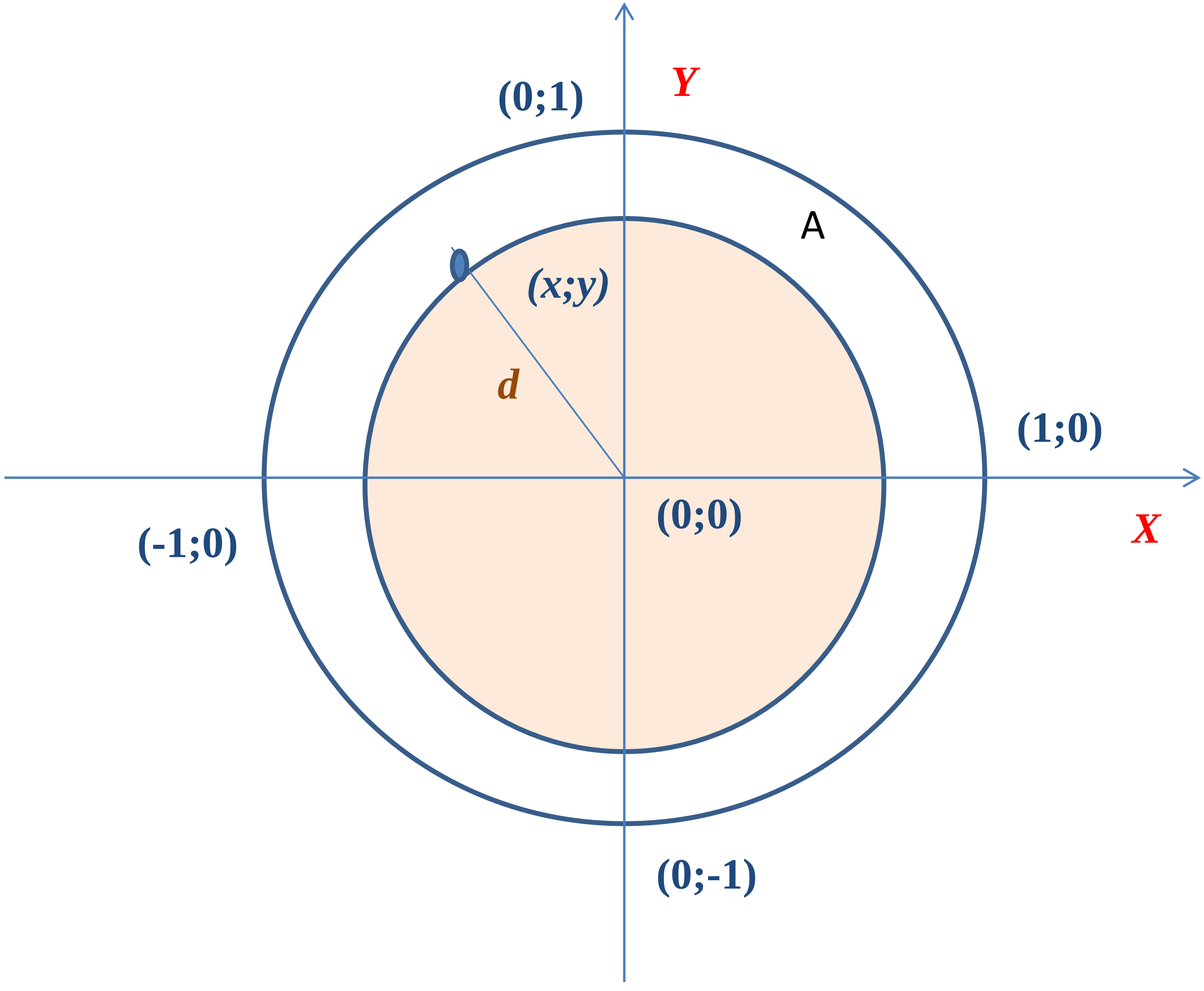

**De:** David Cox <david.cox@nuffield.ox.ac.uk>
**Data:** 12 de maio de 2017 06:56:57 BRT
**Para:** Basilio de Bragança Pereira
(basilio@hucff.ufrj.br) <basilio@hucff.ufrj.br>
**Assunto: your book**

**Dear Basilio, I have just received the copy of your book. Very warm congratulations on it.**
**It is great to have such a comprehensive account of this topic.**
**It is good to have the Bayesian and non-Bayesian approaches both so well presented. I deeply appreciate your kind remarks and the dedication.**
**For me the years at Imperial College were wonderful ones, good colleagues, outstanding doctoral students from so many countries and visitors too**

Best wishes
David


(0;1)
Y
(x;y)
d
(1;0)
χ
(0;0)
X
(-1;0)
(0;-1)

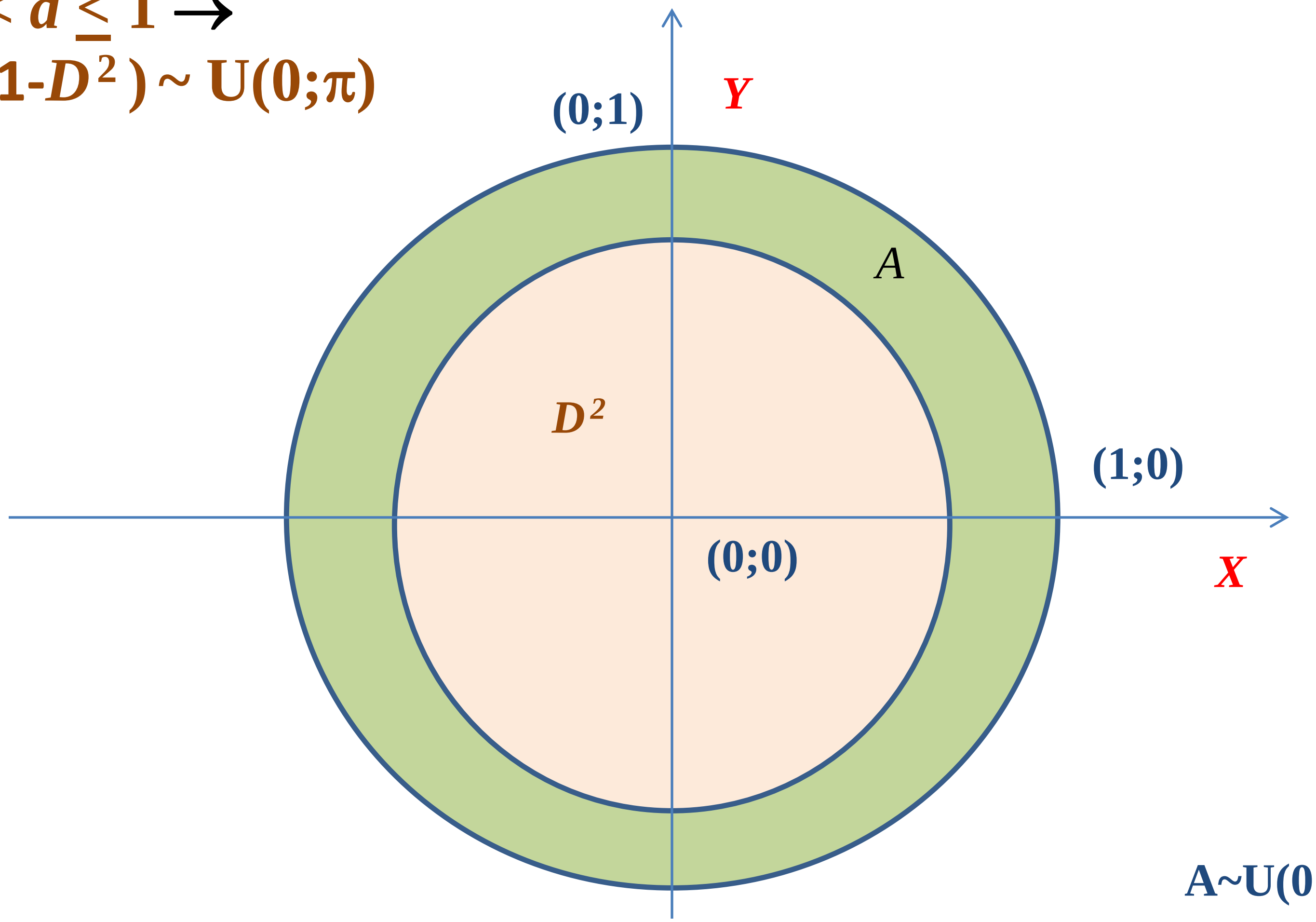
$$\Pr\{D \leq d\} = d^2 \rightarrow f(d) = 2d \text{ for } 0 < d \leq 1$$



$\Pr\{D \leq d\} = d^2 \rightarrow f(d) = 2d$

$for\ 0 < d \leq 1 \rightarrow$

$A = \pi * (1 - D^2) \sim U(0;\pi)$

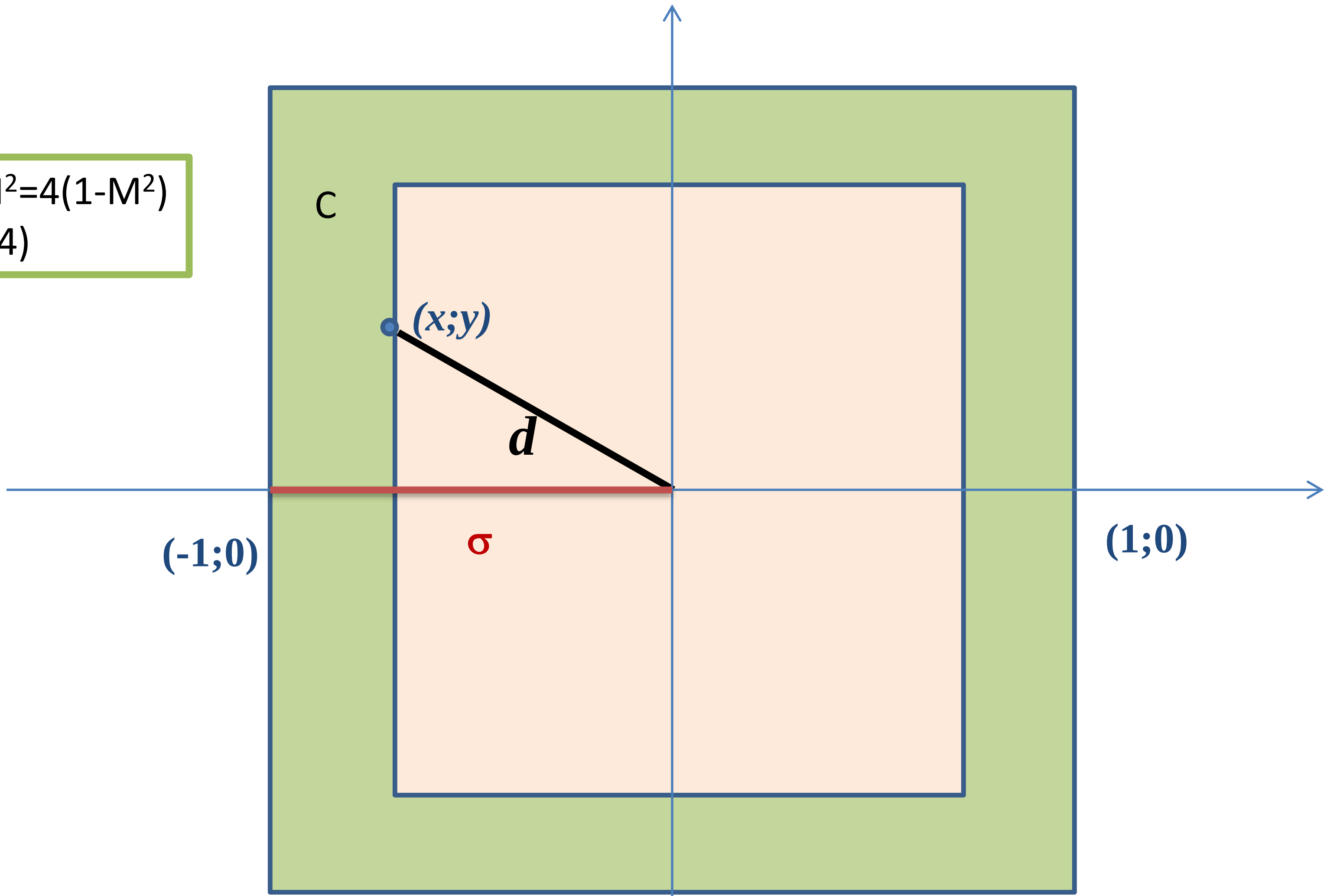



$A \sim U(0;\pi)$

$A = \pi * (1 - D^2) = 1 - F(D) \sim U(0;\pi)$

$C=4-4M^2=4(1-M^2)$

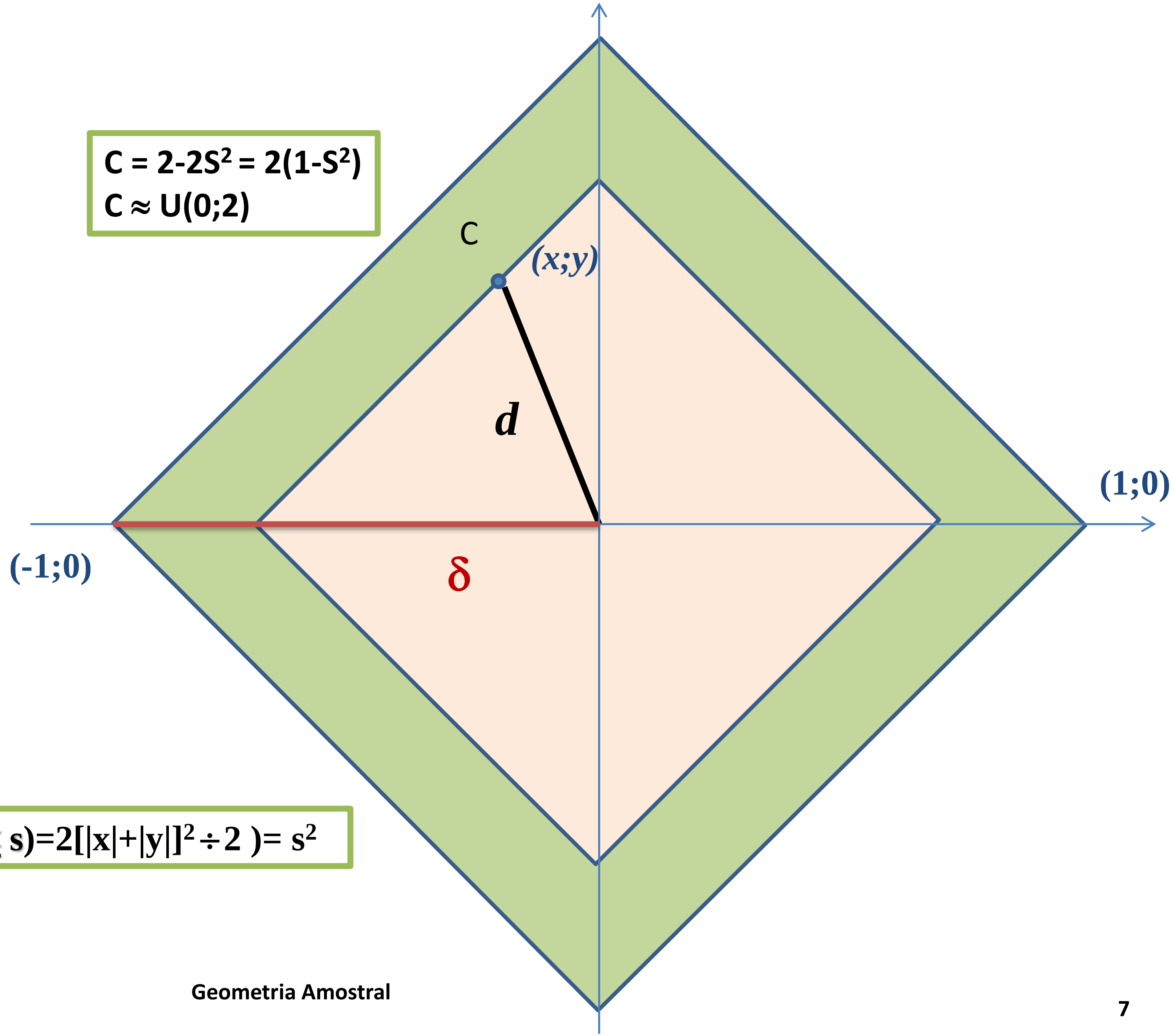
$C \approx U(0;4)$




$Pr(M<m)=[2Max(|x|;|y|)]^2 \div 4\ )=[Max(|x|;|y|)]^2=m^2$

$C = 2 - 2S^2 = 2(1 - S^2)$

$C \approx U(0;2)$



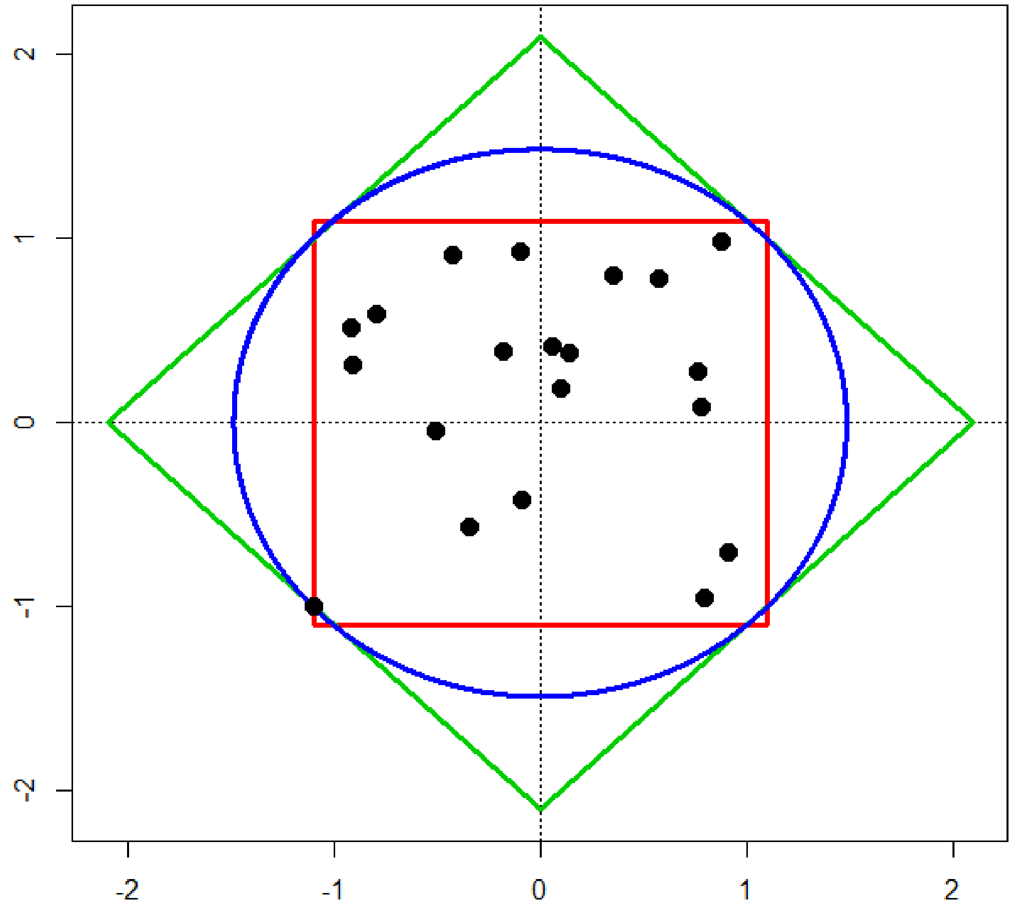

$\Pr(S \leq s) = 2[|x|+|y|]^2 \div 2\ ) = s^2$

| Sample number | QUADRADO | | | σ | δ | χ |
|---|---|---|---|---|---|---|
| | x | y | xy | | | |
| 1 | -0.500 | 0.903 | -0.452 | 0.903 | 1.403 | 1.032 |
| 2 | 0.554 | 0.765 | 0.424 | 0.765 | 1.320 | 0.945 |
| 3 | -0.244 | 0.347 | -0.085 | 0.347 | 0.592 | 0.425 |
| 4 | 0.754 | 0.236 | 0.178 | 0.754 | 0.990 | 0.790 |
| 5 | 0.875 | 0.988 | 0.864 | 0.988 | 1.862 | 1.319 |
| 6 | -1.009 | 0.268 | -0.271 | 1.009 | 1.278 | 1.044 |
| 7 | 0.007 | 0.381 | 0.002 | 0.381 | 0.387 | 0.381 |
| 8 | 0.774 | 0.031 | 0.024 | 0.774 | 0.805 | 0.774 |
| 9 | 0.056 | 0.138 | 0.008 | 0.138 | 0.193 | 0.148 |
| 10 | -0.144 | -0.510 | 0.073 | 0.510 | 0.654 | 0.530 |
| 11 | 0.909 | -0.812 | -0.738 | 0.909 | 1.721 | 1.219 |
| 12 | -0.151 | 0.921 | -0.139 | 0.921 | 1.072 | 0.934 |
| 13 | 0.321 | 0.792 | 0.255 | 0.792 | 1.114 | 0.855 |
| 14 | 0.100 | 0.343 | 0.034 | 0.343 | 0.443 | 0.357 |
| 15 | -0.889 | 0.565 | -0.502 | 0.889 | 1.454 | 1.053 |
| 16 | 0.789 | -1.072 | -0.846 | 1.072 | 1.862 | 1.331 |
| 17 | -0.587 | -0.110 | 0.064 | 0.587 | 0.697 | 0.597 |
| 18 | -1.017 | 0.487 | -0.495 | 1.017 | 1.504 | 1.127 |
| 19 | -0.415 | -0.665 | 0.276 | 0.665 | 1.080 | 0.784 |
| 20 | -1.094 | -1.015 | 1.110 | 1.094 | 2.109 | 1.492 |
| **Stat** | **-0.046** | **0.149** | **-0.011** | 1.094 | 2.109 | 1.492 |
| **Var** | **0.434** | **0.403** | **0.220** | | | |
| **Cov** | | | **-0.004** | | | |
| **Área** | | | | **4.788** | **8.895** | **6.996** |

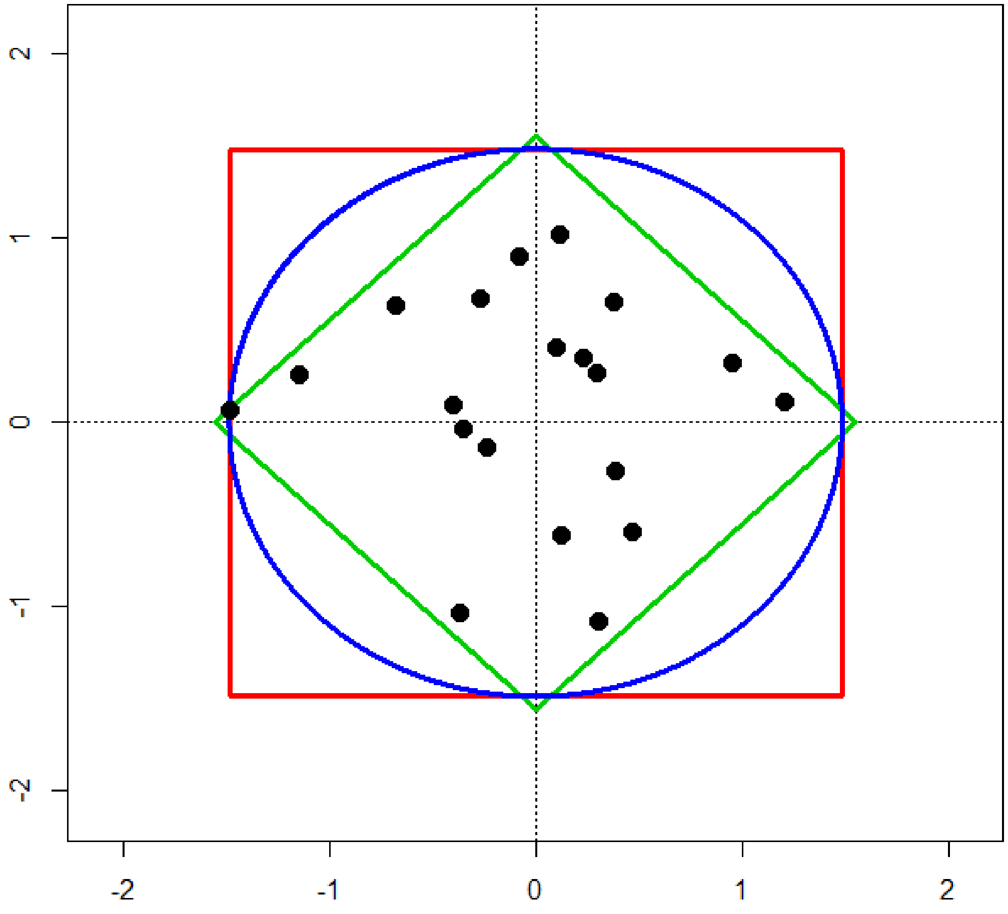
| Sample number | DIAMANTE | | | σ | δ | χ |
|---|---|---|---|---|---|---|
| | x | y | xy | | | |
| 1 | 0.951 | 0.326 | 0.310 | 0.951 | 1.277 | 1.006 |
| 2 | 0.466 | -0.597 | -0.278 | 0.597 | 1.063 | 0.757 |
| 3 | 0.124 | -0.614 | -0.076 | 0.614 | 0.738 | 0.627 |
| 4 | 1.204 | 0.112 | 0.135 | 1.204 | 1.316 | 1.209 |
| 5 | 0.232 | 0.355 | 0.082 | 0.355 | 0.587 | 0.424 |
| 6 | -0.267 | 0.676 | -0.181 | 0.676 | 0.944 | 0.727 |
| 7 | -0.403 | 0.092 | -0.037 | 0.403 | 0.495 | 0.413 |
| 8 | 0.294 | 0.268 | 0.079 | 0.294 | 0.562 | 0.398 |
| 9 | 0.384 | -0.260 | -0.100 | 0.384 | 0.644 | 0.464 |
| 10 | 0.306 | -1.079 | -0.330 | 1.079 | 1.385 | 1.122 |
| 11 | -0.680 | 0.636 | -0.433 | 0.680 | 1.317 | 0.931 |
| 12 | -0.077 | 0.906 | -0.070 | 0.906 | 0.983 | 0.909 |
| 13 | -0.365 | -1.032 | 0.377 | 1.032 | 1.397 | 1.095 |
| 14 | -1.147 | 0.264 | -0.303 | 1.147 | 1.411 | 1.177 |
| 15 | 0.103 | 0.411 | 0.042 | 0.411 | 0.514 | 0.424 |
| 16 | -0.237 | -0.131 | 0.031 | 0.237 | 0.367 | 0.270 |
| 17 | -0.354 | -0.038 | 0.013 | 0.354 | 0.391 | 0.356 |
| 18 | 0.384 | 0.659 | 0.253 | 0.659 | 1.043 | 0.763 |
| 19 | 0.118 | 1.026 | 0.121 | 1.026 | 1.143 | 1.032 |
| 20 | -1.485 | 0.071 | -0.105 | 1.485 | 1.556 | 1.487 |
| **Stat** | **-0.023** | **0.103** | **-0.023** | 1.485 | 1.556 | 1.487 |
| **Var** | **0.383** | **0.329** | **0.044** | | | |
| **Cov** | | | **-0.021** | | | |
| **Área** | | | | **8.820** | **4.840** | **6.943** |

| Sample number | CÍRCULO | | | σ | δ | χ |
|---|---|---|---|---|---|---|
| | x | y | xy | | | |
| 1 | 0.153 | 1.044 | 0.160 | 1.044 | 1.197 | 1.055 |
| 2 | 0.914 | 0.071 | 0.065 | 0.914 | 0.985 | 0.917 |
| 3 | -0.664 | 0.068 | -0.045 | 0.664 | 0.733 | 0.668 |
| 4 | -1.040 | -0.214 | 0.223 | 1.040 | 1.254 | 1.061 |
| 5 | 0.001 | -0.505 | 0.000 | 0.505 | 0.505 | 0.505 |
| 6 | 0.792 | 0.531 | 0.420 | 0.792 | 1.322 | 0.953 |
| 7 | 1.183 | 0.056 | 0.066 | 1.183 | 1.239 | 1.184 |
| 8 | 0.399 | -0.942 | -0.376 | 0.942 | 1.341 | 1.023 |
| 9 | 0.180 | 0.524 | 0.095 | 0.524 | 0.705 | 0.554 |
| 10 | -0.546 | -0.588 | 0.321 | 0.588 | 1.133 | 0.802 |
| 11 | 0.124 | -0.269 | -0.033 | 0.269 | 0.392 | 0.296 |
| 12 | 0.915 | 0.108 | 0.099 | 0.915 | 1.023 | 0.921 |
| 13 | 0.088 | -1.289 | -0.114 | 1.289 | 1.377 | 1.292 |
| 14 | -0.068 | -1.032 | 0.071 | 1.032 | 1.101 | 1.034 |
| 15 | 0.494 | 0.317 | 0.157 | 0.494 | 0.811 | 0.587 |
| 16 | -0.786 | -0.139 | 0.109 | 0.786 | 0.925 | 0.798 |
| 17 | 0.978 | -0.113 | -0.110 | 0.978 | 1.091 | 0.984 |
| 18 | -0.870 | 0.190 | -0.165 | 0.870 | 1.060 | 0.891 |
| 19 | -1.123 | -1.123 | 1.261 | 1.123 | 2.246 | 1.588 |
| 20 | 1.458 | 0.105 | 0.153 | 1.458 | 1.563 | 1.462 |
| **Stat** | **0.129** | **-0.160** | **0.118** | 1.458 | 2.246 | 1.588 |
| **Var** | **0.569** | **0.347** | **0.098** | | | |
| **Cov** | | | **0.138** | | | |
| **Área** | | | | **8.508** | **10.090** | **7.925** |

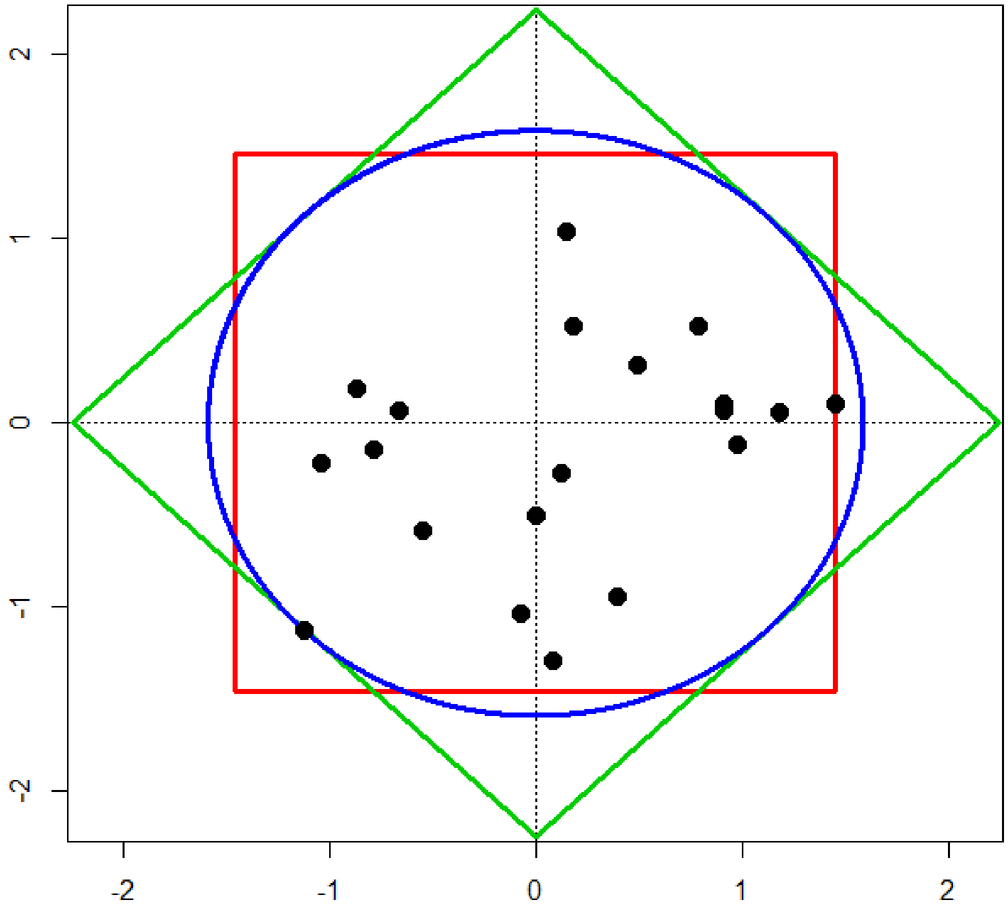
# Square



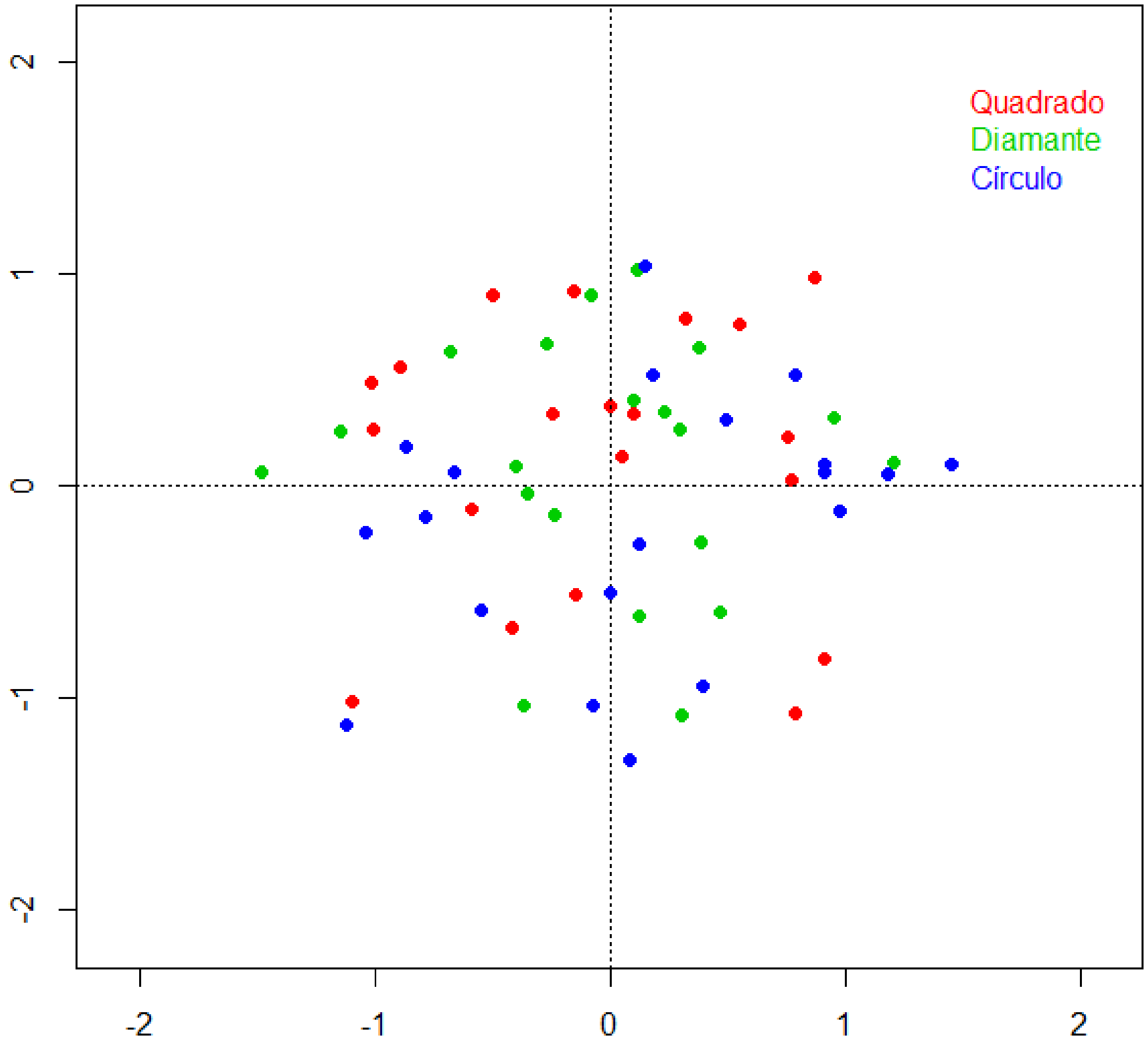
# Diamond



# Circle



## Samples



## Bayesian Operation: Prior to posterior.
## Priors, likelihoods and Posteriors

- ***Sufficient Statistics, LIKELIHOODS, and Priors***
  - $c = Max\sqrt{x_i^2 + y_i^2}\ \Delta\ d = Max(|x_i| + |y_i|)$

    $s = Max\big(Max(|x_i|; |y_i|)\big)$
    - **Square:** $L(\sigma|s) \propto \left(4\sigma^2\right)^{-n}\ for\ s \leq \sigma$
    - **Diamond:** $L(\delta|d) \propto \left(2\delta^2\right)^{-n}\ for\ d \leq \delta$
    - **Circle:** $L(\chi|c) \propto \left(\pi\theta^2\right)^{-n}\ for\ c \leq \theta$
- **Priors:** $g(\sigma) \propto \left(\sigma^2\right)^{-1}; g(\delta) \propto \left(\delta^2\right)^{-1}\ \&\ g(\theta) \propto \left(\theta^2\right)^{-1}$
  - **Posteriors are Pareto Distributions with parameters:**
    - **Shape =** $2n + 1$ **and Scales=**$\sigma, \delta, \&\ \theta$,

**respectively to square, diamond and Circle.**

# Pareto densities

$$f(\sigma|S=s)=\frac{(2n+1)s^{(2n+1)}}{\sigma^{(2n+2)}};\sigma\geq s$$

$$f(\delta|D=d)=\frac{(2n+1)d^{(2n+1)}}{\delta^{(2n+2)}}\ ;\ \delta\geq d$$

$$f(\boldsymbol{\theta}|C=c)=\frac{(2n+1)c^{(2n+1)}}{\boldsymbol{\theta}^{(2n+2)}}\ ;\ \ \theta\geq c.$$

$$\left(\boldsymbol{mo_C=c};\quad \boldsymbol{m_c}=\left(\boldsymbol{1}+\frac{\boldsymbol{1}}{\boldsymbol{2n}}\right)\boldsymbol{r};\quad \boldsymbol{me_c}=\left(\sqrt[\boldsymbol{2n+1}]{\boldsymbol{2}}\right)\boldsymbol{c}\right);$$

$$\left(\boldsymbol{mo_S=s};\quad \boldsymbol{m_S}=\left(\boldsymbol{1}+\frac{\boldsymbol{1}}{\boldsymbol{2n}}\right)\boldsymbol{s};\ \ \boldsymbol{me_s}=\left(\sqrt[\boldsymbol{2n+1}]{\boldsymbol{2}}\right)\boldsymbol{s}\right);$$

$$\left(\boldsymbol{mo_D=d};\ \ \boldsymbol{m_D}=\left(\boldsymbol{1}+\frac{\boldsymbol{1}}{\boldsymbol{2n}}\right)\boldsymbol{d};\quad \boldsymbol{me_D}=\left(\sqrt[\boldsymbol{2n+1}]{\boldsymbol{2}}\right)\boldsymbol{d}\right).$$

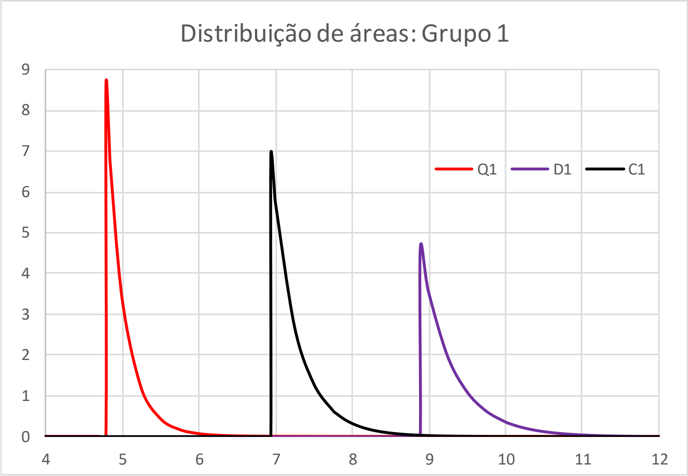
# Area densities

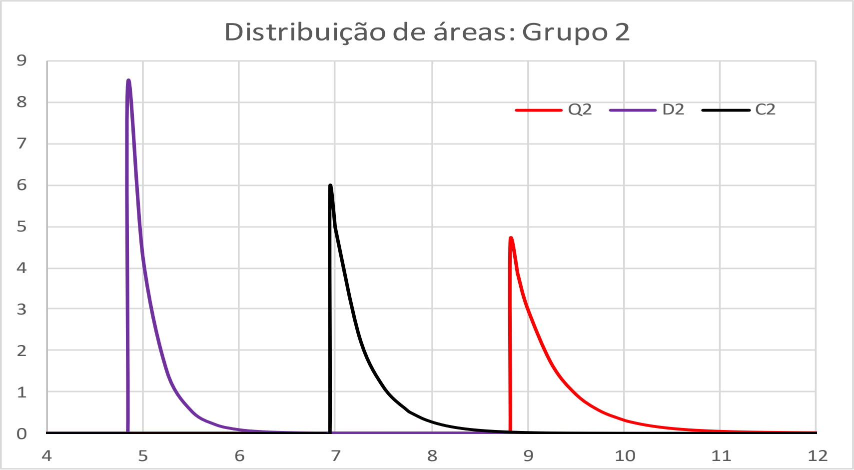
$$f(\boldsymbol{Q}|S=s)=\frac{(2n+1)4^{(n+.5)}s^{(2n+1)}}{\boldsymbol{Q}^{(n+1.5)}};\boldsymbol{Q}\geq 4s^2$$

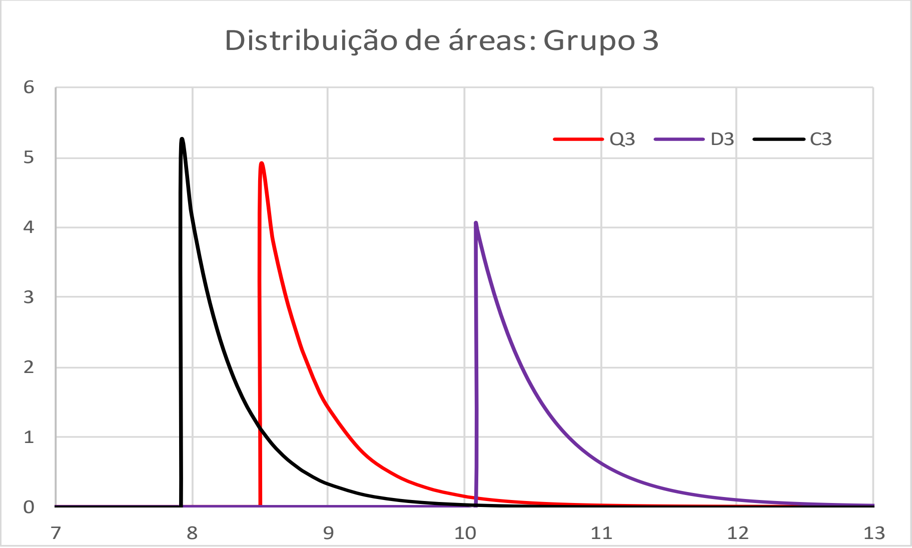
$$f(\boldsymbol{L}|D=d)=\frac{(2n+1)2^{(n+.5)}d^{(2n+1)}}{2\boldsymbol{L}^{(n+1.5)}};\boldsymbol{L}\geq 2d^2$$

$$f(\boldsymbol{A}|C=c)=\frac{(2n+1)\pi^{(n+.5)}c^{(2n+1)}}{2\boldsymbol{A}^{(n+1)}};\ \ \boldsymbol{A}\geq \pi c^2.$$

# Gráficos das densidades das áreas



# Esclerose Múltipla 1

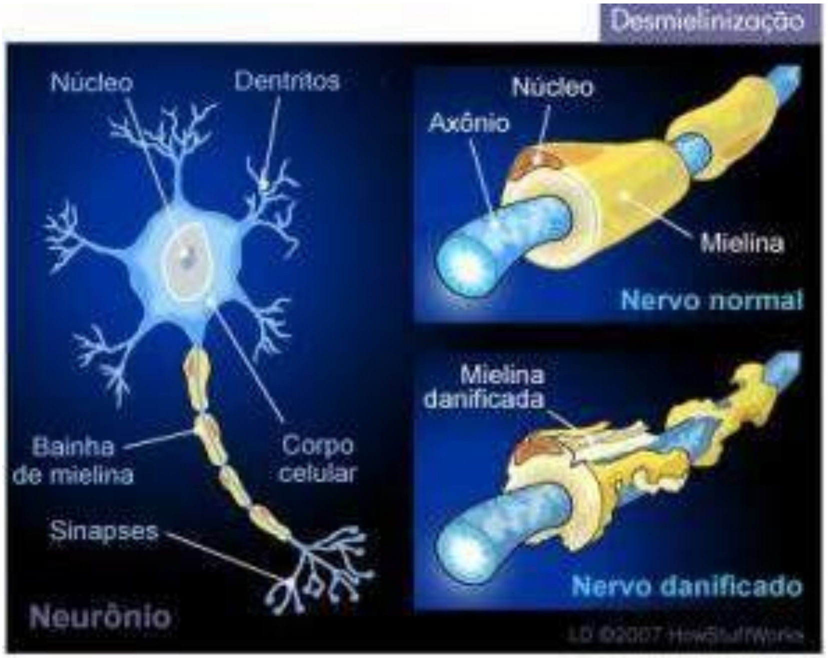
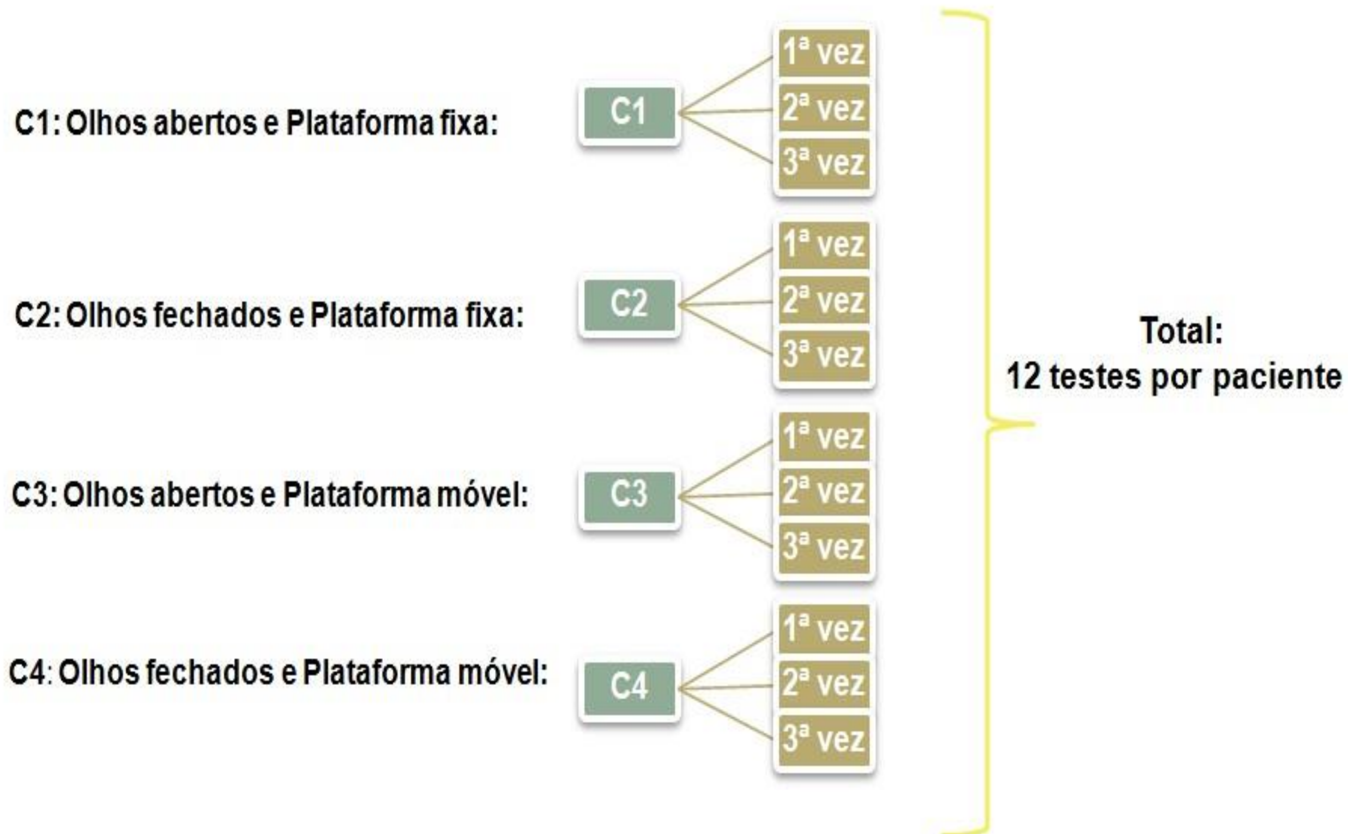



**Figura 1** - Processo de desmielinização

**Figura 2** – Plataforma utilizada nos testes de PDC

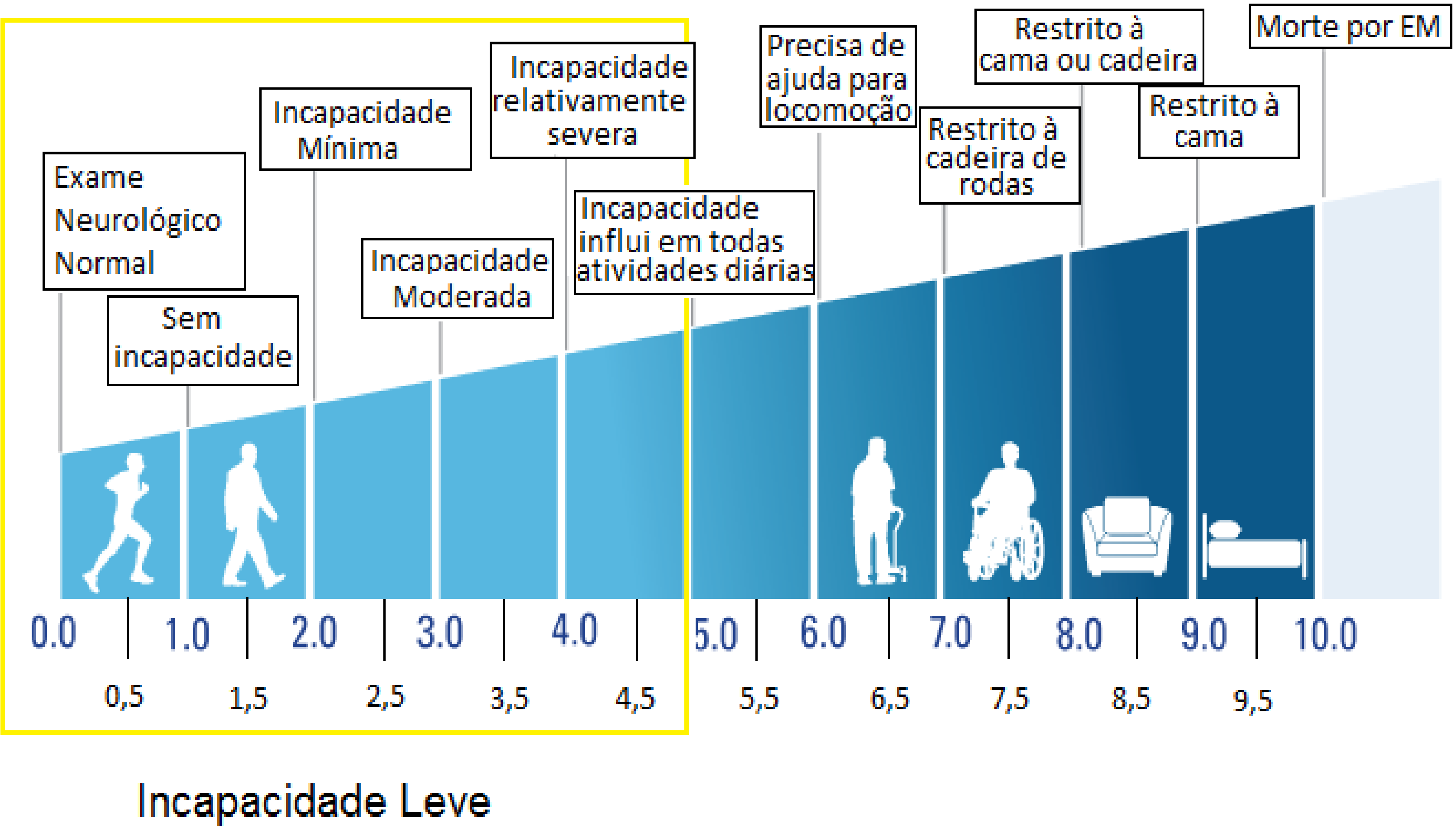
# Esclerose Múltipla 2

**Figura 3** - Condições aplicadas no exame



# Esclerose Múltipla 3:

**Figura 4** – Classificação de incapacidade dos pacientes através da EDSS



# Esclerose Múltipla 4

**Figura 5** - Construção do *convex hull* para uma trajetória

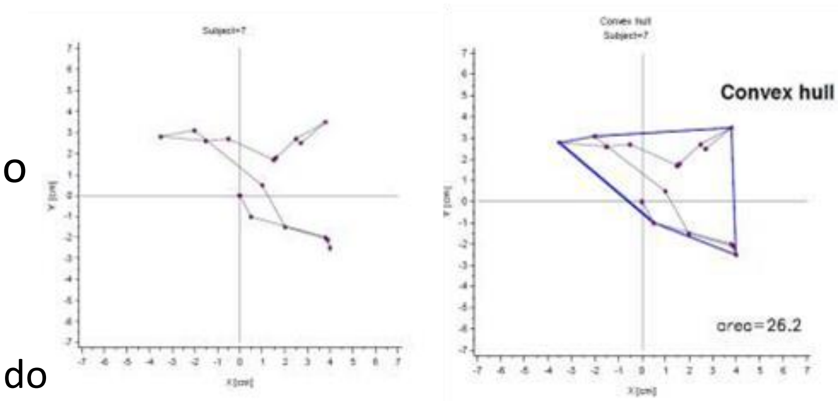
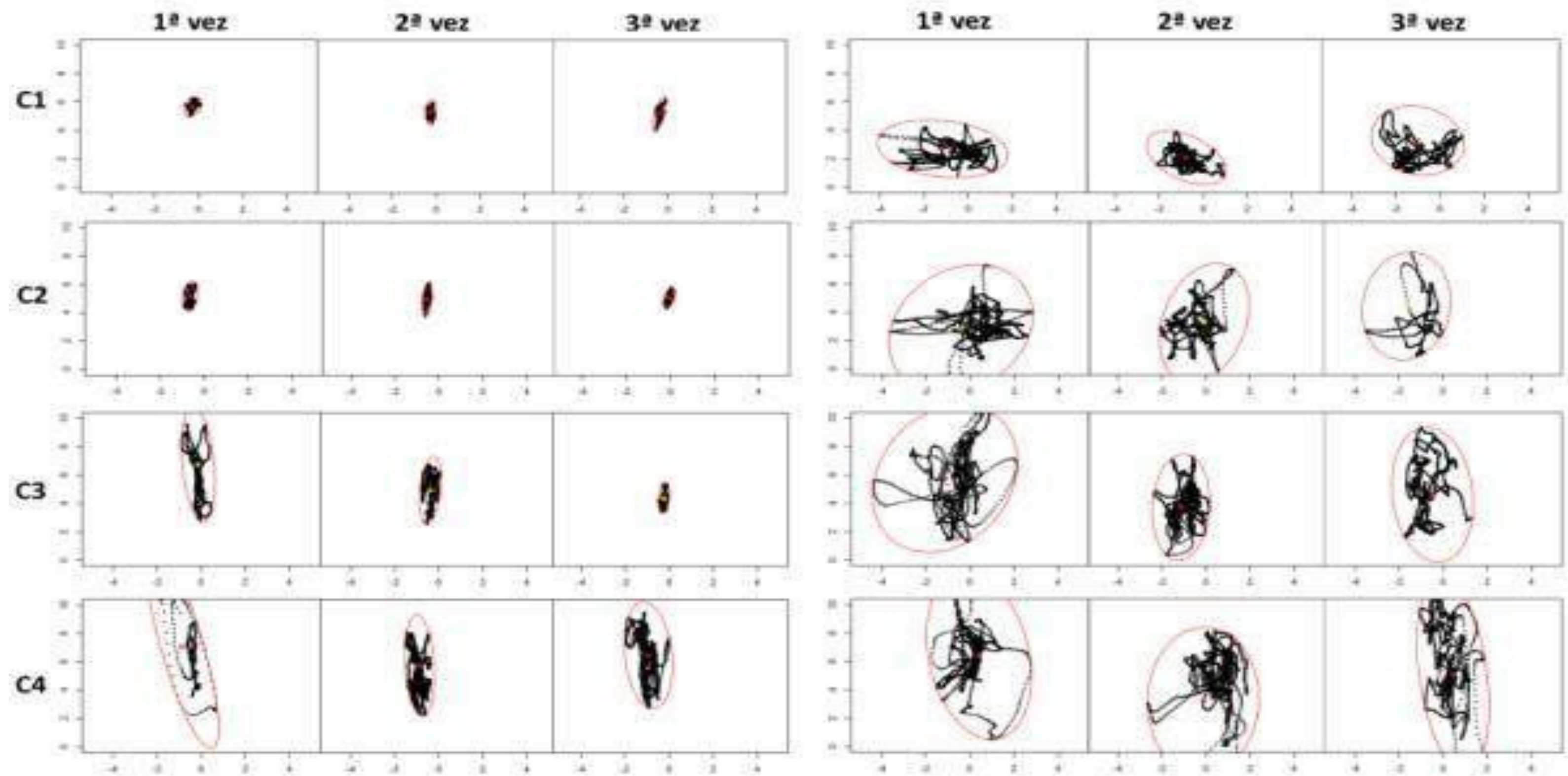


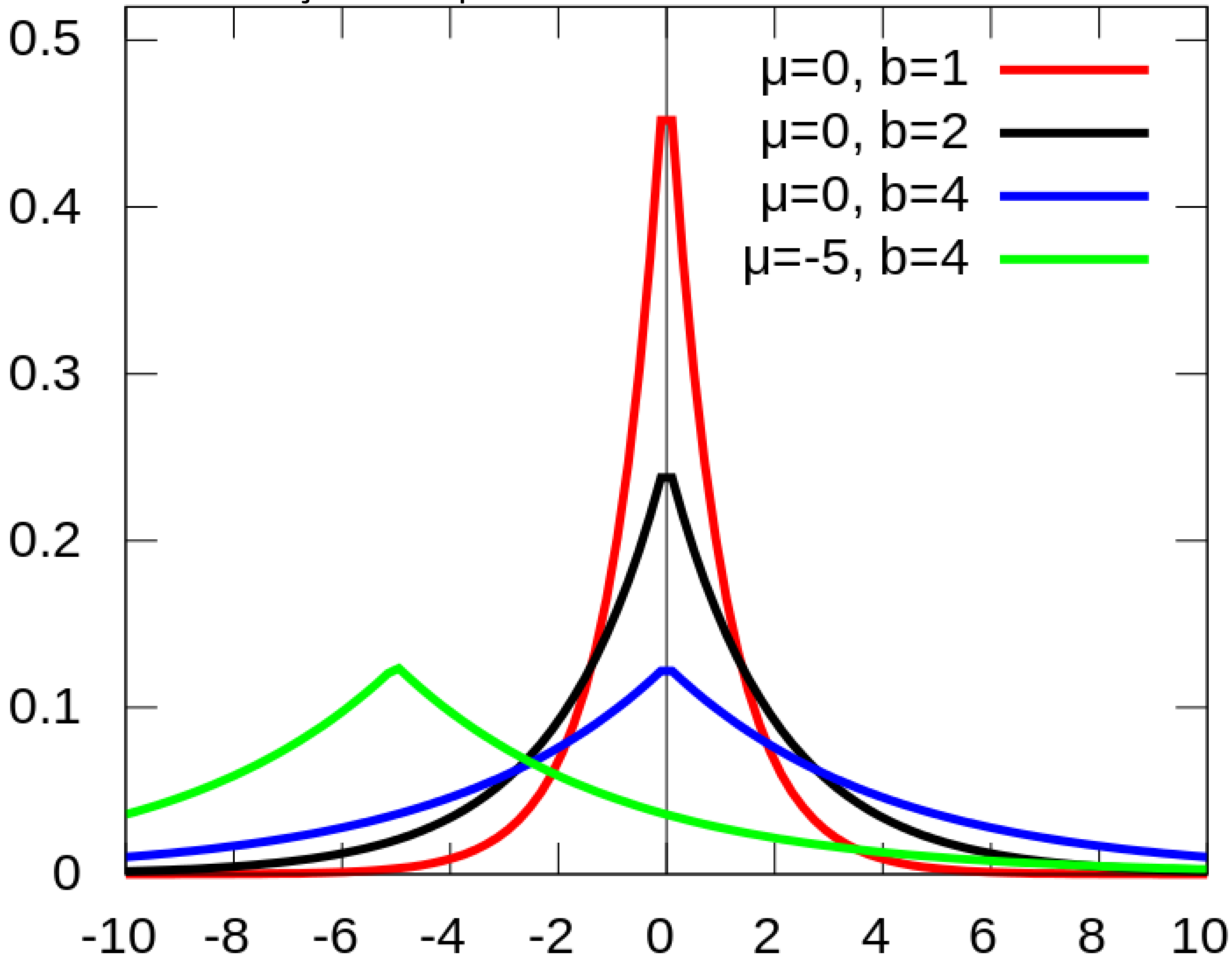

**Figura 7** - Exame posturo-gráfico de um paciente do GSem e um paciente do GQueixa utilizando a elipse.



# Nosso futuro neste problema: Sugestão

- **Temos aqui um plano com os pontos de maior peso quando a máquina está trabalhando.**
- **Para a análise estatística consideramos a área das menores elipses ajustadas. Os interessados aprovaram nossas análises.**
- **Para um análise mais interessante gostaríamos de procurar pelo melhor modelo que pudesse se aproximar da geração dos dados.**
- **Mostraremos a seguir como uma composição de distribuições de Laplace poderia nos ajudar a ter uma boa análise estatística**

Distribuição de Laplace



**Densidade com média zero e parâmetro $\beta$**

$$f(x|\beta) = \frac{1}{2\beta} exp\left(-\frac{|x|}{\beta}\right) = \begin{cases} \frac{1}{2\beta} exp\left(\frac{x}{\beta}\right); \quad x \leq 0 \\ \frac{1}{2\beta} exp\left(-\frac{x}{\beta}\right); x > 0 \end{cases}$$

**Nossa nova proposta é tentar definir para 3 parâmetros**

$$\boldsymbol{f(x|\beta, \rho, \pi) = \begin{cases} \frac{\pi}{\beta} exp\left(\frac{x}{\beta}\right); \quad x \leq 0 \\ 0 \leq \pi \leq 1 \\ \frac{1-\pi}{\rho} exp - \left(\frac{x}{\rho}\right); x > 0 \end{cases}}$$

## Cobrindo o plano

**Consideremos 3 variáveis aleatórias independentes e com distribuição de Laplace Padrão (poderão ser a modificada)**

***X, Y e Z***

Suponhamos que

$$X|\alpha \approx Lap(0;\alpha);\ \ Y|\beta \approx Lap(0;\beta)\ ;$$

$$\&$$

$$Z \approx Lap(0;\delta)$$

# Cobrindo o plano

**Estas darão origem as variáveis de geração dos modelos**

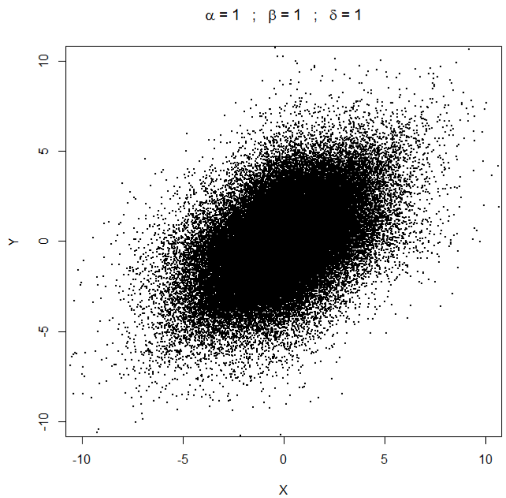
**Modelo 1:** $U = X + Z$ e $V = Y + Z$ ***quando Cov(U,V)>0***

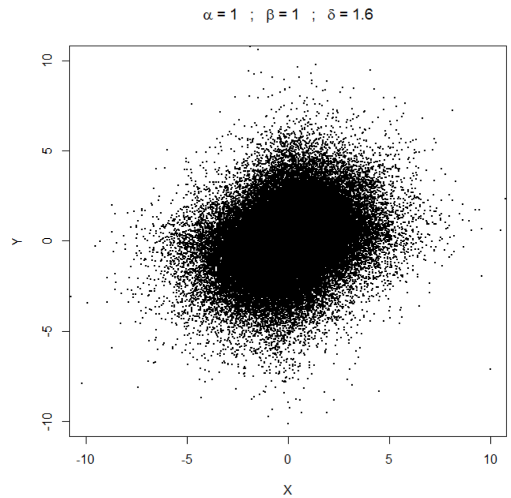
**Modelo 2:** $U = \frac{X+Y+2Z}{\sqrt{2}}$ e $V = \frac{X-Y}{\sqrt{2}}$ ***quando Var(U)>Var(V)***

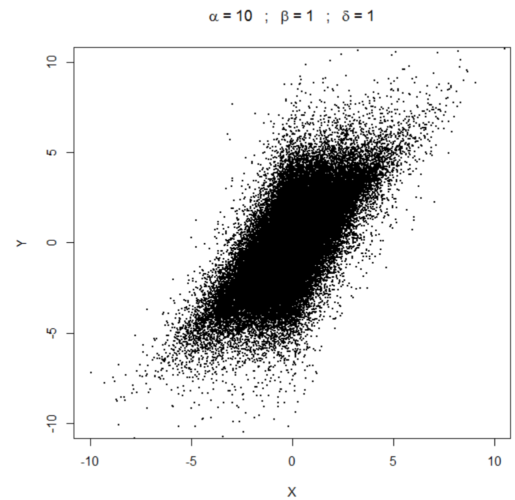
**Modelo 3:** $U = X + \mathrm{Z}$ e $V = Y - Z$ ***quando Cov(U,V)<0***

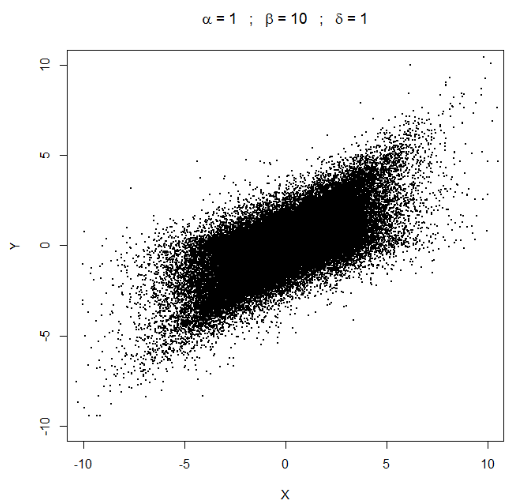
**Modelo 4:** $U = \frac{X+Y}{\sqrt{2}}$ e $Y = \frac{X-Y+2z}{\sqrt{2}}$ ***quando Var(U)<Var(V)***

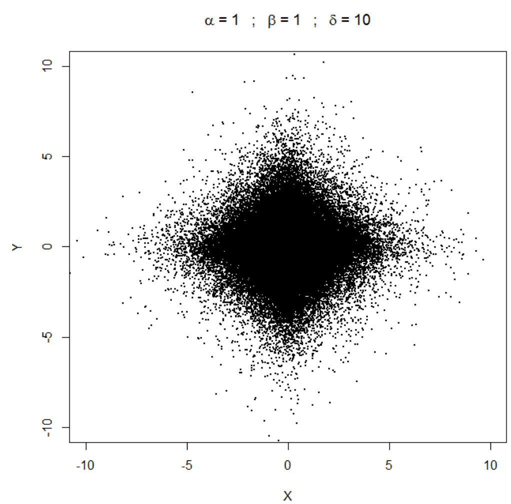
# Modelo 1 (Cov positiva)

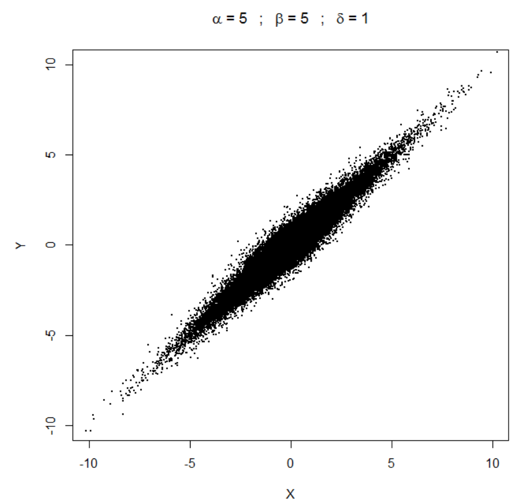


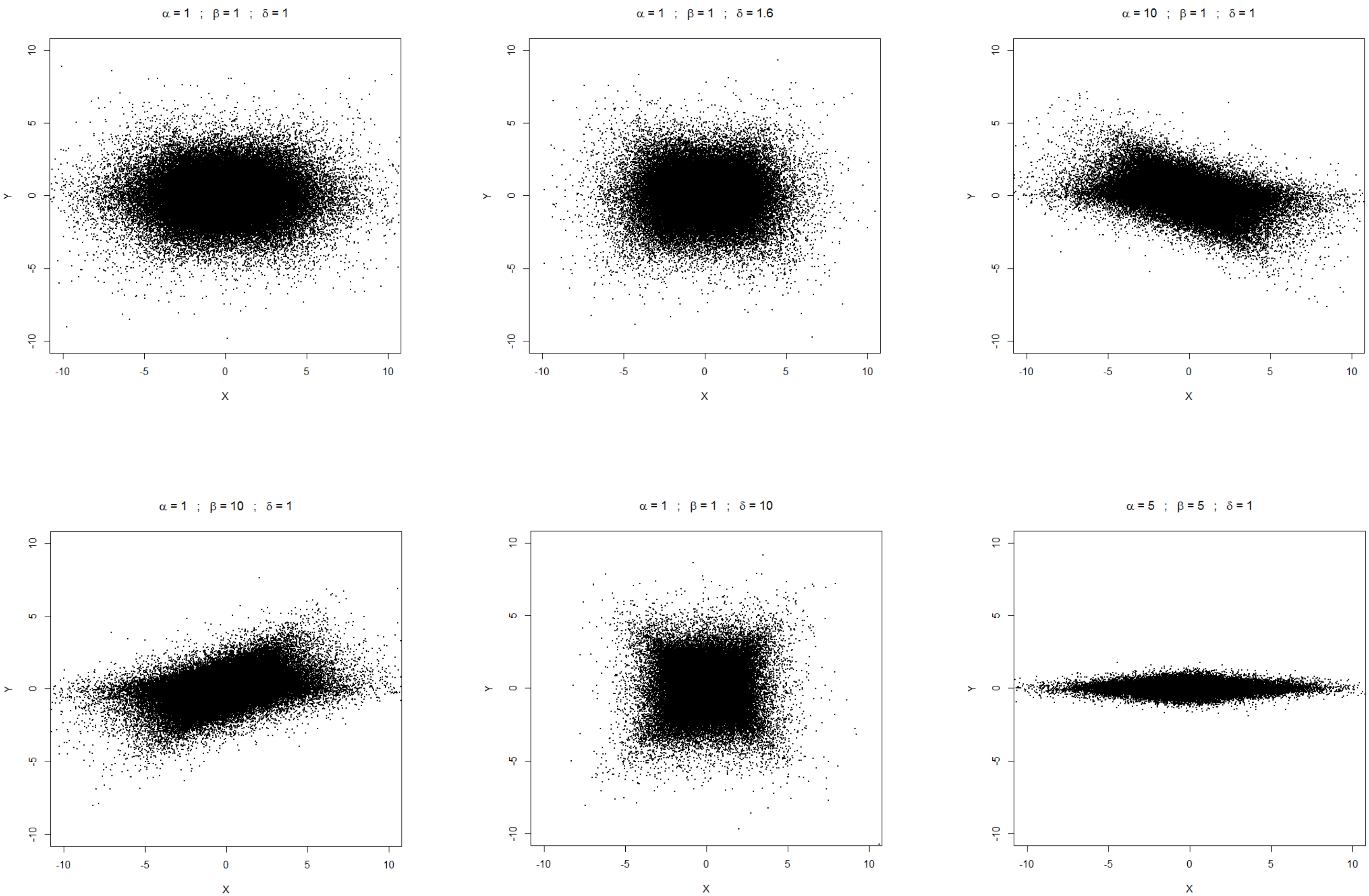



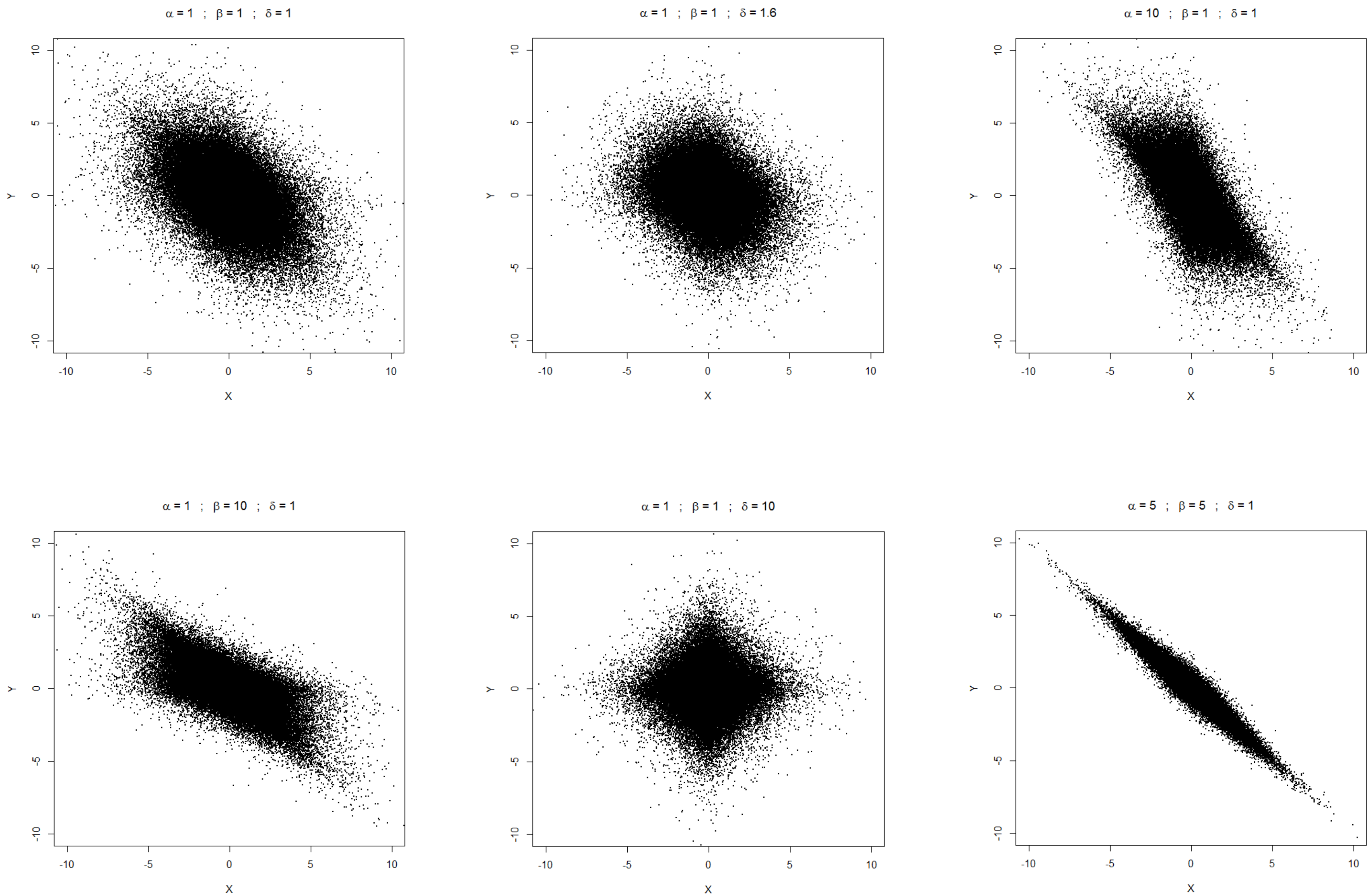



# Modelo 2
(rotação 45º no Modelo 1)

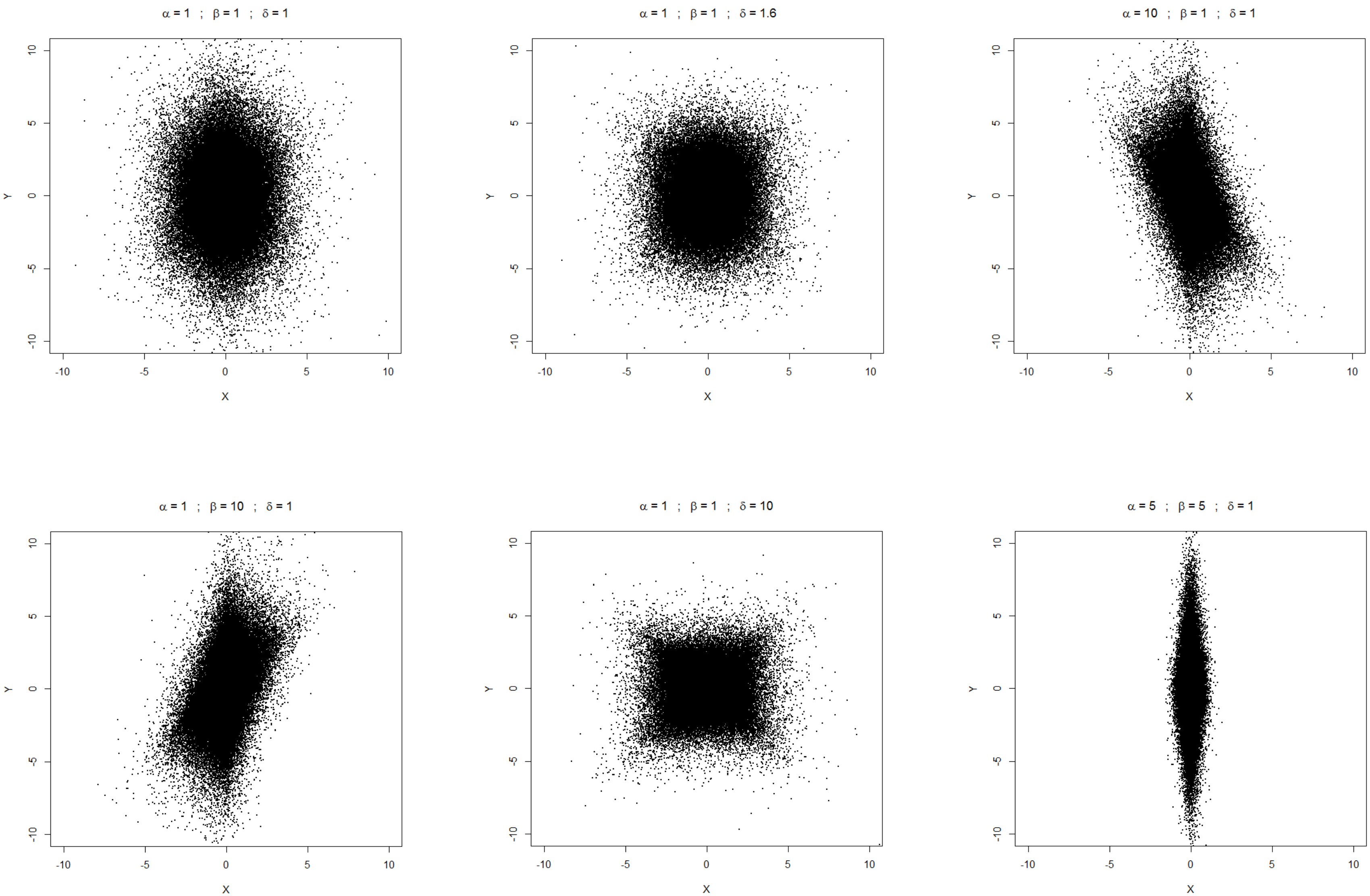


# Modelo 3 (Cov negativa)



# Modelo 4

(rotação 45º no Modelo 3)



Laplace Padrão simétrica em torno de zero

Moda, Média e Mediana = 0; Variância = $2\beta^2$

## Estimativas dos parâmetros dos modelos

**Modelo 1:** $\boldsymbol{U} = x + z$ e $\boldsymbol{V} = y + z$

$$Var(U) = Var(X) + Var(Z) = 2\alpha^2 + Cov(U,V) \Rightarrow$$

$$\hat{\alpha} = \sqrt{\frac{Var(U) - Cov(U,V)}{2}}$$

$$Var(U) = Var(Y+Z) = Var(Y) + Var(Z) = 2\beta^2 + Cov(U,V) \Rightarrow$$

$$\hat{\beta} = \sqrt{\frac{Var(U) - Cov(U,V)}{2}} =$$

$$Cov(U,V) = Cov(X,Y) + Cov(X,Z) + Cov(Y,Z) + Var(Z) = 2\delta^2 \Rightarrow$$

$$\hat{\delta} = \sqrt{\frac{Cov(U,V)}{2}}$$

**Restrição:** $Cov(\boldsymbol{U},\boldsymbol{V}) > 0$

Laplace Padrão simétrica em torno de zero

Moda, Média e Mediana = 0; Variância = $2\beta^2$

## Estimativas dos parâmetros dos modelos

**Modelo 2:** $U = \frac{X+Y+2Z}{\sqrt{2}}$ e $V = \frac{X-Y}{\sqrt{2}}$

$$Var(U) = \tfrac{1}{2}[Var(X+Y+2Z) = \frac{Var(X)+Var(Y)+4Var(Z)}{2} = \alpha^2 + \beta^2 + 4\delta^2$$

$$Var(V) = \tfrac{1}{2}Var(X-Y) = \frac{Var(X)+Var(Y)}{2} = \alpha^2 + \beta^2$$

$$Cov(U,V) = \frac{Cov(X+Y+2Z;X-Y)}{2} = \frac{Var(X)-Var(Y)}{2} = \alpha^2 - \beta^2$$

$$\hat{\alpha} = \sqrt{\frac{Var(V)+Cov(U,V)}{2}} \qquad \hat{\beta} = \sqrt{\frac{Var(V)-Cov(U;V)}{2}} \qquad \hat{\delta} = \frac{\sqrt{Var(U)-Var(V)}}{2}$$

**Restrição:** $Cov(U+V,U-V) > 0 \quad \rightarrow \quad Var(U) > Var(V)$

Laplace Padrão simétrica em torno de zero

Moda, Média e Mediana = 0; Variância = $2\beta^2$

## Estimativas dos parâmetros dos modelos

**Modelo 3:** $U = X + Z$ e $V = Y - Z$

$$Var(U) = Var(X) + Var(Z) = 2\alpha^2 + Cov(U,V) \Rightarrow$$

$$\alpha^2 = \sqrt{\frac{Var(U)+Cov(U,V)}{2}}$$

$$Var(V) = Var(Y-Z) = Var(Y) + Var(Z) = 2\beta^2 + Cov(U,V) \Rightarrow$$

$$\hat{\beta} = \sqrt{\frac{Var(V)+Cov(U,V)}{2}}$$

$$Cov(U,V) = Cov(X,Y) - Cov(X,Z) + Cov(Y,Z) - Var(Z) = -Var(Z) = -2\delta^2$$

$$\hat{\delta} = \sqrt{-\frac{Cov(U,V)}{2}}$$

**Restrição:** $Cov(U,V) < 0$

Laplace Padrão simétrica em torno de zero

Moda, Média e Mediana = 0; Variância = $2\beta^2$

## Estimativas dos parâmetros dos modelos

**Modelo 4:** $U = \frac{X+Y}{\sqrt{2}}$ e $V = \frac{X-Y+2Z}{\sqrt{2}}$

$$Var(U) = \frac{Var(X+Y)}{2} = \frac{Var(X)+Var(Y)}{2} = \alpha^2 + \beta^2$$

$$Var(V) = \frac{Var(X-Y+2Z)}{2} = \frac{Var(X)+Var(Y)+4Var(Z)}{2} = \alpha^2 + \beta^2 + 4\delta^2$$

$$Cov(U,V) = \frac{Cov(X+Y;X-Y+2Z)}{2} = \frac{Var(X)-Var(Y)}{2} = \alpha^2 - \beta^2$$

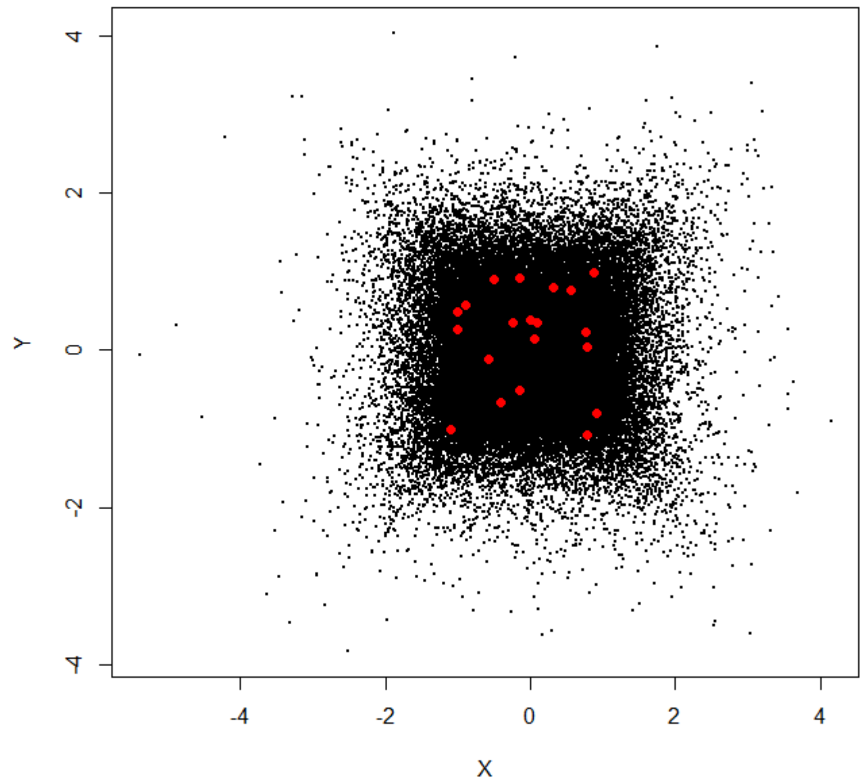
$$\hat{\alpha} = \sqrt{\frac{Var(U)+Cov(U;V)}{2}} \qquad \hat{\beta} = \sqrt{\frac{Var(U)-Cov(U;Z)}{2}} \qquad \hat{\delta} = \frac{\sqrt{Var(V)-Var(U)}}{2}$$

**Restrição:** $Cov(U+V,U-V) < 0 \rightarrow Var(U) < Var(V)$

# Exemplo do QUADRADO – Modelo 2 ( var[U]>var[V])

| Sample number | QUADRADO | | |
|---|---|---|---|
| | x | y | xy |
| 1 | -0,500 | 0,903 | -0,452 |
| 2 | 0,554 | 0,765 | 0,424 |
| 3 | -0,244 | 0,347 | -0,085 |
| 4 | 0,754 | 0,236 | 0,178 |
| 5 | 0,875 | 0,988 | 0,864 |
| 6 | -1,009 | 0,268 | -0,271 |
| 7 | 0,007 | 0,381 | 0,002 |
| 8 | 0,774 | 0,031 | 0,024 |
| 9 | 0,056 | 0,138 | 0,008 |
| 10 | -0,144 | -0,510 | 0,073 |
| 11 | 0,909 | -0,812 | -0,738 |
| 12 | -0,151 | 0,921 | -0,139 |
| 13 | 0,321 | 0,792 | 0,255 |
| 14 | 0,100 | 0,343 | 0,034 |
| 15 | -0,889 | 0,565 | -0,502 |
| 16 | 0,789 | -1,072 | -0,846 |
| 17 | -0,587 | -0,110 | 0,064 |
| 18 | -1,017 | 0,487 | -0,495 |
| 19 | -0,415 | -0,665 | 0,276 |
| 20 | -1,094 | -1,015 | 1,110 |
| **Stat** | **-0,046** | **0,149** | **-0,011** |
| **Var** | **0,434** | **0,403** | **0,220** |
| **Cov** | | | **-0,004** |
| **Área** | | | |

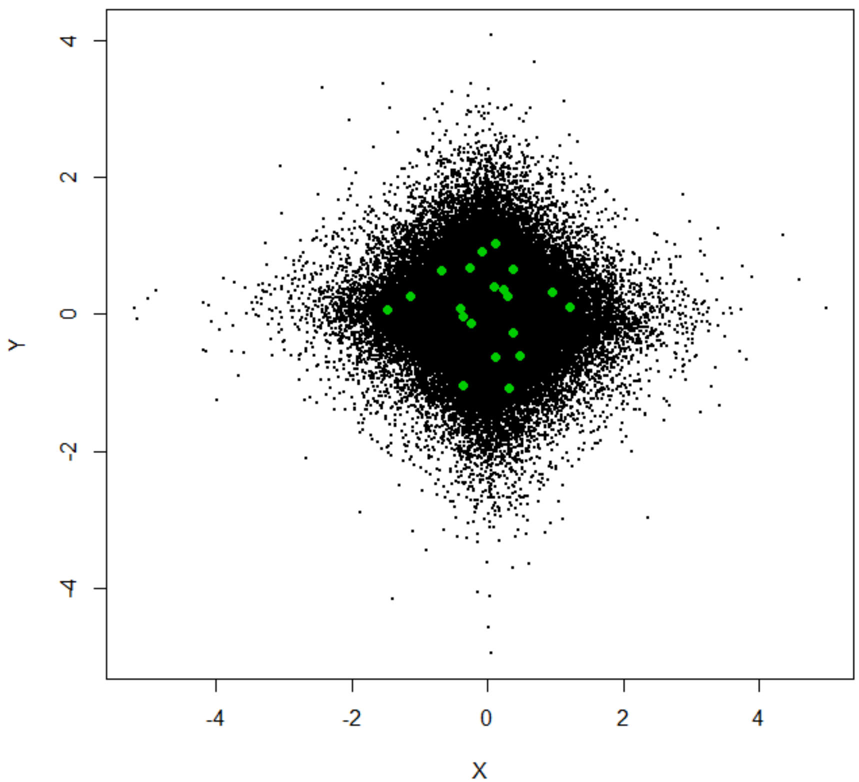
$$\hat{\alpha} = \sqrt{\frac{Var(V)+Cov(U,V)}{2}} = 0{,}4637 \qquad \hat{\beta} = \sqrt{\frac{Var(V)-Cov(U,V)}{2}} = 0{,}4511 \qquad \hat{\delta} = \frac{\sqrt{Var(U)-Var(V)}}{2} = 0{,}0880$$



# Exemplo do DIAMANTE – Modelo 3 ( Cov[U,V]<0 )

| Sample number | DIAMANTE | | |
|---|---|---|---|
| | x | y | xy |
| 1 | 0,951 | 0,326 | 0,310 |
| 2 | 0,466 | -0,597 | -0,278 |
| 3 | 0,124 | -0,614 | -0,076 |
| 4 | 1,204 | 0,112 | 0,135 |
| 5 | 0,232 | 0,355 | 0,082 |
| 6 | -0,267 | 0,676 | -0,181 |
| 7 | -0,403 | 0,092 | -0,037 |
| 8 | 0,294 | 0,268 | 0,079 |
| 9 | 0,384 | -0,260 | -0,100 |
| 10 | 0,306 | -1,079 | -0,330 |
| 11 | -0,680 | 0,636 | -0,433 |
| 12 | -0,077 | 0,906 | -0,070 |
| 13 | -0,365 | -1,032 | 0,377 |
| 14 | -1,147 | 0,264 | -0,303 |
| 15 | 0,103 | 0,411 | 0,042 |
| 16 | -0,237 | -0,131 | 0,031 |
| 17 | -0,354 | -0,038 | 0,013 |
| 18 | 0,384 | 0,659 | 0,253 |
| 19 | 0,118 | 1,026 | 0,121 |
| 20 | -1,485 | 0,071 | -0,105 |
| **Stat** | **-0,023** | **0,103** | **-0,023** |
| **Var** | **0,383** | **0,329** | **0,044** |
| **Cov** | | | **-0,021** |
| **Área** | | | |

$$\hat{\alpha} = \sqrt{\frac{Var(U)+Cov(U,V)}{2}} = 0{,}4254 \qquad \hat{\beta} = \sqrt{\frac{Var(V)+Cov(U,V)}{2}} = 0{,}3924 \qquad \hat{\delta} = \sqrt{-\frac{Cov(U,V)}{2}} = 0{,}1025$$



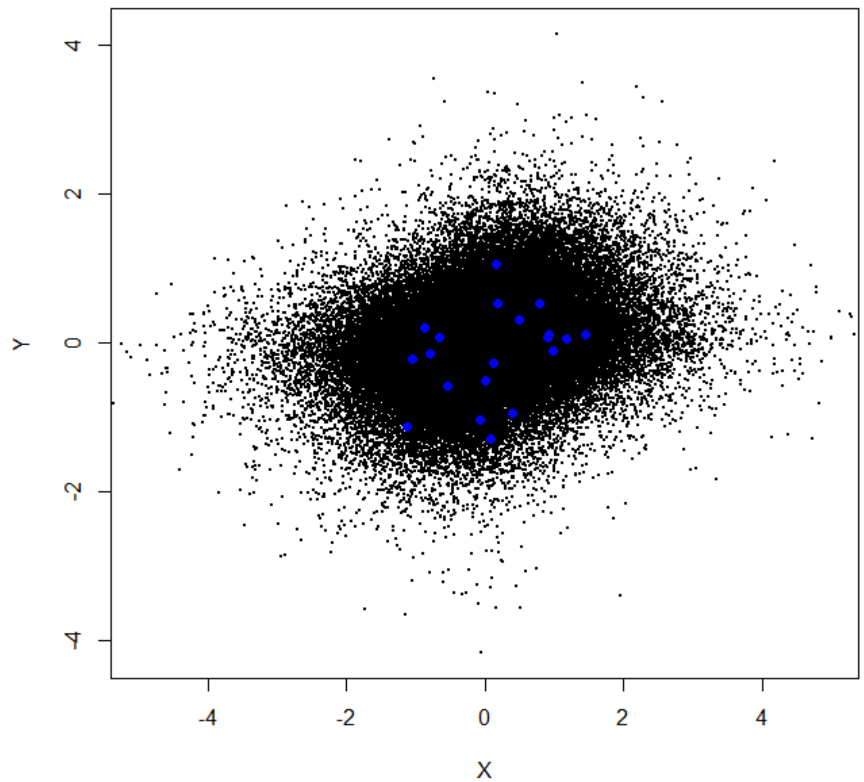
# Exemplo do CÍRCULO – Modelo 1 ( Cov[U,V]>0 )

| Sample number | CÍRCULO | | |
|---|---|---|---|
| | x | y | xy |
| 1 | 0,153 | 1,044 | 0,160 |
| 2 | 0,914 | 0,071 | 0,065 |
| 3 | -0,664 | 0,068 | -0,045 |
| 4 | -1,040 | -0,214 | 0,223 |
| 5 | 0,001 | -0,505 | 0,000 |
| 6 | 0,792 | 0,531 | 0,420 |
| 7 | 1,183 | 0,056 | 0,066 |
| 8 | 0,399 | -0,942 | -0,376 |
| 9 | 0,180 | 0,524 | 0,095 |
| 10 | -0,546 | -0,588 | 0,321 |
| 11 | 0,124 | -0,269 | -0,033 |
| 12 | 0,915 | 0,108 | 0,099 |
| 13 | 0,088 | -1,289 | -0,114 |
| 14 | -0,068 | -1,032 | 0,071 |
| 15 | 0,494 | 0,317 | 0,157 |
| 16 | -0,786 | -0,139 | 0,109 |
| 17 | 0,978 | -0,113 | -0,110 |
| 18 | -0,870 | 0,190 | -0,165 |
| 19 | -1,123 | -1,123 | 1,261 |
| 20 | 1,458 | 0,105 | 0,153 |
| **Stat** | **0,129** | **-0,160** | **0,118** |
| **Var** | **0,569** | **0,347** | **0,098** |
| **Cov** | | | **0,138** |
| **Área** | | | |

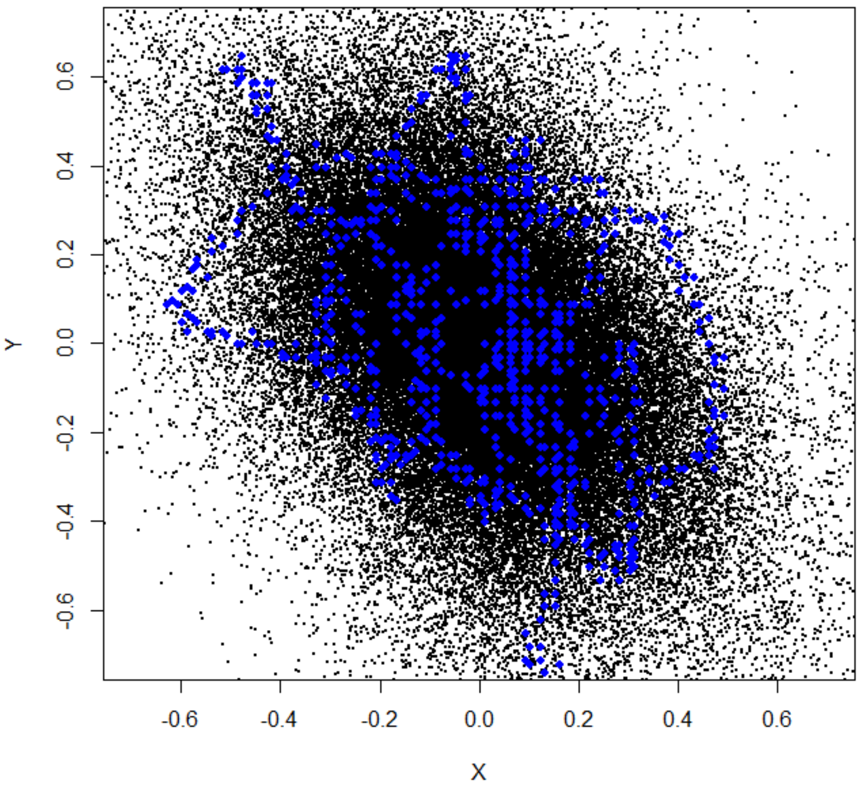
$$\hat{\alpha} = \sqrt{\frac{Var(U) - Cov(U,V)}{2}} = 0{,}4642$$

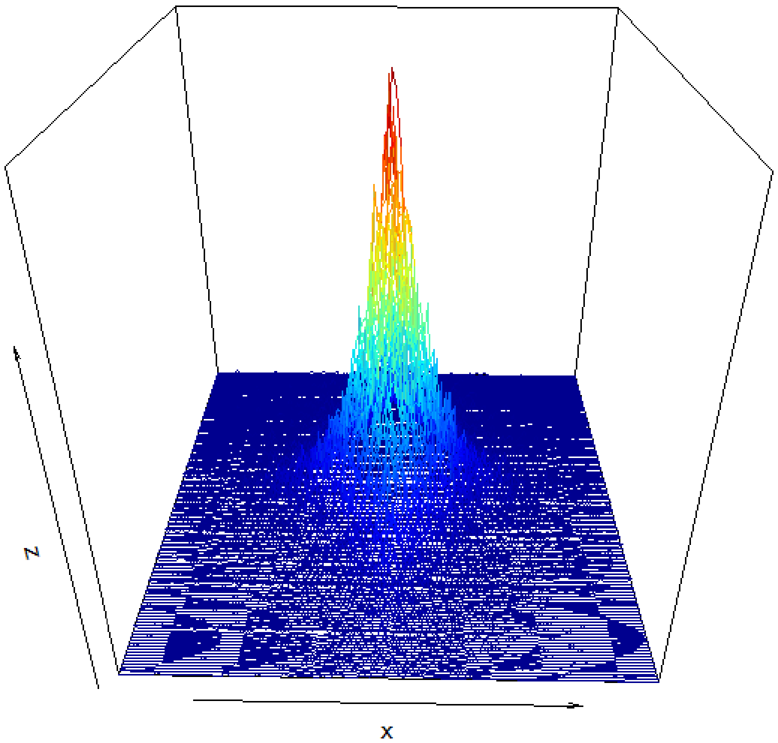
$$\hat{\beta} = \sqrt{\frac{Var(V) - Cov(U,V)}{2}} = 0{,}3233$$

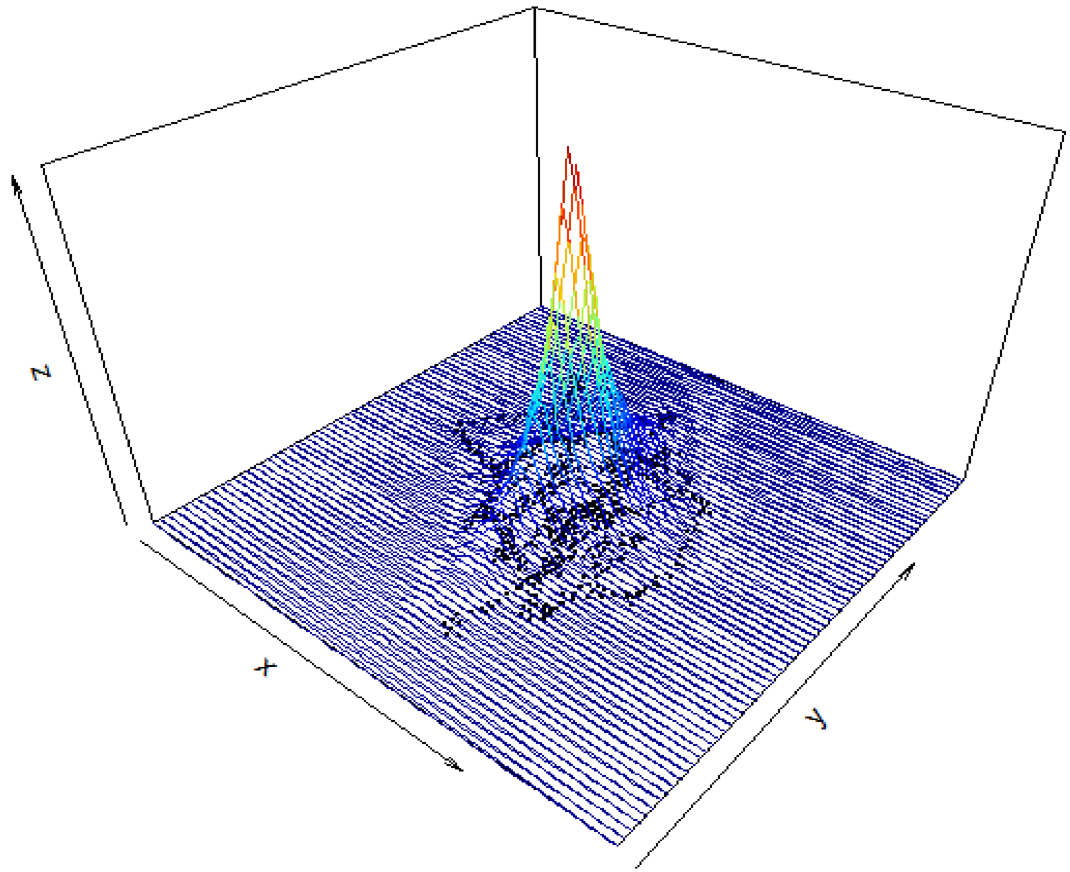
$$\hat{\delta} = \sqrt{\frac{Cov(X,Y)}{2}} = 0{,}2627$$



# Ajuste do modelo em dados de um paciente




z
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